Proceedings of the 2nd International Conference on Computer and Information Science and Technology (CIST'16)
Ottawa, Canada - May 11 - 12 2016
Paper No. 116

Collaborated Beamforming for Bidirectional Relay Networks in the Presence of Interference

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Abstract - In this paper, we obtain the beamforming vector as well as the users' transmit powers for an amplify-and-forward (AF)-based two-way relaying network in the presence of interference. To this end, the total transmit power consumed in the whole network is minimized subject to two constraints on the users' received signal-to-interference-plus-noise ratios (SINRs). Our technique is distinct from the published works in the sense that we jointly obtain the optimal relay beamforming weights and user transmit powers under the influence of interference, whereas the reported algorithms in the literature have not addressed the effect of interference. Numerical experiments confirm efficiency of the proposed approach.

Keywords: co-channel interference, distributed beamforming, power allocation, two-way relay network

1. Introduction

Two-way relaying methods (bidirectional communications) have received a lot of attention in recent years due to their capability of supporting communications in two directions with improved spectral efficiency [1, 2]. The advantages of two-way relaying networks can be achieved through collaborative (distributed) relay beamforming techniques, where a set of relaying nodes cooperate to build a beam towards the intended receiver [3]. In particular, collaborative beamforming techniques have been developed under the amplify-and-forward (AF) protocol.

The bidirectional relaying network might be employed in the presence of interference, where the interference may come from the other transmitter(s) using the same frequency band and the interference typically exists in heterogeneous network (HetNet). Specifically, all the proposed beamforming approaches in the literature; see [3, 4, 5, 6, 7] and the references therein, were limited to an idealistic assumption of the network, where there is no interference. Recognizing the fact that the interference can not be avoided in typical wireless networks for higher spectrum efficiency, this paper concentrates on the important and general scenario where some or all of the two users and relays are affected by interference, and we design optimal beamforming schemes for bidirectional networks. To the best of our knowledge, this is the first paper that addresses the problem of network beamforming for two-way relaying scenario under the influence of interference.

Obtaining the optimal user transmit powers and beamforming coefficients of the relays represents the main focus of this work. In particular, we aim to jointly design the optimal power allocation and beamforming technique such that the total transmit power consumed in the whole network is minimized, subject to two constraints on the quality of service (QoS) at the two users in terms of the signal-to-interference-plus-noise ratio (SINR). Our technique is distinct from the published works in the sense that we jointly obtain the optimal relay beamforming weights and user transmit powers under the influence of interference, whereas the reported algorithms in the literature have not addressed the effect of interference.

Notation: Throughout this paper, bold upper case symbols denote matrices and bold lower case symbols denote vectors. Subscripts $(\cdot)^*$, $(\cdot)^T$, and $(\cdot)^H$ stand for complex conjugate, transpose, and Hermitian, respectively. Also, we use $\mathscr{N}(\mu,\sigma^2)$ to denote complex Guassian distribution with mean μ and variance σ^2. $|z|$ and $\measuredangle z$ represent the amplitude and the phase of the complex number $z=|z|e^{\measuredangle z}$, respectively. Furthermore, $\mathrm{diag}(\mathbf{x})$ represents a diagonal matrix with the elements of the vector $\mathbf{x}$ as its diagonal entries. Also, $\|\mathbf{x}\|$ stands for the Euclidean norm of the vector $\mathbf{x}$. $\mathbf{I}_N$ is an $N\times N$ identity matrix. We use x_i

*This work was supported in part by National Sciences and Engineering Research Council (NSERC) and in part by the National Research Foundation of Korea (NRF) grant funded by the Korean government (MSIP) (2013R1A2A2A01067195).

and $[\mathbf{A}]_{i,j}$ to denote the i-th element of the vector $\mathbf{x}$ and the (i,j) element of the matrix $\mathbf{A}$, respectively. For two Hermitian matrices $\mathbf{A}$ and $\mathbf{B}$, $\mathbf{A} \succcurlyeq \mathbf{B}$ means that $\mathbf{A}-\mathbf{B}$ is positive semi-definite. $\lambda_{\max}(\mathbf{A})$ is used to represent the largest eigenvalue of matrix $\mathbf{A}$. We define $\delta(k) = \begin{cases} 1, & k=2 \\ 2, & k=1 \end{cases}$.

