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Synchronization of Complex Dynamic Networks for Differential Wheel Mobile Robots

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Abstract - This paper studies the trajectory tracking of a network composed of two differential-drive mobile robots. For this purpose, the kinematic model is utilized, and a synchronization scheme based on the master-slave configuration is developed. Networks were proposed to analyze different cases in the trajectory-tracking problem. To guarantee asymptotic synchronization, the network used Lyapunov's theorem to conduct a stability analysis.

Keywords**:** mobile robot, synchronization, trajectory tracking, complex network.

1. Introduction

This work proposes the trajectory-tracking problem for a network composed of two differential-wheel robots. The robots in the network will go from an initial position p_i to a final position p_f grouping together on a path defined by kinematic modeling and synchronization of complex networks, assuming a mobile robot is a master with a desired path and a mobile robot is a slave.

Some techniques attack the path-tracking problem. In the work of [8], path tracking is carried out using feedback control based on the backstepping law for a kinematic and dynamic model. Meanwhile, [6] carries out a study on the stability of differential-wheel mobile robots, focusing on the periodic solution based on the kinematic model of the robot based on Lyapunov stability.

In general, the kinematic behavior of a mobile robot is characterized by a mechanical description. The kinematic model estimates position and velocity, so the problem of direct kinematics in a differential wheel robot consists of estimating the mobile robot's position and orientation from the geometry and speeds of the wheels with respect to a reference frame [5].

This paper is organized as follows: Section 2 describes the kinematic model of the mobile robot considered. Section 3 explains the process and theory for the synchronization of dynamic networks. Section 4 presents a solution to the trajectory Tracking problem using the theory of synchronization of complex networks. Section 5 studies the desired position control problem for a complex network. Finally, the conclusion is made in Section 6.

2. Kinematic model

2.1. Differential wheel mobile robot

The structural properties of mobile robots have been studied by [3]. This work considers a differential wheel robot with a pair of wheels with independent angular rotation velocities and a spherical wheel for support (Figure 1).

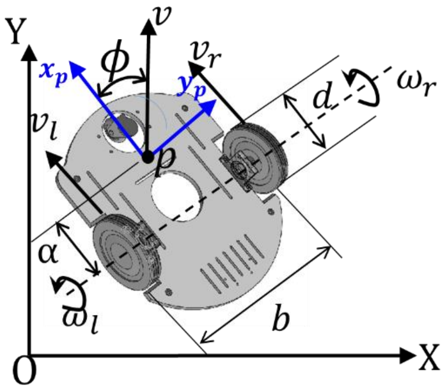

Figure 1: Differential robot with non-holonomic constraints.

The reference plane of the robot is defined by position coordinates and an orientation angle (x_p, x_p, ϕ) concerning the center of mass at point p, where the orientation angle ϕ is considered as the angular difference between the global reference frame relative to the origin (X_o, Y_o) of the Cartesian plane of Figure 1 [3]. The rotational speed on the left and right wheels ω_l and ω_r, is directly related to the linear and angular velocities in the form: $v_l = \omega_l d$, $v_r = \omega_r d$, where d is the diameter of the robot wheels, b is the distance between the wheel axles and α is the distance at p point. Therefore, the relation between the body velocities and the wheel velocities is given by:

$$v = \frac{v_r + v_l}{2} = \frac{(\omega_r + \omega_l)d}{2}; \qquad \omega = \frac{v_r - v_l}{b} = \frac{(\omega_r - \omega_l)d}{b} \tag{1}$$

⇒

$$\omega_l = \frac{v - (\frac{b}{2})\omega}{d}; \qquad \omega_r = \frac{v + (\frac{b}{2})\omega}{d} \tag{2}$$

⇒

$$v_l = v - (\frac{b}{2})\omega; \qquad v_r = v + (\frac{b}{2})\omega \tag{3}$$

The kinematic model for a differential wheel robot is given:

$$\dot{p} = J(p)q = \begin{bmatrix} \cos\phi & -\alpha\sin\phi \\ \sin\phi & \alpha\cos\phi \\ 0 & 1 \end{bmatrix} q \tag{4}$$

