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Design of a PD Controller Based on Eigenvalue Assignment for Active Magnetic Bearings-Rigid Rotor Systems

Tianhao Zhou[1], Changsheng Zhu[1], Wengheng Li[1]
[1]Zhejiang University
Zheda Road No.38, Zhejiang Province, China
zhouth_419@zju.edu.cn; zhu_zhang@zju.edu.cn; 3150103804@zju.edu.cn

Abstract **-** In order to acquire satisfactory dynamic performance of the active magnetic bearings (AMBs) -rigid rotor system, a design method for the proportional-differential (PD) feedback controller based on eigenvalue assignment is proposed. Firstly, the cross-differential feedback control scheme was introduced, and the four-degree-of-freedom (4-DOF) AMBs-rigid rotor system was transformed into two identical 2-DOF sub-systems. Then, aiming at one 2-DOF sub-system, the numerical relation between the PD parameters and the closed-loop eigenvalues was obtained through eigenvalue assignment. Moreover, the appropriate eigenvalues of the closed-loop 2-DOF sub-system were given, and the related PD parameters were obtained. Finally, the designed PD controller based on eigenvalue assignment for the AMBs-rigid rotor system was verified to be effective in numerical simulations.

Keywords**:** Active magnetic bearing(AMB), rigid rotor, eigenvalue assignment, PD control

1. Introduction

AMBs completely eliminate the mechanical friction between the bearings and the rotor, which dramatically expand the lifespans of rotating machineries. Due to the absence of the mechanical contacts, various rotating machines supported by the AMBs can operate at a very high rotational speed. Nowadays, AMBs have been widely used in various rotating machineries, such as high-speed motors, air compressors, flywheel energy systems, etc [1].

The AMBs-rotor system is inherently unstable, and the design of its controller is necessary and vital to the normal operation of the system. Many control methods, such as sliding-mode control [2], robust H_∞ control [3], fuzzy control [4], LQG control [5], and so on, have been applied to the AMBs-rotor system to obtain good stability and dynamical performance.

In this paper, a simple design method based on eigenvalue assignment for the PD controller is proposed. Firstly, in order to simplify the design process of the PD controller, the 4-DOF AMBs-rigid rotor system is transformed into two identical 2-DOF sub-systems by applying cross-differential feedback. Then, aiming at one 2-DOF sub-system, the numerical relation between the parameters of the PD controller and the closed-loop eigenvalues is derived through eigenvalue assignment. By setting proper eigenvalues, the PD controller with desire performance is obtained. The design method of the PD controller for the AMBs-rigid rotor system is verified to be feasible and effective in numerical simulations.

2. Model of the AMBs-rigid rotor system

The structure diagram of the AMBs-rigid rotor system is shown in Fig. 1.

In Fig. 1, the points *O* and *C* denote the geometric and mass center of the rotor, respectively. Since the unbalance mass is far smaller than the rotor mass, *O* and *C* are almost coincident. l_a, l_b, l_{sa}, and l_{sb} are the axial distances from the AMB-A/-B and the sensor-A/-B to *C*, respectively. f_{xa}, f_{xb}, f_{ya}, and f_{yb} are the electromagnetic forces provided by the AMB-A and -B in x- and y-directions, respectively. The rotor position can be described by the generalized coordinate vector of *C*, $\boldsymbol{q}_1=[\theta_y\ x\ \theta_x\ y]^T$, where θ_x and θ_y denote the deflection angles of the rotor around *x*- and *y*-axes, *x* and *y* denote the displacements of *C* in *x*- and *y*-directions, respectively.

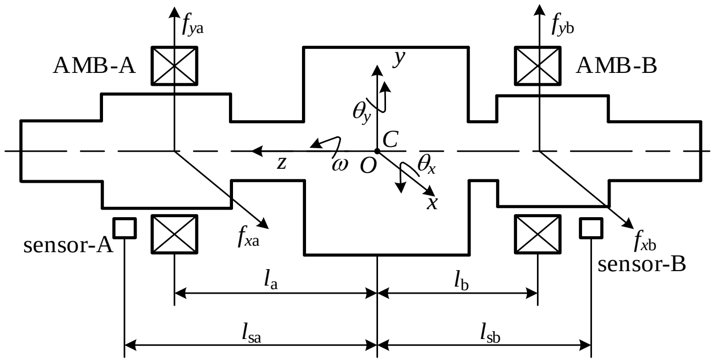


Fig. 1: The structure diagram of the AMBs-rigid rotor system.

