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Development of a Detailed Drop Tower Impact Model Tuned via Particle Swarm Optimization

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Abstract **-** This paper presents a systematic framework for modeling, simulation, and parameter identification of a two-degree-of-freedom impact model of a drop tower setup. The model emulates the dynamics of a falling carriage impacting a reaction surface, with both the impactor and the reaction surface layers explicitly represented by mass-spring-damper elements. In contrast to the existing models that simplify the impact surface as a single layer, the proposed model offers a more detailed and realistic representation of a drop test setup, capturing the role of individual layers in shaping the carriage's impact response. A target impact acceleration profile, represented by a standard half-sine pulse, is used as a reference for parameter identification. Particle Swarm Optimization is utilized to identify the stiffness and damping characteristics of each layer, allowing the simulated acceleration response to match the target half-sine pulse. The optimized impact model has successfully reproduced the shock pulse, with the corresponding identified parameters providing insights into material selection for each layer. The proposed approach provides a suitable framework for drop test design.

***Keywords*:** drop tower, parameter identification, particle swarm optimization (PSO), 2DOF impact modeling

1. Introduction

Mechanical shock pulses present major concerns to fragile components in mechanical and electronic systems. These abrupt transients can induce high acceleration loads, potentially damaging internal assemblies of such systems. Engineers commonly use drop towers to replicate such events in a controlled laboratory environment, allowing them to assess the product durability under standardized shock test conditions. Hence, accurate modeling of impact events is critical for designing protective solutions.

Traditionally, engineers have mitigated these forces using shock isolators or elastomer layers that dissipate energy during impacts [1], [2]. However, designing such protective layers has remained largely pragmatic and often relies on trial and error due to the complex interactions between material properties and system dynamics. To simplify analysis, lumped-mass impact models, especially one-degree-of-freedom (1DOF) systems, have been extensively used in literature to approximate the drop tower impact behaviour, as demonstrated by Zhou et al. [3].

A typical 1DOF impact model, presented in Fig. 1 consists of a carriage mass m_c falling from a height h, impacting the pulse modulator represented by a spring k_{pu} and a damper c_{pu}. While computationally simple, such 1DOF models often fail to capture the nuanced dynamics that arise from interactions between multiple masses and layered materials.

This study introduces a more realistic two-degree-of-freedom (2DOF) model to better represent the impact dynamics observed in drop tower tests. The model simulates the dynamic interaction between a falling carriage with a polyurethane attachment and a reaction mass supported by a base, while incorporating both a pulse modulator and an impact isolator. Unlike prior works, the system parameters are not assumed and instead are identified using Particle Swarm Optimization (PSO) to match a target impact response profile. The target acceleration is modeled as a damped half-sine pulse, mimicking a typical sharp, high-g shock pulse observed in standard drop tests [4].

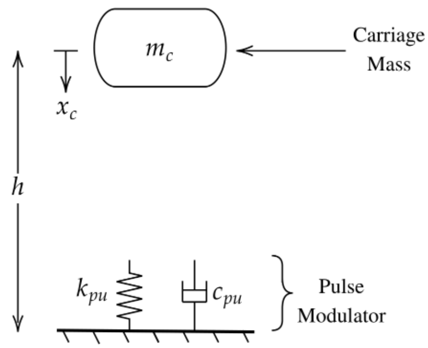

Fig. 1 1DOF impact model.

Through simulation, this research provides a simplified 2DOF impact system combined with an optimization technique for identifying optimal design parameters. These insights aim to guide material selection and design decisions for real-world impact applications.

2. Impact System Modeling

To replicate an actual drop tower experiment via simulation, a realistic impact model with 2DOF system representation is constructed. This system captures the dynamic interaction between the falling carriage and the impact surface, incorporating key physical elements such as the polyurethane attachment, pulse modulator, impact isolator, and reaction mass.