2. System Model

Here, we consider a bidirectional relaying network which consists of two users and some relays, where each node is equipped with a single antenna. Throughout the paper, U_1, U_2, and R_l stands for the first user, the second user, and the l-th relay for $l=1,\ldots,L$, respectively. It is also assumed that U_1, U_2, and relays are respectively affected by n_{A}, n_{B}, and n_{C} interferers. Throughout the paper, we use $\zeta_{1,j}$ $(j=1,\ldots,n_{\mathrm{A}})$, $\zeta_{2,m}$ $(m=1,\ldots,n_{\mathrm{B}})$, and $\zeta_{R,i}$ $(i=1,\ldots,n_{\mathrm{C}})$ to denote the interferers affecting U_1, U_2, and relays, respectively. In addition, $h_{l,k}$ represents the fading coefficient of the channel between U_k $(k=1,2)$ and R_l $(l=1,\ldots,L)$. It is assumed that that channel reciprocity holds for all user-relay links. Also, $g_{1,j}$, $g_{2,m}$, and $g_{R_l,i}$ stand for the channel coefficients from $\zeta_{1,j}$ $(j=1,\ldots,n_A)$ to U_1, from $\zeta_{2,m}$ $(m=1,\ldots,n_B)$ to U_2, and from $\zeta_{R,i}$ $(i=1,\ldots,n_C)$ to R_l $(l=1,\ldots,L)$, respectively. In this work, similar to related literature, it is assumed that the perfect channel knowledge is available. However, in [8], we have considered the scenario where channel coefficients are subject to estimation errors.

For such a system, the total transmission consists of two consecutive equal-duration time-slots. In the first time-slot, both users transmit their own signals to the relays. The resulting signal at the relay is then given by [8]

$$\mathbf{r}_R = \sqrt{P_1}\mathbf{h}_1 s_1 + \sqrt{P_2}\mathbf{h}_2 s_2 + \sum_{i=1}^{n_C} \sqrt{P_{\xi_{R,i}}}\mathbf{g}_{R,i}\xi_{R,i} + \vartheta_R, \tag{1}$$

where $\mathbf{r}_R$ represents an $L\times 1$ vector whose l-th entry stands for the signal received by R_l. Also, the $L\times 1$ vector $\mathbf{h}_k \triangleq [h_{1,k},\ h_{2,k},\ \ldots,\ h_{L,k}]^T$ denotes the channel coefficients between U_k and relays and the $L\times 1$ vector $\mathbf{g}_{R,i} \triangleq [g_{R_1,i},\ g_{R_2,i},\ \ldots,\ g_{R_L,i}]^T$ represents the channel coefficients from $\zeta_{R,i}$ to relays. In addition, the $L\times 1$ vector $\vartheta_R = [\vartheta_{R_1}, \vartheta_{R_2}, \ldots, \vartheta_{R_L}]^T$ is the additive white Gaussian noises at relays. Note that the elements of ϑ_R are independently and identically distributed (i.i.d.) as $\mathcal{N}(0,\sigma_\eta^2)$. In (1), s_1 and s_2 represent the transmitted signals by U_1 and U_2, respectively. It is assumed that $E\{|s_1|^2\} = E\{|s_2|^2\} = 1$. Also, $\xi_{R,i}$ denotes the interference signal generated by $\zeta_{R,i}$ $(i=1,\ldots,n_C)$, where $E\{|\xi_{R,i}|^2\} = 1$. In addition, we use P_1 and P_2 to represent the transmit powers of U_1 and U_2, respectively. Also, $P_{\xi_{R,i}}$ denotes the power of the interference signal $\zeta_{R,i}$ $(i=1,\ldots,n_C)$.