According to rotor dynamic theory, the motion differential equation of the AMBs-rigid rotor system can be described as

$$\boldsymbol{M}_1\ddot{\boldsymbol{q}}_1+\boldsymbol{G}_1\dot{\boldsymbol{q}}_1=\boldsymbol{F}_{1\mathrm{AMB}}+\boldsymbol{F}_{1\mathrm{G}}+\boldsymbol{F}_{1\varepsilon} \tag{1}$$

where $\boldsymbol{M}_1$ and $\boldsymbol{G}_1$ are the generalized mass matrix and the gyroscopic matrix of the AMBs-rotor system, respectively. $\boldsymbol{F}_{1\mathrm{G}}$ is the rotor gravity vector, $\boldsymbol{F}_{1\mathrm{AMB}}$ is the generalized electromagnetic force vector of the two AMBs, and $\boldsymbol{F}_{1\varepsilon}$ is the unbalance force vector, defined as

$$\boldsymbol{M}_1=\begin{bmatrix} J_r & 0 & 0 & 0\\ 0 & m & 0 & 0\\ 0 & 0 & J_r & 0\\ 0 & 0 & 0 & m \end{bmatrix},\ \boldsymbol{G}_1=\begin{bmatrix} 0 & 0 & -J_z\omega & 0\\ 0 & 0 & 0 & 0\\ J_z\omega & 0 & 0 & 0\\ 0 & 0 & 0 & 0 \end{bmatrix},\ \boldsymbol{F}_{1\mathrm{G}}=\begin{bmatrix} 0\\ 0\\ 0\\ -mg \end{bmatrix},$$

$$\boldsymbol{F}_{1\mathrm{AMB}}=\begin{bmatrix} f_{xa}l_{\mathrm{a}}-f_{xb}l_{\mathrm{b}}\\ f_{xa}+f_{xb}\\ -f_{ya}l_{\mathrm{a}}+f_{yb}l_{\mathrm{b}}\\ f_{ya}+f_{yb} \end{bmatrix},\ \boldsymbol{F}_{1\varepsilon}=\begin{bmatrix} U_{\mathrm{u}}(\dot{\varphi}^2\cos\varphi+\ddot{\varphi}\sin\varphi)\varepsilon_z\\ U_{\mathrm{u}}(\dot{\varphi}^2\cos\varphi+\ddot{\varphi}\sin\varphi)\\ -U_{\mathrm{u}}(\dot{\varphi}^2\sin\varphi-\ddot{\varphi}\cos\varphi)\varepsilon_z\\ U_{\mathrm{u}}(\dot{\varphi}^2\sin\varphi-\ddot{\varphi}\cos\varphi) \end{bmatrix}.$$

where m is the rotor mass. J_z and J_r are the moments of inertia of the rotor around z- and x-(y-)axes, respectively. $U_u=m_u\varepsilon$ is the amount of the rotor unbalance, where m_u is the unbalance mass. ε and ε_z are the radial and axial offset of the unbalance, respectively. ω and φ are the angular speed and the rotation angle of the rotor, respectively, which satisfy $\omega=\mathrm{d}\varphi/\mathrm{d}t$.

$\boldsymbol{F}_{1\mathrm{AMB}}$ can also be written as follows:

$$\boldsymbol{F}_{1\mathrm{AMB}}=\boldsymbol{L}_1\left(\boldsymbol{K}_{\mathrm{i}1}\boldsymbol{I}_{\mathrm{c}1}+\boldsymbol{K}_{\mathrm{h}1}\boldsymbol{q}_{\mathrm{b}1}\right) \tag{2}$$

where $\boldsymbol{K}_{\mathrm{i}1}=\mathrm{diag}[k_{\mathrm{i}}\ k_{\mathrm{i}}\ k_{\mathrm{i}}\ k_{\mathrm{i}}]$ and $\boldsymbol{K}_{\mathrm{h}1}=\mathrm{diag}[k_{\mathrm{h}}\ k_{\mathrm{h}}\ k_{\mathrm{h}}\ k_{\mathrm{h}}]$ are the current and displacement stiffness matrix of the AMBs in the 4-DOF AMBs-rotor system, respectively, where k_{i} and k_{h} are the current and displacement stiffness coefficients, respectively. $\boldsymbol{I}_{\mathrm{c}1}$ is the control current vector. $\boldsymbol{q}_{\mathrm{b}1}=\boldsymbol{L}_1^{\mathrm{T}}\boldsymbol{q}$ is the rotor displacement vector at AMBs position. $\boldsymbol{L}_1$ is the coordinate transformation matrix, defined as