In the proposed drop tower impact setup, Fig. 2 presents the impactor assembly, where a carriage of mass m_c is connected to a polyurethane attachment of mass m_{po}. Additionally, Fig. 3 presents a detailed configuration of the corresponding impact surface, showcasing its stacked (layered) arrangement. Each layer serves a distinct role during the impact event. The impact surface consists of reaction mass m_r, which supports a pulse modulator of mass m_{pu} and has an impact isolator of mass m_i beneath it. This entire layered stack is positioned on a rigid base of mass m_b.

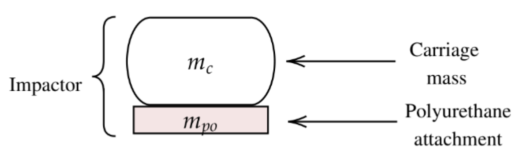

Fig. 2 Proposed impactor assembly.

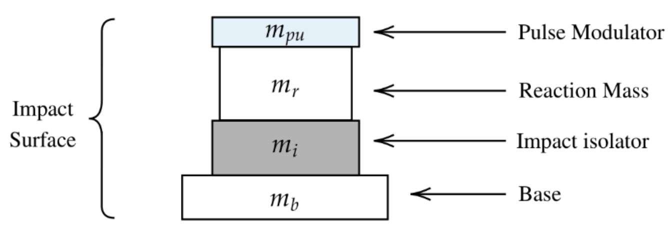

Fig. 3 Proposed impact surface with pulse modulator.

2.1. System Components and Layer Description

The impact surface in the proposed drop tower setup consists of a sequence of dynamically interacting stacked layers, each serving a specific role in absorbing energy and shaping the system's acceleration response.

1. Impactor:
    a. Carriage: The primary test mass undergoing free fall during the experiment. It initiates the impact event by imparting kinetic energy to the layered impact surface upon impact.
    b. Polyurethane attachment: Attached to the base of the carriage, this compliant layer protects the carriage mass and any sensitive components in it by deforming and damping upon impact, thereby contributing to the initial energy absorption during impact.

2. Impact Surface:
    a. Pulse modulator: Placed above the reaction mass, it modulates and tweaks the impact pulse to a desired shape. It helps in controlling the pulse duration and peak amplitude of the impact acceleration pulse.
    b. Reaction mass: Primary impact layer responsible for generating the impact.
    c. Impact isolator: Positioned under the reaction mass, it attenuates the impact shocks and prevents the force from transmitting to the base of the drop tower. It adds both stiffness and damping for energy dissipation, ensuring structural integrity of the base.
    d. Base: This is a fixed integrated component of a drop tower supporting the entire drop tower structure. It is assumed to be non-deformable and represents a grounded platform.

2.2. 2DOF Impact System

In a drop tower experiment the impactor freely falls on the impact surface. Upon contacting the reaction surface, the system transitions to a 2DOF interaction between the impactor and the layered reaction surface. Fig. 4 illustrates the complete 2DOF dynamic model of impact on a drop tower. The material properties (compliant behaviour) of the polyurethane attachment, pulse modulator, and impact isolator are captured using spring-damper elements. The parameters of the proposed drop tower system shown in the figure are:

1. Masses:
    a. Impactor mass: $m_1 = m_c + m_{po}$
    b. Impact surface mass $m_2 = m_r + m_{pu} + m_i$
2. Spring-damper layer representation: Each layer is modeled by spring-damper elements, described by a pair of parameters: stiffness (k) and damping coefficient (c)
    a. Polyurethane attachment: k_{po}, c_{po}
    b. Pulse modulator: k_{pu}, c_{pu}
3. Displacements:
    a. Carriage displacement: x_c
    b. Reaction mass displacement: x_r

Upon contact, the impactor mass m_1 interacts with the reaction surface of mass m_2 through a series spring-damper connections representing the polyurethane attachment and pulse modulator. The reaction mass is further supported by an additional spring-damper pair, representing the impact isolator. Additionally, all displacements downward are considered positive.