Where $p = [x_p \quad y_p \quad \phi]^T$ is the global coordinate vector at point p and $q = [v \quad \omega]^T = [v_p \quad \omega_p]^T$ is a vector that contains the linear and angular velocities, respectively, $J(p)$ is known as the Jacobian matrix and satisfy the relationship given in [1]. Then, from (4) one has

$$q = [(\mathrm{J(p)})^T J(p)]^{-1}\dot{p} \tag{5}$$

⇒

$$\begin{bmatrix} \dot{x}_p \\ \dot{y}_p \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \cos\phi & -\alpha\sin\phi \\ \sin\phi & \alpha\cos\phi \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \tag{6}$$

For the differential wheel mobile robot with non-holonomic constraints, in this work is considered the case in which the geometric center of the robot is separated at a distance α from the position reference point illustrated in Figure 1. From the relationship (6), it is possible to obtain a kinematic representation with the following structure.

$$\begin{bmatrix} \dot{x}_p \\ \dot{y}_p \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \cos\phi & -\alpha\sin\phi \\ \sin\phi & \alpha\cos\phi \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{(\omega_r + \omega_l)d}{2} \\ \frac{(\omega_r - \omega_l)d}{b} \end{bmatrix} \tag{7}$$

2.2. Synchronization condition for complex network

Let´s consider a complex dynamic network composite by N identical nodes (x_i) coupled linearly through the first state variable of each node described by [7]:

$$\dot{x}_i = f(x_i) + u_{i1} \tag{8}$$

With

$$u_{i1} = c\sum_{j=1}^{N} a_{ij}\Gamma x_{j1} \tag{9}$$

Where $x_i = (x_{i1}, x_{i2}, \cdots, x_{iN})^T \in \mathbb{R}^n$ is the vector state for the node i, $\dot{x}_i$ represents a dynamic system in the $node\ i$. In this work $\dot{x}_i = (\dot{x}_1, \dot{x}_2)^T$ and represent the dynamic model for the master and slave nodes respectively, $u_{j1} \in \mathbb{R}^n$ is the diffusive coupling and represents the control signal for the first variable state on the $node\ i$, $c > 0$ represents the coupling constant between the input signal and the state, $\Gamma = diag(r_1, r_2, r_3, \dots, r_N)$ is a matrix with constant coefficients and relates the state variables coupled in the network.

For the complex network described in equation (8), the coupling matrix network is defined by:

$$Ac = A(g) - D(g) \tag{10}$$

The coupling matrix $A_c \in \mathbb{R}^{n\times n}$ represents the network topology in which if there exists a connection between node i and node j of the network, the elements $a_{ij} \in A_c$ are defined in equation (11):

$$a_{ii} = -\sum_{\substack{j=1\\ j\neq i}}^{N} a_{ij} = -\sum_{\substack{j=1\\ j\neq i}}^{N} a_{ji} = -d_i, \quad for \qquad i = 1,2,\cdots,N. \tag{11}$$

The adjacency matrix $A(g)$ relate the links between the nodes and take the values: $a_{ij} = 1$, otherwise $a_{ij} = 0$ for $i \neq j$. The degree matrix$D(g)$ is a diagonal matrix which elements d_{ij} given by:

$$d_{ij} = \begin{cases} d_i; & i = j \\ 0; & i \neq j \end{cases} \tag{12}$$

Where d_i is the degree of i-th node and are studied by [2], [7]. Suppose there are no isolated nodes in the network (8), then Ac is a symmetrical and irreducible matrix, in this case Ac will have an eigenvalue in $\lambda_1 = 0$ with multiplicity one and the other eigenvalues will be strictly negative, so the synchronization of the states of the network (8) can be characterized by the non-zero eigenvalues of the matrix Ac. According to [6] the states of the network (8) are asymptotically synchronized if.

$$x_1(t) = x_2(t) = \cdots = x_N(t) \quad when \quad t \to \infty \tag{13}$$

Where the diffusive coupling condition (13) guarantees that; the state synchronization is a solution $s(t) \in \mathbb{R}^n$ of an isolated node, that is:

$$\dot{s}(t) = f(s(t)) \tag{14}$$

Where $s(t)$ can be an equilibrium point, a periodic orbit, or a strange attractor. So, the stability of the state synchronization:

$$x_1(t) = x_2(t) = \cdots = s(t) \tag{15}$$

This stability will be determined by the dynamics of an isolated node, that is; the function f, the coupling constant c, the coefficient matrix Γ and the coupling matrix A_c.