$$\boldsymbol{L}_1=\begin{bmatrix} l_{\mathrm{a}} & -l_{\mathrm{b}} & 0 & 0\\ 1 & 1 & 0 & 0\\ 0 & 0 & -l_{\mathrm{a}} & l_{\mathrm{b}}\\ 0 & 0 & 1 & 1 \end{bmatrix} \tag{3}$$

Due to the gyroscopic effect, the rotational motion around x-axis is coupled with that around y-axis. Besides, it may decrease the stability of the AMBs-rotor system when the rotor rotates at a high rotational speed. In order to decouple the 4-DOF AMBs-rotor system and suppress the gyroscopic effect, a cross-differential feedback is introduced to the control system, whose matrix can be expressed as

$$\boldsymbol{K}_{\mathrm{g}} = \left(\boldsymbol{L}_1\boldsymbol{K}_{\mathrm{i}1}\right)^{-1}\boldsymbol{G}_1\boldsymbol{L}_1^{-\mathrm{T}} = \boldsymbol{K}_{\mathrm{i}1}^{-1}\boldsymbol{L}_1^{-1}\boldsymbol{G}_1\boldsymbol{L}_1^{-\mathrm{T}} \tag{4}$$

According to rotor dynamic theory, by applying the cross-differential feedback, the 4-DOF AMBs-rigid rotor system system can be decomposed into two 2-DOF sub-systems. After linearizing the electromagnetic force of the AMBs and using the PD controller, one of the 2-DOF sub-systems can be written as follows:

$$\boldsymbol{M}\ddot{\boldsymbol{q}} - \boldsymbol{K}_{\mathrm{ss}}\boldsymbol{q} = -\boldsymbol{L}\boldsymbol{K}_{\mathrm{i}}\begin{bmatrix}\boldsymbol{K}_{\mathrm{p}} & \boldsymbol{K}_{\mathrm{d}}\end{bmatrix}\begin{bmatrix}\boldsymbol{L}^{\mathrm{T}} & 0_{2\times2}\\ 0_{2\times2} & \boldsymbol{L}^{\mathrm{T}}\end{bmatrix}\begin{bmatrix}\boldsymbol{q}\\ \dot{\boldsymbol{q}}\end{bmatrix} \tag{5}$$

where $\boldsymbol{M}$=diag[J_r m] and $\boldsymbol{q}$=[θ_y x]$^{\mathrm{T}}$ are the generalized mass matrix and coordinate vector of the 2-DOF sub-system, respectively. $\boldsymbol{K}_{\mathrm{i}}$=diag[$k_{\mathrm{i}}$ k_{i}] is the current stiffness matrix. $\boldsymbol{K}_{\mathrm{p}}$=diag[$p_1$,$p_2$] and $\boldsymbol{K}_{\mathrm{d}}$=diag[$d_1$,$d_2$] are the proportional and differential feedback matrices, respectively. $\boldsymbol{K}_{\mathrm{ss}}$ is the negative displacement stiffness matrix, and $\boldsymbol{L}$ is the coordinate transformation matrix, which are defined as

$$\boldsymbol{K}_{\mathrm{ss}} = k_{\mathrm{h}}\begin{bmatrix} l_{\mathrm{a}}^2 + l_{\mathrm{b}}^2 & l_{\mathrm{a}} - l_{\mathrm{b}}\\ l_{\mathrm{a}} - l_{\mathrm{b}} & 2\end{bmatrix}, \quad \boldsymbol{L} = \begin{bmatrix} l_{\mathrm{a}} & -l_{\mathrm{b}}\\ 1 & 1\end{bmatrix}.$$

The structure of the closed-loop 2-DOF sub-system under the PD control is shown in Fig. 2, where $x_{\mathrm{aref}}(s)$=$x_{\mathrm{bref}}(s)$=0.