2.3. Impact Dynamics

From Fig. 4 at the first instance of impact, the spring-damper elements representing the polyurethane layer and pulse modulator form a series connection. This interaction produces equivalent constants k_{pp} and c_{pp}, representing the effective stiffness and damping in series, respectively. These are given by

$$\frac{1}{k_{pp}} = \frac{1}{k_{po}} + \frac{1}{k_{pu}} \tag{1}$$

$$\frac{1}{c_{pp}} = \frac{1}{c_{po}} + \frac{1}{c_{pu}} \tag{2}$$

Before impact, the impactor is in freefall and moves with a velocity v_c, while the reaction mass remains The velocity of the carriage right until it contacts the pulse modulator of the impact surface is given by

$$v_c = \dot{x}_c = \sqrt{2gh} \tag{3}$$

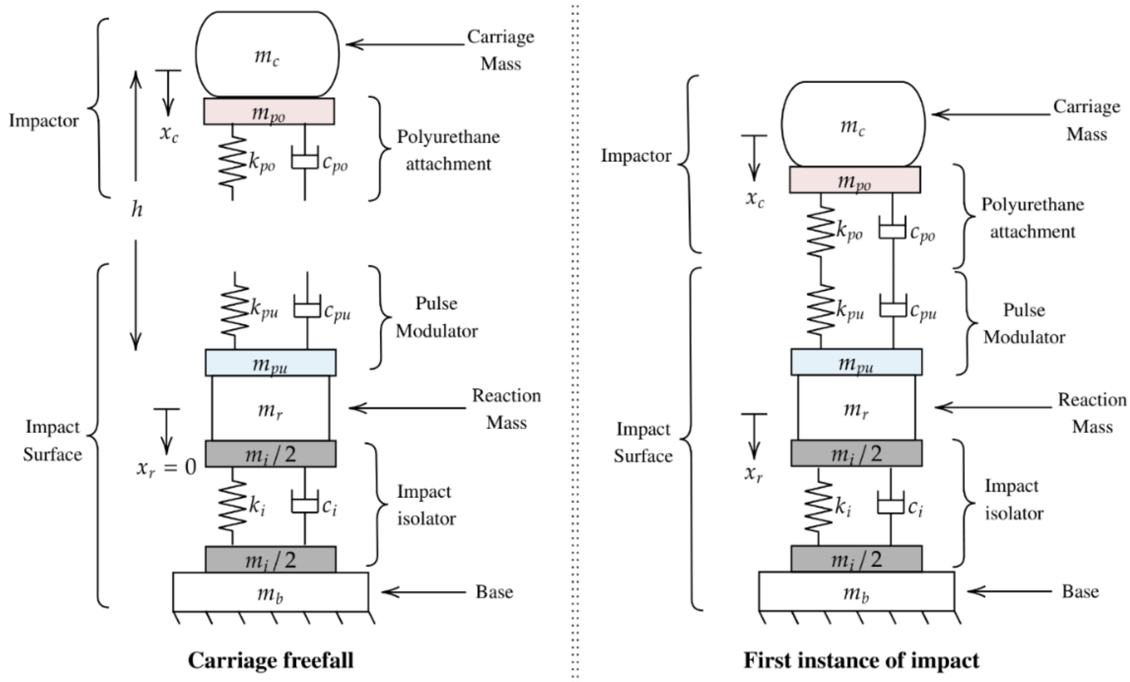

Fig. 4 Schematic of the 2DOF impact model in a drop tower setup.

where h is the drop height and g is the acceleration due to gravity. Fig. 5 presents the free-body diagrams of the impactor and impact surface, depicting the forces experienced at the instance of contact. Following the free-body diagram in Fig. 5, the dynamics of the masses during the contact phase (impact) can be represented by the differential equations

$$m_1\ddot{x}_c = m_1 g - k_{pp}(x_c - x_r) - c_{pp}(\dot{x}_c - \dot{x}_r) \tag{4}$$

$$m_2\ddot{x}_r = k_{pp}(x_c - x_r) + c_{pp}(\dot{x}_c - \dot{x}_r) - k_i x_r - c_i\dot{x}_r \tag{5}$$

where variables are as previously defined in Section 2.2. The system of Equations (4) and (5) in matrix form is