During the second time-slot, the l-th relay first multiplies its received signal by a complex weight w_l^*, and then transmits the obtained signal to both users. The transmitted signals by all relays can be represented as an $L\times 1$ vector

$$\mathbf{z} = \mathbf{W}^H \mathbf{r}_R, \tag{2}$$

where $\mathbf{W} \triangleq \mathrm{diag}(\mathbf{w})$ and $\mathbf{w} = [w_1,\ w_2,\ \ldots\ ,w_L]^T$. Finally, the signal y_k received by U_k, $k=1,2$, can be expressed as

$$y_k = \sqrt{P_k}\mathbf{h}_k^T\mathbf{W}^H\mathbf{h}_k s_k + \sqrt{P_{\delta(k)}}\mathbf{h}_k^T\mathbf{W}^H\mathbf{h}_{\delta(k)} s_{\delta(k)} + \sum_{i=1}^{n_C}\sqrt{P_{\xi_{R,i}}}\mathbf{h}_k^T\mathbf{W}^H\mathbf{g}_{R,i}\xi_{R,i} + \mathbf{g}_k^T\xi_k + \mathbf{h}_k^T\mathbf{W}^H\vartheta_R + v_k,\ k=1,2, \tag{3}$$

where the $n_A\times 1$ vector $\mathbf{g}_1 \triangleq [g_{1,1},\ g_{1,2},\ \ldots,\ g_{1,n_A}]^T$ stands for the channel coefficients from $\left\{\zeta_{1,j}\right\}_{j=1}^{n_A}$ to U_1 and the $n_B\times 1$ vector $\mathbf{g}_2 \triangleq [g_{2,1},\ g_{2,2},\ \ldots,\ g_{2,n_B}]^T$ denotes the channel coefficients from $\{\zeta_{2,m}\}_{m=1}^{n_B}$ to U_2. Also, v_k is the noise at $\mathrm{U}_k \sim \mathcal{N}(0,\sigma_{v_k}^2)$. Note that $\xi_1 \triangleq [\sqrt{P_{\xi_{1,1}}}\xi_{1,1}, \ldots, \sqrt{P_{\xi_{1,n_A}}}\xi_{1,n_A}]^T$, where $\xi_{1,j}$ and $P_{\xi_{1,j}}$ are the interference signal transmitted from $\zeta_{1,j}$ and its corresponding interference power, respectively. Similarly, $\xi_2 \triangleq [\sqrt{P_{\xi_{2,1}}}\xi_{2,1}, \ldots, \sqrt{P_{\xi_{2,n_B}}}\xi_{2,n_B}]^T$, where $\xi_{2,m}$ and $P_{\xi_{2,m}}$ represent the interference signal transmitted by $\zeta_{2,m}$ and its corresponding interference power, respectively. Moreover, we assume that $E\{|P_{\xi_{1,j}}|^2\} = E\{|P_{\xi_{2,m}}|^2\} = 1$ for all $j=1,\ldots,n_A$ and $m=1,\ldots,n_B$.

It can be seen from (3) that the received signal at U_k includes the corresponding self-signal earlier transmitted to the relays. Since U_k knows its own transmitted signal, s_k, it can subtracts the resulting self-interference term $\sqrt{P_k}\mathbf{h}_k^T\mathbf{W}^H\mathbf{h}_k s_k$ from the received signal. Accordingly, the remaining signal for U_k, $k=1,2$, is

$$\widetilde{y}_k = \widetilde{y}_{S,k} + \widetilde{y}_{I,k} + \widetilde{y}_{N,k},\ k=1,2, \tag{4}$$

where $\widetilde{y}_{S,k}$, $\widetilde{y}_{I,k}$, and $\widetilde{y}_{N,k}$ are respectively the intended signal component, interference component, and noise component of $\widetilde{y}_k$. Considering the fact that $\mathbf{W} = \text{diag}(\mathbf{w})$ is a diagonal matrix, we have $\mathbf{h}_k^T \mathbf{W}^H = \mathbf{w}^H \text{diag}(\mathbf{h}_k)$, $k = 1,2$. Therefore, $\widetilde{y}_{S,k}$, $\widetilde{y}_{I,k}$, and $\widetilde{y}_{N,k}$ can be expressed as

$$\widetilde{y}_{S,k} = \sqrt{P_{\delta(k)}}\mathbf{w}^H \mathbf{H}_k \mathbf{h}_{\delta(k)} s_{\delta(k)},\ k = 1,2 \tag{5}$$