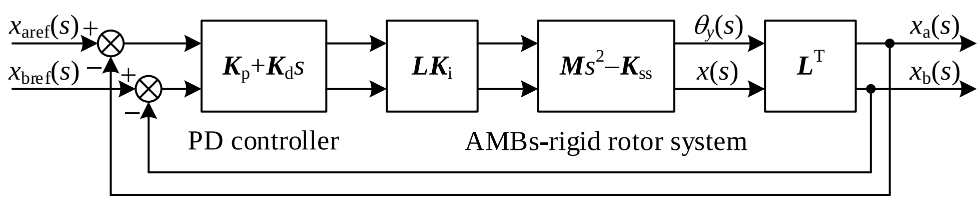


Fig. 2: The structure diagram of the AMBs-rigid rotor system.

3. Eigenvalue assignment

According to Eq.**Error! Reference source not found.**, the closed-loop eigenstructure problem of the 2-DOF sub-system can be described as

$$\left(s_i^2\boldsymbol{M} - \boldsymbol{K}_{\mathrm{ss}}\right)\boldsymbol{v}_i = -\boldsymbol{L}\boldsymbol{K}_{\mathrm{i}}\boldsymbol{w}_i,\ i = 1,2,3,4 \tag{6}$$

where s_i is the eigenvalue. $\boldsymbol{v}_i$ and $\boldsymbol{w}_i$ are the right eigenvector and control vector of the system with respect to s_i. $\boldsymbol{w}_i$ is defined as

$$\boldsymbol{w}_i = \begin{bmatrix}\boldsymbol{K}_1 & \boldsymbol{K}_2\end{bmatrix}\begin{bmatrix}\boldsymbol{L}^{\mathrm{T}} & \boldsymbol{0}_{2\times2}\\ \boldsymbol{0}_{2\times2} & \boldsymbol{L}^{\mathrm{T}}\end{bmatrix}\begin{bmatrix}\boldsymbol{v}_i\\ s_i\boldsymbol{v}_i\end{bmatrix} \tag{7}$$

From (6) we obtain

$$\begin{bmatrix}s_i^2\boldsymbol{M} - \boldsymbol{K}_{\mathrm{ss}} & \boldsymbol{L}\boldsymbol{K}_{\mathrm{i}}\end{bmatrix}\begin{bmatrix}\boldsymbol{v}_i\\ \boldsymbol{w}_i\end{bmatrix} = \boldsymbol{0}_{2\times2} \tag{8}$$

Suppose that $\boldsymbol{H}_i = [s_i^2\boldsymbol{M} - \boldsymbol{K}_{\mathrm{ss}} \quad \boldsymbol{L}\boldsymbol{K}_{\mathrm{i}}]$. $\boldsymbol{H}_i$ is full-rank since rank($\boldsymbol{L}\boldsymbol{K}_{\mathrm{i}}$) =2. According to PBH-rank criterion [6], the system is controllable. Besides, it is obvious that the vector [$\boldsymbol{v}_i$ $\boldsymbol{w}_i$]$^{\mathrm{T}}$ lies in the null space of $\boldsymbol{H}_i$.

The singular value decomposition (SVD) is applied to $\boldsymbol{H}_i$ and we have

$$\boldsymbol{H}_i = \boldsymbol{P}_i\boldsymbol{\Lambda}_{\mathrm{s}i}\boldsymbol{Q}_i^{\mathrm{H}} \tag{9}$$

where $\boldsymbol{P}_i \in C^{2\times2}$ and $\boldsymbol{Q}_i \in C^{4\times4}$ are both unitary matrices. $(\cdot)^{\mathrm{H}}$ denotes the conjugate transpose of the matrix $(\cdot)$. $\boldsymbol{\Lambda}_{si}$=[diag($\sigma_{i1}$, σ_{i2}) $\boldsymbol{0}_{2\times2}$], where σ_{i1} and σ_{i2} are the singular values of $\boldsymbol{H}_i$.

The matrix $\boldsymbol{Q}_i$ can be partitioned as follows:

$$\boldsymbol{Q}_i = \begin{bmatrix} \boldsymbol{E}_i & \boldsymbol{N}_i \\ \boldsymbol{C}_i & \boldsymbol{D}_i \end{bmatrix} \tag{10}$$

where $\boldsymbol{E}_i \in C^{2\times 2}$, $\boldsymbol{N}_i \in C^{2\times 2}$, $\boldsymbol{C}_i \in C^{2\times 2}$, and $\boldsymbol{D}_i \in C^{2\times 2}$ are the submatrices of $\boldsymbol{Q}_i$.