Theorem **1** *Consider the dynamic network (8) and let*

$$0 = \lambda_1 > \lambda_2 \geq \lambda_3 \geq \cdots \geq \lambda_N \tag{16}$$

The *eigenvalues of the coupling matrix A_c. Suppose there exists a diagonal matrix $D^{n\times n} > 0$ and two constants $\bar{\eta} < 0$* and $\tau > 0$ *such that:*

$$\left[Df\left(s(t)\right) + \eta\Gamma\right]^T D + D\left[Df\left(s(t)\right) + \eta\Gamma\right] \leq -\tau I_n \tag{17}$$

for all $\eta \leq \bar{\eta}$ where $\mathrm{I_n} \in \mathbb{R}^{n\times n}$ is the identity matrix, such that:

$$c\lambda_2 \leq \bar{\eta} \tag{18}$$

Then, the synchronization of the states (15) of the dynamic network (8) will be exponentially stable. Where $\lambda_2 < 0$, so the inequality (18) is equivalent to:

$$c \geq \left|\frac{\bar{\eta}}{\lambda_2}\right|. \tag{19}$$

3. Synchronization network for mobile robots

In this section, the fundamental aspects for the synchronization of mobile robot networks will be described, based on the synchronization of dynamic networks which have been recently studied by [1] and [6], among others. According to (8) and (9), a master-slave network is depicted in Figure 2, whit kinematic model for the master node described as follow:

$$\begin{bmatrix}\dot{x}_{11}\\ \dot{x}_{12}\\ \dot{x}_{13}\end{bmatrix} = \begin{bmatrix}(1/2)\alpha(\cos\phi)(\omega_r+\omega_l)-(a/b)\alpha(\sin\phi)(\omega_r-\omega_l)+u_{11}\\ (1/2)\alpha(\sin\phi)(\omega_r+\omega_l)+(a/b)\alpha(\cos\phi)(\omega_r-\omega_l)+u_{12}\\ (1/b)\alpha(\omega_r-\omega_l)+u_{13}\end{bmatrix} \tag{20}$$

And the slave node:

$$\begin{bmatrix}\dot{x}_{21}\\ \dot{x}_{22}\\ \dot{x}_{23}\end{bmatrix} = \begin{bmatrix}(1/2)\alpha(\cos\phi)(\omega_r+\omega_l)-(a/b)\alpha(\sin\phi)(\omega_r-\omega_l)+u_{21}\\ (1/2)\alpha(\sin\phi)(\omega_r+\omega_l)+(a/b)\alpha(\cos\phi)(\omega_r-\omega_l)+u_{22}\\ (1/b)\alpha(\omega_r-\omega_l)+u_{23}\end{bmatrix} \tag{21}$$

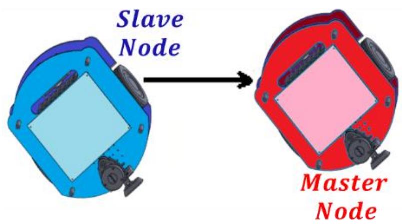

Figure 2: Master Slave Network configuration for differential wheel robots.

With the coupling matrix defined as:

$$A_c = \begin{bmatrix}0 & 0\\ -1 & -1\end{bmatrix} \tag{22}$$

Which is a symmetric and irreducible matrix and eigenvalues $\lambda_1 = 0$ and $\lambda_2 = -1 < 0$ [1]. Where the diffusive coupling condition for this case will be: $u_{11} = u_{12} = u_{13} = 0$, $c \neq 0$ since $a_{11} = a_{12} = 0$, and $u_{21} = c(a_{21}x_{11} + a_{22}x_{21})$, $u_{22} = c(a_{21}x_{12} + a_{12}x_{22})$, $u_{23} = c(a_{21}x_{13} + a_{22}x_{23})$,with $\Gamma = diag(1,1)$. So, the states of the slave robot (21) will be asymptotically synchronized with the states of the master robot (20) if $x_1(t) = x_2(t)\ when\ t \to \infty$.