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{Bmatrix} \ddot{x}_c \\ \ddot{x}_r \end{Bmatrix} = \begin{bmatrix} -k_{pp} & k_{pp} \\ k_{pp} & -k_{pp} - k_i \end{bmatrix}\begin{Bmatrix} x_c \\ x_r \end{Bmatrix} + \begin{bmatrix} -c_{pp} & c_{pp} \\ c_{pp} & -c_{pp} - c_i \end{bmatrix}\begin{Bmatrix} x_c \\ x_r \end{Bmatrix} + \begin{bmatrix} m_1 \\ 0 \end{bmatrix} g \tag{6}$$

To simulate the impact acceleration response of the carriage, the system is expressed using first-order ordinary differential equations (ODEs) by defining $x_c = x_1$, and $x_r = x_3$. Upon substitution, the system of Equations (4) and (5) can be rewritten as a set of first-order ODEs

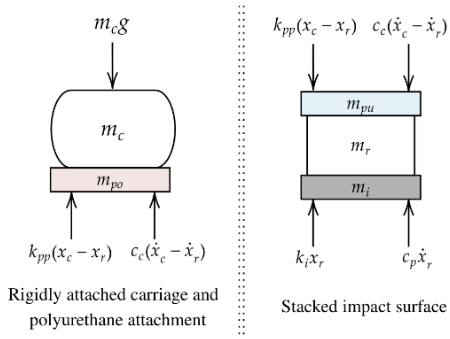

Fig. 5 Impactor and impact surface free-body diagrams at the instant of impact.

$$\dot{x}_c = \dot{x}_1 = x_2 \tag{7}$$

$$\dot{x}_r = \dot{x}_3 = x_4 \tag{8}$$

$$\dot{x}_2 = \frac{-k_{pp}x_1 + k_{pp}x_3 - c_{pp}x_2 + c_{pp}x_4 + m_1 g}{m_1} \tag{9}$$

$$\dot{x}_4 = \frac{-k_{pp}x_1 + (k_{pp} + k_i)x_3 - k_i x_5 + c_{pp}x_2 + (c_{pp} + c_i)x_4 + c_i x_6}{m_2} \tag{10}$$

3. PSO-Based Identification of Impact System Parameters

The characteristics of the impact acceleration experienced by the carriage is heavily influenced by the material properties of the layers in the impact model, mainly the pulse modulator. One conventional approach to selecting appropriate materials for each layer involves trial and error. However, this method is time-consuming and may not yield optimal results, especially when dealing with a complex layered system like the one considered in this study.

To overcome these limitations, PSO is employed in this study as a systematic method for identifying the spring and damping parameters of the 2DOF impact model. The PSO algorithm minimizes the difference between the simulated acceleration response and a predefined target pulse, offering a reliable approach to an automated iterative parameter estimation. PSO is renowned for its global search capabilities, making it particularly suitable for solving multi-parameter optimization problems, like those encountered in the proposed impact system. Therefore, in this context, PSO is tasked with identifying optimal stiffness and damping parameters for each layer, such that the simulated acceleration response of the 2DOF impact model matches the target acceleration pulse.

3.1. Target Acceleration Response

Upon impact, a falling object may experience several types of acceleration profiles, of which the half-sine is one of the most common [4]. It is characterized by a smooth half-sine curve that spans the duration of the impact event. The half-sine impact waveform is frequently used in shock testing and mechanical drop simulations due to its resemblance to real-world impact behaviour and its simplicity in analytical modeling.

In this study, the half-sine acceleration pulse defines the desired response experienced by the carriage and serves as the reference objective for the optimization routine. Mathematically, the half-sine acceleration pulse is given by

$$a_{target} = A \sin\left(\frac{\pi t}{T}\right), \qquad 0 \leq t \leq T \tag{11}$$

where A is the peak acceleration and T is the impact pulse duration. This half-sine waveform peaks at $t = \frac{T}{2}$ and returns to zero at the end of the event. Fig. 6 Half-sine target impact acceleration curve. presents the target acceleration curve, with a peak amplitude of -22g. The negative sign follows the sign convention used in this study, where upward acceleration is considered negative.