Since each column of the matrix $[\boldsymbol{N}_i^{\mathrm{T}} \quad \boldsymbol{D}_i^{\mathrm{T}}]^{\mathrm{T}}$ is the base vector in the null space of $\boldsymbol{H}_i$, $[\boldsymbol{v}_i^{\mathrm{T}} \quad \boldsymbol{w}_i^{\mathrm{T}}]^{\mathrm{T}}$ is a linear of the columns of $[\boldsymbol{N}_i^{\mathrm{T}} \quad \boldsymbol{D}_i^{\mathrm{T}}]^{\mathrm{T}}$, expressed as

$$\begin{bmatrix} \boldsymbol{v}_i \\ \boldsymbol{w}_i \end{bmatrix} = \begin{bmatrix} \boldsymbol{N}_i \\ \boldsymbol{D}_i \end{bmatrix} \boldsymbol{g}_i \tag{11}$$

where $\boldsymbol{g}_i \in C^{2\times 1}$ are free complex vectors.

Combining (7) and (11), we obtain

$$\begin{cases} \begin{bmatrix} p_1 & d_1 \end{bmatrix} \begin{bmatrix} l_{\mathrm{a}} & 1 & 0 & 0 \\ 0 & 0 & l_{\mathrm{a}} & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{N}_i \\ s_i \boldsymbol{N}_i \end{bmatrix} \boldsymbol{g}_i = \boldsymbol{D}_{i1} \boldsymbol{g}_i \\ \begin{bmatrix} p_2 & d_2 \end{bmatrix} \begin{bmatrix} -l_{\mathrm{b}} & 1 & 0 & 0 \\ 0 & 0 & -l_{\mathrm{b}} & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{N}_i \\ s_i \boldsymbol{N}_i \end{bmatrix} \boldsymbol{g}_i = \boldsymbol{D}_{i2} \boldsymbol{g}_i \end{cases} \tag{12}$$

where $\boldsymbol{D}_{i1}$ and $\boldsymbol{D}_{i2}$ are the first and second rows of $\boldsymbol{D}_i$, respectively.

Suppose that

$$\boldsymbol{X}_{i1} = \begin{bmatrix} l_{\mathrm{a}} & 1 & 0 & 0 \\ 0 & 0 & l_{\mathrm{a}} & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{N}_i \\ s_i \boldsymbol{N}_i \end{bmatrix} \boldsymbol{g}_i, \quad \boldsymbol{X}_{i2} = \begin{bmatrix} -l_{\mathrm{b}} & 1 & 0 & 0 \\ 0 & 0 & -l_{\mathrm{b}} & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{N}_i \\ s_i \boldsymbol{N}_i \end{bmatrix} \boldsymbol{g}_i, \quad Y_{i1} = \boldsymbol{D}_{i1} \boldsymbol{g}_i, \quad Y_{i2} = \boldsymbol{D}_{i2} \boldsymbol{g}_i, \quad \boldsymbol{X}_1 = \begin{bmatrix} \boldsymbol{X}_{11} & \boldsymbol{X}_{21} & \boldsymbol{X}_{31} & \boldsymbol{X}_{41} \end{bmatrix},$$

$$\boldsymbol{X}_2 = \begin{bmatrix} \boldsymbol{X}_{12} & \boldsymbol{X}_{22} & \boldsymbol{X}_{32} & \boldsymbol{X}_{42} \end{bmatrix}, \quad \boldsymbol{Y}_1 = \begin{bmatrix} Y_{11} & Y_{21} & Y_{31} & Y_{41} \end{bmatrix}, \quad \boldsymbol{Y}_2 = \begin{bmatrix} Y_{12} & Y_{22} & Y_{32} & Y_{42} \end{bmatrix},$$

(12) can be expressed as

$$\begin{cases} \begin{bmatrix} p_1 & d_1 \end{bmatrix} \boldsymbol{X}_1 = \boldsymbol{Y}_1 \\ \begin{bmatrix} p_2 & d_2 \end{bmatrix} \boldsymbol{X}_2 = \boldsymbol{Y}_2 \end{cases} \tag{13}$$