Definition. The states of the system nodes (20) and (21) are asymptotically synchronized if:

$$\lim_{t\to\infty} \|e(t)\| = 0 \tag{23}$$

Regardless of the initial conditions of $x_1(0)$ and $x_2(0)$, where $e(t) = (e_1(t), e_2(t), e_3(t))^T$ represents the error synchronization with:

$$\begin{aligned} e_1(t) &= x_{11}(t) - x_{21}(t)\\ e_2(t) &= x_{12}(t) - x_{22}(t)\\ e_3(t) &= x_{13}(t) - x_{23}(t)\end{aligned} \tag{24}$$

Then, the error dynamics defined by (23) will be:

$$\begin{aligned} \dot{e}_1(t) &= \dot{x}_{11}(t) - \dot{x}_{21}(t)\\ \dot{e}_2(t) &= \dot{x}_{12}(t) - \dot{x}_{22}(t)\\ \dot{e}_3(t) &= \dot{x}_{13}(t) - \dot{x}_{23}(t)\end{aligned} \tag{25}$$

Replacing the equation (25) on to (20) and (21) we have the error dynamic as:

$$\left\{\begin{bmatrix}\dot{e}_{11}\\ \dot{e}_{12}\\ \dot{e}_{13}\end{bmatrix} = \begin{bmatrix}-2ce_1 + (d\omega_r + d\omega_l 2\sin(((x_{23}+x_{13})/2))\cos(((-e_3)/2)) - ce_1\\ (d\omega_r + d\omega_l)(\cos(x_{13}) - \cos(x_{23})) - ce_2\\ -e_3\end{bmatrix}\right. \tag{26}$$

Thus, the asymptotic stability at zero of the error dynamics (23) can be determinate by the following theorem.

Theorem 2 *The state vectors of the slave robot (21) and the master robot (20) are asymptotically synchronized for the condition (13) if:*

$$\lim_{t\to\infty} \|e(t)\| = 0 \qquad (27)$$

Where the error synchronization vector is considered in equations (24) and (25).

Proof**.** A stability analysis by Lyapunov theorem's, is carried on the equation (26) for the network composed of two mobile robots in order to guarantee (27):

$$\left\{ \begin{bmatrix} \dot{e}_{11} \\ \dot{e}_{12} \\ \dot{e}_{13} \end{bmatrix} = \begin{bmatrix} -2ce_1 + (d\omega_r + d\omega_l 2\sin(((x_{23} + x_{13})/2))\cos(((-e_3)/2)) \\ (d\omega_r + d\omega_l)(\cos(x_{13}) - \cos(x_{23})) \\ 0 \end{bmatrix} - \begin{bmatrix} -ce_1 & 0 & 0 \\ 0 & -ce_2 & 0 \\ 0 & 0 & -e_3 \end{bmatrix} \right. \qquad (28)$$

Which is satisfied for the origin with $\omega_r = \omega_l = 0$. Proposing a candidate Lyapunov function as:

$$V(e,t) = \frac{1}{2}\sum_{i=1}^{3} e_i \qquad (29)$$

Whose time derivative is given as:

$$\dot{V}(e,t) = \frac{1}{2}\sum_{i=1}^{3} e_i \dot{e}_i = -3c{e_1}^2 - c{e_2}^2 - c{e_3}^2 \qquad (30)$$

To satisfy stability asymptotically it is necessary that:

$$\dot{V}(e,t) = V(e,t) = -e^T Q e = -e \begin{bmatrix} -ce_1 & 0 & 0 \\ 0 & -ce_2 & 0 \\ 0 & 0 & -e_3 \end{bmatrix} e \qquad (31)$$

From condition (29), the function it is negative definite for any c > 0, so it follows that the origin of the dynamical system defined by the error (25) is asymptotically stable in the Lyapunov sense. This guarantees that the synchronization will be exponentially stable. For the condition (18), there exists a coupling constant $c \geq \bar{\eta}$, such that the dynamic network composed of (20) and (21) can synchronize. To satisfy the condition (19) is necessary chose a constant $\eta > 0$, such that zero is an equilibrium point of the dimensionless system.

$$Df(s(t)) + \eta\Gamma \qquad (32)$$

Which is equivalent to find an isolated node with constant $\eta > 0$, such that the feedback system with $-\eta x_1$ can stabilize the isolated node [1]:

$$\begin{aligned} \dot{x}_1 &= f_1(x) - \eta x_1 \\ \dot{x}_2 &= f_2(x) - \eta x_2 \\ \dot{x}_3 &= f_3(x) - \eta x_3 \end{aligned} \qquad (33)$$

which for a particular value $\eta = 1$ can be stabilized ■.

4. Experimental results

In order to synchronize the network for mobile robots in the master slave configuration (20), (21) with kinematic model, suppose in the master network (20) a parametric variation on the left wheel, in order to create a trajectory using the function:

$$\omega_i = f(\omega_i) = \begin{cases} 1.2; & t \leq 10 \\ 0; & 10 < t < 25 \\ 2; & 25 < t < 40 \\ 0.6; & t > 40 \end{cases} \qquad (34)$$

Figure 3 shows the master node with a circular path and the slave robot following a straight line path with a coupling constant zero ($c = 0$), in this case, there will be no state synchronization, as show in third column of Figure 3.