$$\widetilde{y}_{I,k} = \sum_{i=1}^{n_C} \sqrt{P_{\xi_{R,i}}}\mathbf{w}^H \mathbf{H}_k \mathbf{g}_{R,i} \xi_{R,i} + \mathbf{g}_k^T \xi_k,\ k = 1,2 \tag{6}$$

$$\widetilde{y}_{N,k} = \mathbf{w}^H \mathbf{H}_k \vartheta_R + v_k,\ k = 1,2, \tag{7}$$

where $\mathbf{H}_k \triangleq \text{diag}(\mathbf{h}_k)$, $k = 1,2$.

3. Joint Power Allocation and Beamforming Design

Our problem is to minimize the total transmit power P_T of the whole network while the received SINRs at U_1 and U_2, which are respectively denoted as Γ_1 and Γ_2, are kept above pre-defined certain thresholds γ_1 and γ_2, respectively. Mathematically, we aim to solve the following optimization problem:

$$\begin{aligned} \min_{P_1,\, P_2,\, \mathbf{w}} \quad & P_{\text{T}} \\ \text{s. t.} \quad & \Gamma_1 \geq \gamma_1 \\ & \Gamma_2 \geq \gamma_2. \end{aligned} \tag{8}$$

The total transmit power P_T can be written as:

$$P_T = P_R + P_1 + P_2. \tag{9}$$

The relay transmit power $P_R \triangleq E\left\{|\mathbf{z}|^2\right\}$ is given by

$$P_R = \mathbf{w}^H \left(P_1 \mathbf{D}_1 + P_2 \mathbf{D}_2 + \sum_{i=1}^{n_C} P_{\xi_{R,i}} \mathbf{D}_{\xi_{R,i}} + \sigma_\eta^2 \mathbf{I}_L \right) \mathbf{w}, \tag{10}$$

where $\mathbf{D}_k \triangleq \mathbf{H}_k^H \mathbf{H}_k$, $k = 1,2$, and $\mathbf{D}_{\xi_{R,i}} \triangleq \text{diag}(\mathbf{g}_{R,i})^H \text{diag}(\mathbf{g}_{R,i})$, $i = 1,...,n_C$. The received SINR at U_k, $k = 1,2$, can be written as

$$\Gamma_{\text{k}} = \frac{P_{\delta(k)} \mathbf{w}^H \mathbf{f}\mathbf{f}^H \mathbf{w}}{\mathbf{w}^H \left(\sum\limits_{i=1}^{n_C} P_{\xi_{R,i}} \bar{\mathbf{f}}_{k,i} \bar{\mathbf{f}}_{k,i}^H + \sigma_\eta^2 \mathbf{H}_k \mathbf{H}_k^H \right) \mathbf{w} + \mathbf{g}_k^H \mathbf{P}_{\xi_k} \mathbf{g}_k + \sigma_{v_k}^2}, \tag{11}$$

where $\mathbf{f} \triangleq \mathbf{H}_k \mathbf{h}_{\delta(k)}$, $k = 1,2$, and $\bar{\mathbf{f}}_{ki} \triangleq \mathbf{H}_k \mathbf{g}_{R,i}$, $k = 1,2$ and $i = 1,...,n_C$. In addition, $\mathbf{P}_{\xi_1} \triangleq \text{diag}\left([P_{\xi_{1,1}},\ P_{\xi_{1,2}}, \ldots,\ P_{\xi_{1,n_A}}] \right)$ and $\mathbf{P}_{\xi_2} \triangleq \text{diag}\left([P_{\xi_{2,1}},\ P_{\xi_{2,2}}, \ldots,\ P_{\xi_{2,n_B}}] \right)$.