(13) is an overdetermined linear system of equation for p_1, d_1, p_2, and d_2. Thus, in order to make it valid, the following conditions should be satisfied:

$$\begin{cases} \operatorname{rank}\begin{pmatrix} \boldsymbol{X}_1 \\ \boldsymbol{Y}_1 \end{pmatrix} = \operatorname{rank}(\boldsymbol{X}_1) \\ \operatorname{rank}\begin{pmatrix} \boldsymbol{X}_2 \\ \boldsymbol{Y}_2 \end{pmatrix} = \operatorname{rank}(\boldsymbol{X}_2) \end{cases} \tag{14}$$

From (14) we know that $\boldsymbol{Y}_1$ must be the linear combination of each row of $\boldsymbol{X}_1$, and $\boldsymbol{Y}_2$ must be the linear combination of each row of $\boldsymbol{X}_2$, which can be expressed as

$$\begin{cases} \boldsymbol{D}_{11}\boldsymbol{g}_1 = (c_1 + s_1 c_2)\begin{bmatrix} l_{\mathrm{a}} & 1 \end{bmatrix} \boldsymbol{N}_1 \boldsymbol{g}_1 \\ \boldsymbol{D}_{21}\boldsymbol{g}_2 = (c_1 + s_2 c_2)\begin{bmatrix} l_{\mathrm{a}} & 1 \end{bmatrix} \boldsymbol{N}_2 \boldsymbol{g}_2 \\ \boldsymbol{D}_{31}\boldsymbol{g}_3 = (c_1 + s_3 c_2)\begin{bmatrix} l_{\mathrm{a}} & 1 \end{bmatrix} \boldsymbol{N}_3 \boldsymbol{g}_3 \\ \boldsymbol{D}_{41}\boldsymbol{g}_4 = (c_1 + s_4 c_2)\begin{bmatrix} l_{\mathrm{a}} & 1 \end{bmatrix} \boldsymbol{N}_4 \boldsymbol{g}_4 \end{cases}, \quad \begin{cases} \boldsymbol{D}_{12}\boldsymbol{g}_1 = (c_3 + s_1 c_4)\begin{bmatrix} -l_{\mathrm{b}} & 1 \end{bmatrix} \boldsymbol{N}_1 \boldsymbol{g}_1 \\ \boldsymbol{D}_{22}\boldsymbol{g}_2 = (c_3 + s_2 c_4)\begin{bmatrix} -l_{\mathrm{b}} & 1 \end{bmatrix} \boldsymbol{N}_2 \boldsymbol{g}_2 \\ \boldsymbol{D}_{32}\boldsymbol{g}_3 = (c_3 + s_3 c_4)\begin{bmatrix} -l_{\mathrm{b}} & 1 \end{bmatrix} \boldsymbol{N}_3 \boldsymbol{g}_3 \\ \boldsymbol{D}_{42}\boldsymbol{g}_4 = (c_3 + s_4 c_4)\begin{bmatrix} -l_{\mathrm{b}} & 1 \end{bmatrix} \boldsymbol{N}_4 \boldsymbol{g}_4 \end{cases} \tag{15}$$

Let

$$\begin{bmatrix} \boldsymbol{g}_1 & \boldsymbol{g}_2 & \boldsymbol{g}_3 & \boldsymbol{g}_4 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ x_1 + jy_1 & x_1 - jy_1 & x_2 + jy_2 & x_2 - jy_2 \end{bmatrix} \tag{16}$$

If s_1, s_2, s_3, and s_4 are given, c_1, c_2, c_3, c_4, x_1, y_1, x_2, and y_2 are uniquely determined, and thus $\boldsymbol{g}_1$ and $\boldsymbol{g}_2$ are also uniquely determined. After substituting $\boldsymbol{g}_1$ and $\boldsymbol{g}_2$ into (13), the PD parameters p_1, d_1, p_2, and d_2 can be derived.

4. Numerical simulation

The parameters of the AMBs-rigid rotor system are shown in Table 1, where J_z is the moment of inertia of the rotor around *z*-axis, and i_0 is the bias current.