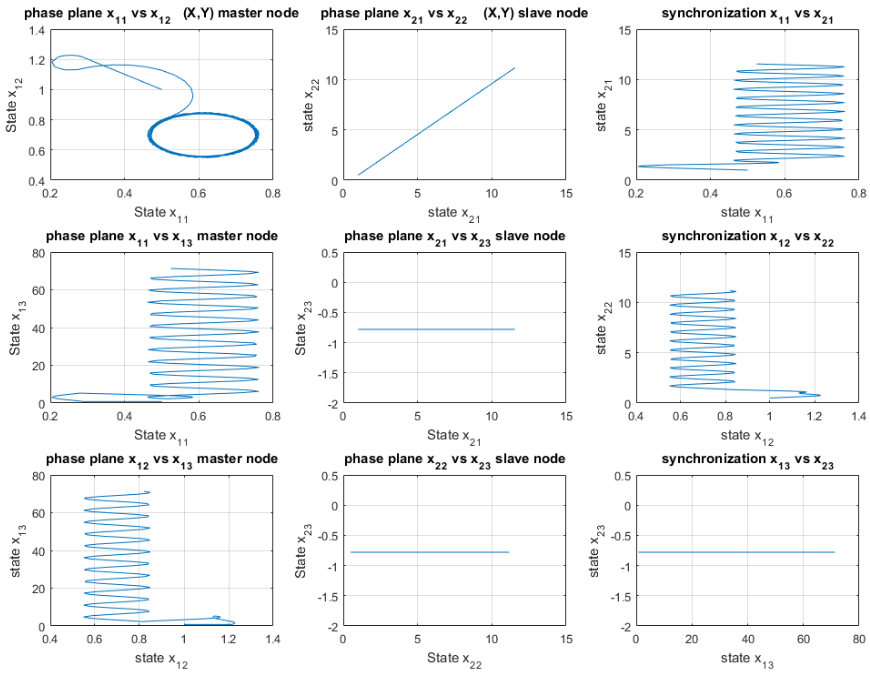

Figure 3: The first and second columns show the individual trajectories of each robot in the network; the third column corresponds to the lack of synchronization for c = 0.

For values of the coupling constant $c > 0$, the states of both mobile robots begin to couple, for values slightly higher than 0 ($c = 0.01$). When a coupling constant $c = 1$ is considered, and an error synchronization is observed in the third column on Figure 4, with the convergence to zero in condition (27) for the trajectory errors $e_{1(t)}$; $e_{2(t)}$; $e_{3(t)}$ in the network (8) composed by (20) and (21).

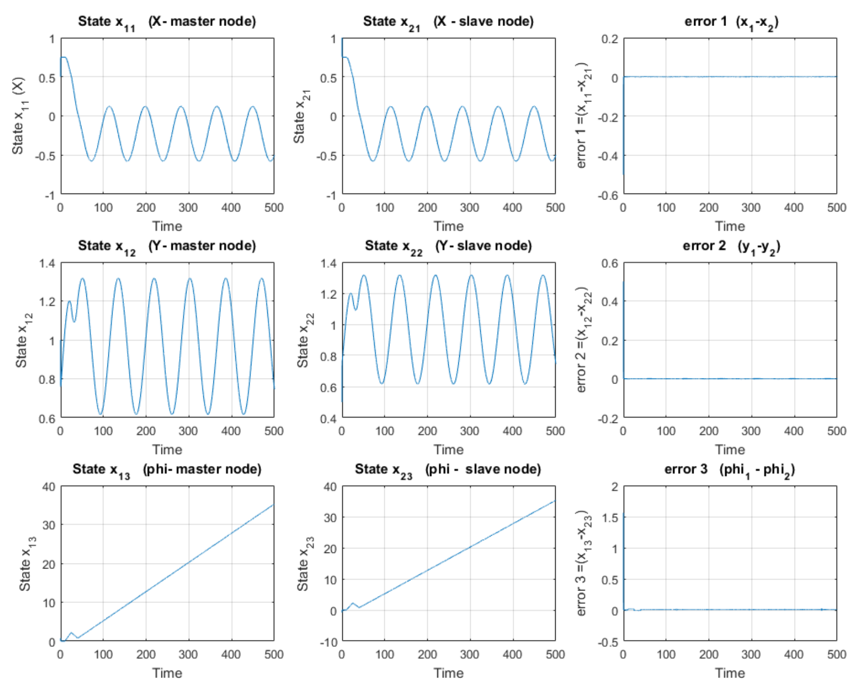

Figure 4. Trajectories and error for the tracking problem in network of two nodes.