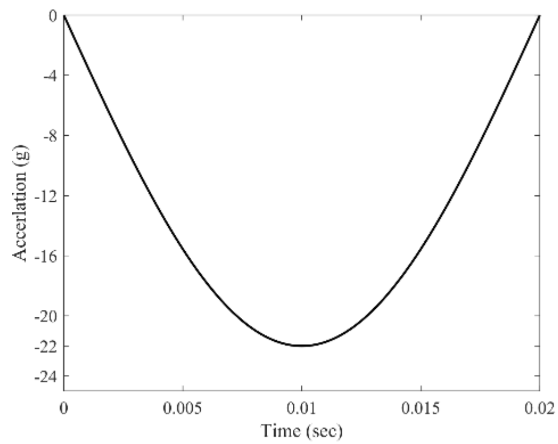

Fig. 6 Half-sine target impact acceleration curve.

3.2. PSO Formulation and Optimization Process

In this study, PSO is employed as the principal optimization algorithm to identify the optimal layer parameters that best replicate the desired impact response of the 2DOF impact model. PSO is a population-based algorithm inspired by the collective behaviour of swarms in nature. It is particularly effective for navigating multi-dimensional search spaces without relying on gradient or Hessian information. This makes it well-suited for tackling problems, such as the one presented in this study.

The implementation here follows standard PSO formulation as described in [5]. Detailed information about the PSO workings, its algorithm and implementation can be found in the original work by Kennedy and Eberhart [5]. In the PSO implementation, each particle's position in the swarm represents a candidate solution defined by a set of stiffness and damping parameters associated with the polyurethane attachment, pulse modulator, and impact isolator. Collectively, this parameter set forms a multidimensional search space. The full set of bulk stiffness and damping parameters in the search space includes

- k_{pp}, c_{pp}: Effective serial impact interface of the polyurethane attachment and pulse modulator. The polyurethanes are industrial materials which can be purchased and whose properties can be experimentally identified. Thereby, leaving only the pulse modulator ($k_{pu} - c_{pu}$ elements), which is the primary layer that is to be tuned to generate the impact pulse as desired.
- k_i, c_i: Impact isolator

Each of these parameter values uniquely influences the characteristics of the carriage's acceleration response. PSO seeks to optimize the solution by minimizing the error between the simulated and target acceleration profiles. By exploring and exploiting the search space, the iterative search continues until the algorithm converges, providing an optimal set of parameters that best replicates the desired acceleration response. The optimization process is driven by a cost function (objective function) given by the root-mean-square error (RMSE) to quantify the error between the simulated acceleration pulse and the target acceleration profile. By defining the carriage acceleration as $a_{sim} = \ddot{x}_c$, the RMSE metric is computed with a convergence tolerance set to $10^{-6}\ \mathrm{m/s^2}$. However, in a multi-variable optimization problem, different parameter sets (solutions) minimize the cost function below the set threshold, resulting in similar

$$J = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left[a_{sim}(i) - a_{target}(i)\right]^2} \tag{12}$$

RMSE values. Despite this, these distinct parameter sets can produce qualitatively different acceleration curves (different curve shapes). Equation (12) represents the cost function J, where a_{sim} is the simulated acceleration profile, a_{target} is the target half-sine pulse, and n is the total number of samples.

In this study, during the initial optimization attempts, the produced carriage acceleration curves peaked prematurely. To address this, a minimum damping level was enforced through a penalty term added to the cost function. This is done to penalize undesirable parameter combinations, discouraging deviations in the simulated curve from the target pulse shape. The resulting modified cost function is given by

$$J = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left[a_{sim}(i) - a_{target}(i)\right]^2} + \text{penalty} \tag{13}$$

where the introduced penalty term guides the optimization process away from the potentially undesirable parameter sets (solutions) that compromise the shape of the carriage's acceleration curve. The penalty is applied when the damping parameter c_{pp} falls below a predefined threshold as

$$\text{penalty} = \begin{cases} 1000, & \text{if } c_{pp} < 100 \\ 0, & \text{otherwise} \end{cases} \tag{14}$$

In addition to the penalty term in the cost function, to ensure physically realistic solutions within physically-meaningful limits, parameter bounds are imposed for each variable to reflect reasonable bulk stiffness and damping values for each layer. The bounds are set as follows:

- $k_{pp} \in [10^3,\ 5 \times 10^5]$ N/m ,
- $c_{pp} \in [10, 2 \times 10^4]$ Ns/m
- $k_i \in [10^3, 10^6]$ N/m
- $c_i \in [10, 10^4]$ Ns/m

Hence, the goal of PSO in this study is to identify optimal parameter values that minimize the cost function J under the defined threshold (10^{-6} m/s^2), while ensuring that the simulated acceleration response closely matches the target pulse. Fig. 7 presents a flowchart of the overall PSO-based parameter identification process employed in this study.