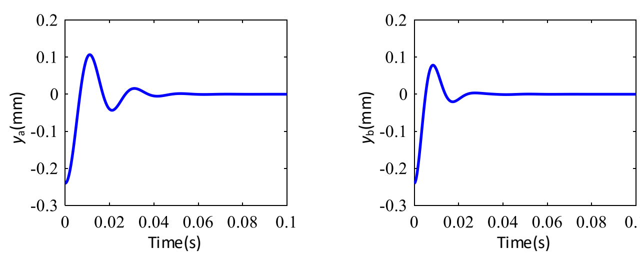
Table 1: Caption for table goes at the top.

Symbol	Quantity	Unit	Symbol	Quantity	Unit
m	18.09	kg	l_{sa}	0.163	m
J_r	0.2	kg·m^2	l_{sb}	0.112	m
J_z	0.0223	kg·m^2	k_i	321.09i_0	N/A^2
l_a	0.155	m	k_h	847600i_0^2	N/(m·A^2)
l_b	0.105	m	I_0	1	A

The eigenvalues of the sub-systems are set as $s_{1,2}$=−100±j300, $s_{3,4}$=−150±j400, and the value of p_1, d_1, p_2, and d_2 are 4564, 3.955, 7564, and 7.822, respectively. The step responses and acceleration responses in the operation are shown as follows.

4.1. Step responses

Suppose that the initial position of the rotor in y-direction at AMB-A and -B, y_{a0} and y_{b0}, are both -0.24mm, and the reference position, y_{aref} and y_{bref}, are both 0mm. After applying the PD control based on eigenvalue assignment, the step responses of the rotor displacement in *y*-direction at AMB-A and -B are shown in Fig. 3.



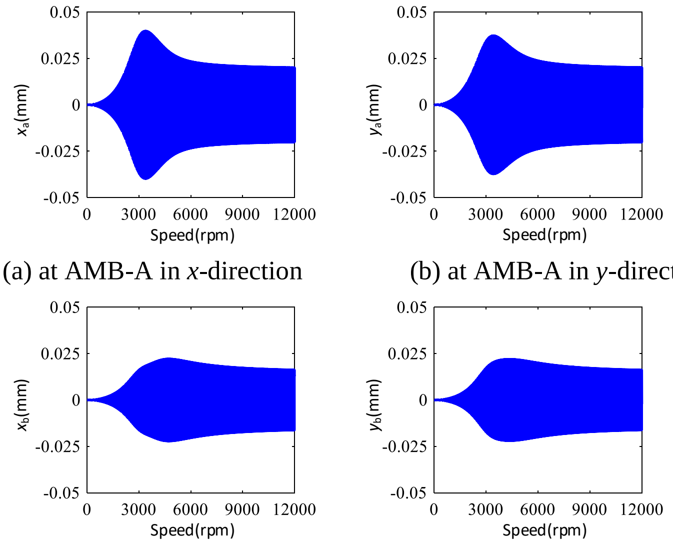
(a) at AMB-A (b) at AMB-B

Fig. 3: The step responses of the rotor displacement in *y*-direction.

As is shown in Fig. 3, the AMBs-rigid rotor system controlled by the designed PD controller can achieve fast and steady suspension, and the overshoots of the step responses are both smaller than 0.11mm, which is far smaller than 0.24mm, the air gap length between the rotor and the backup bearing.

4.2. Acceleration responses

Suppose that the amount of the unbalance is 3×10^{-4}kg·m, and the axial offset of the unbalance is 0.01m. The displacement responses of the rotor in the constant acceleration operation of 100rpm/s are shown in Fig. 4.



(a) at AMB-A in *x*-direction (b) at AMB-A in *y*-direction

(c) at AMB-B in *x*-direction (d) at AMB-B in *y*-direction

Fig. 4: The displacement responses of the rotor in the constant acceleration operation of 100rpm/s.

As is shown in Fig. 4, the peak amplitudes of the rotor displacement at AMB-A and -B in *x*- and *y*-direction are all smaller than 0.048mm, 20% of the air gap length between the rotor and the backup bearing. It is obvious that the AMBs-rigid rotor system can cross the rigid critical speed safely and operate stably and steady with small vibration in full rotational speed range.

5. Conclusion

This paper proposes a design method for the PD feedback controller based on eigenvalue assignment. The cross-differential feedback is applied to simplify the controller design process and obtain the 2-DOF sub-systems. After assigning the closed-loop eigenvalues, the PD parameters are obtained by eigenvalue assignment. Numerical simulation verifies that the design method of the PD controller proposed in this paper is feasible and effective.

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