In addition to coupling constant $c > 0$,angular velocities in the wheels for the master and slave nodes are considered, with: $\omega_{l\_slave} = 1.2$ rad/s, $\omega_{r\_slave} = 1.2$ rad/s, $\omega_{r\_master} = 1.2\ \mathrm{rad/s}$ and $\omega_{l\_master} = f(\omega_l)$ described by the function (34), in order to trace the trajectories of each robot in the network (8).

The trajectory synchronization of mobile robots with kinematic model (20) and (21) is shown in Figure 5 for trajectories with a coupling constant c = 1, in the third column the synchronization of states trace a line inclined at 45 degrees, which indicates that the trajectory of states $x_1(t) = x_2(t)$ is the same in (13). On the other hand, in the first and second columns of Figure 5, the space states for the master node (20) and the slave node (21) are the same as proposed in the condition form equation (13).

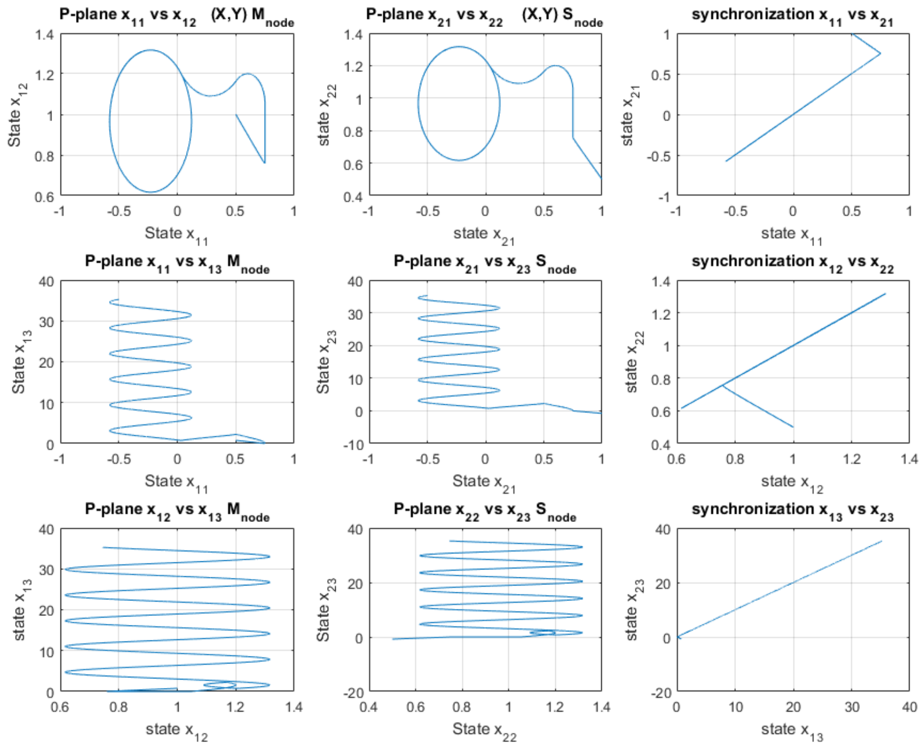

Figure 5. Synchronization of network composed by two differential wheel mobile robots with constant velocities $\omega_{i\_slave} = 1.2$ and $\omega_{r\_slave} = 1.2$ for the slave robot and $\omega_{i\_master} = f(\omega_i)$ and $\omega_{r\_master} = 1.2$ for the master node.

These computational results suggest that state synchronization is possible for a set of mobile robots with a kinematic model.

5. Conclusion

This work shows the possibility of synchronizing mobile robot network trajectories using only the first state of the master mobile robot kinematic model. For the case of path tracking where the working environment is known, it sufficient control of the angular velocity of a single wheel to obtain the velocity, position, and synchronization network of differential-wheel mobile robots. Furthermore, asymptotic trajectory synchronization is possible for a network formed by two differential-wheel mobile robots through the complex network theory. Future work includes experiments for more than two mobile robots, including obstacle avoidance and studies for different network topologies.

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