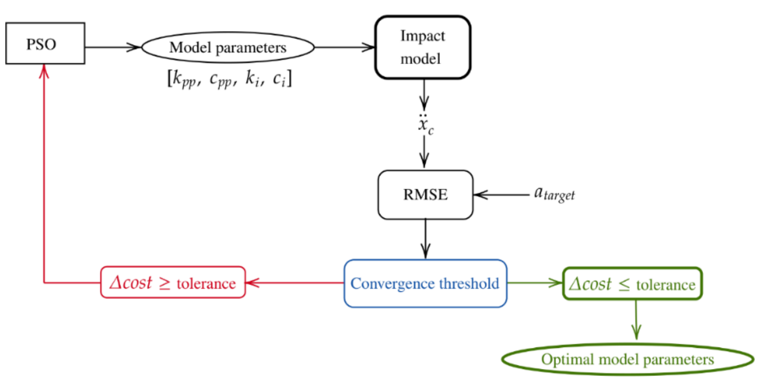

Fig. 7 PSO-based parameter identification of the impact model.

4. Results and Discussion

The objective was to shape the system response such that the mass was brought to rest quickly-thereby producing a sharp acceleration pulse, without bouncing or over-dampening the response. This required identifying parameter values that yield an impact response closely matching the target pulse in both magnitude and pulse duration.

For a fixed drop height of $h = 0.8$ m, target pulse duration of $T = 0.02$ s and target peak acceleration of $A = -22$g, the simulation was conducted using Equations (7)-(10) along with the PSO algorithm described in [5] for the parameter identification process. Additionally, the mass values were set as follows: carriage mass $m_c = 60$ kg, polyurethane attachment mass $m_{po} = 0.5$ kg, pulse modulator mass $m_{pu} = 0.5$ kg, reaction mass $m_r = 10$ kg, and isolator mass $m_i = 0.5$ kg.

Fig. 8 illustrates the results, illustrating a close match between the simulated acceleration response, using PSO-identified parameters, and the target half-sine pulse. The simulation accurately captures both the peak acceleration and pulse duration, demonstrating that the identified parameters effectively represent the underlying impact dynamics.

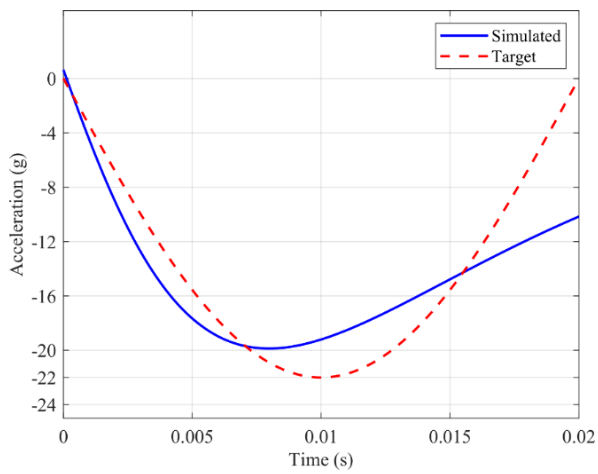

Fig. 8 Simulated acceleration response versus target half-sine pulse.

5. Conclusion

This study presented a 2DOF dynamic model of drop tower impact, representing the layered system with mass-spring-damper elements. Particle Swarm Optimization was used to identify optimal parameters that replicate a target half-sine acceleration pulse. The approach demonstrates a unique integration of physical modeling and optimization, enabling efficient and realistic drop test simulations.

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