Using (9), (10), and (11), the optimization problem in (8) can be rewritten as

$$\begin{aligned} \min_{P_1, P_2, \mathbf{w}} \quad & P_1 + P_2 + \mathbf{w}^H \left(P_1 \mathbf{D}_1 + P_2 \mathbf{D}_2 + \sum_{i=1}^{n_C} P_{\xi_{R,i}} \mathbf{D}_{\xi_{R,i}} + \sigma_\eta^2 \mathbf{I}_L \right) \mathbf{w} \\ \text{s. t.} \quad & \frac{P_2 \mathbf{w}^H \mathbf{f}\mathbf{f}^H \mathbf{w}}{\mathbf{w}^H \left(\sum\limits_{i=1}^{n_C} P_{\xi_{R,i}} \bar{\mathbf{f}}_{1,i} \bar{\mathbf{f}}_{1,i}^H + \sigma_\eta^2 \mathbf{H}_1 \mathbf{H}_1^H \right) \mathbf{w} + \mathbf{g}_1^H \mathbf{P}_{\xi_1} \mathbf{g}_1 + \sigma_{v_1}^2} \geq \gamma_1 \\ & \frac{P_1 \mathbf{w}^H \mathbf{f}\mathbf{f}^H \mathbf{w}}{\mathbf{w}^H \left(\sum\limits_{i=1}^{n_C} P_{\xi_{R,i}} \bar{\mathbf{f}}_{2,i} \bar{\mathbf{f}}_{2,i}^H + \sigma_\eta^2 \mathbf{H}_2 \mathbf{H}_2^H \right) \mathbf{w} + \mathbf{g}_2^H \mathbf{P}_{\xi_2} \mathbf{g}_2 + \sigma_{v_2}^2} \geq \gamma_2. \end{aligned} \tag{12}$$

In this optimization problem, the design parameters include $\mathbf{w}$ as well as P_1 and P_2. This makes finding the global minimum of (12) very challenging. In order to find the global solution to (12), we will develop a method which consists of a two-dimensional search over a sufficiently fine grid that covers all possible values of P_1 and P_2 and an second-order cone-programming (SOCP) problem over $\mathbf{w}$. To that end, we first obtain the feasibility set of (12) in the following.

We rewrite the constraints of the optimization problem (12) as follows

$$\mathbf{w}^H\left(P_{\delta(k)}\mathbf{f}\mathbf{f}^H-\gamma_k\Big(\sum_{i=1}^{n_C}P_{\xi_{R,i}}\bar{\mathbf{f}}_{k,i}\bar{\mathbf{f}}_{k,i}^H+\sigma_\eta^2\mathbf{H}_k\mathbf{H}_k^H\Big)\right)\mathbf{w}\geq\gamma_k\left(\mathbf{g}_k^H\mathbf{P}_{\xi_k}\mathbf{g}_k+\sigma_{v_k}^2\right),k=1,2. \tag{13}$$

It is seen that the optimization problem (12) is infeasible if for a given value of $P_{\delta(k)}$, $k=1$ or 2, the matrix $\left(P_{\delta(k)}\mathbf{f}\mathbf{f}^H-\gamma_k\left(\sum_{i=1}^{n_c}P_{\xi_{R,i}}\bar{\mathbf{f}}_{k,i}\bar{\mathbf{f}}_{k,i}^H+\sigma_\eta^2\mathbf{H}_k\mathbf{H}_k^H\right)\right)$ is negative definite for $k=1$ or 2. In other words, the optimization problem (12) is feasible if and only if for any given pair of P_1 and P_2, the matrixes $\left(P_{\delta(k)}\mathbf{f}\mathbf{f}^H-\gamma_k\left(\sum_{i=1}^{n_C}P_{\xi_{R,i}}\bar{\mathbf{f}}_{k,i}\bar{\mathbf{f}}_{k,i}^H+\sigma_\eta^2\mathbf{H}_k\mathbf{H}_k^H\right)\right)$ are non-negative definite for both $k=1$ and 2. Defining

$$\mathbf{R}_k\triangleq\left(\sum_{i=1}^{n_C}P_{\xi_{R,i}}\bar{\mathbf{f}}_{k,i}\bar{\mathbf{f}}_{k,i}^H+\sigma_\eta^2\mathbf{H}_k\mathbf{H}_k^H\right),\ k=1,2, \tag{14}$$

which can be shown to be a positive definite matrix, for $k=1$ and 2, we have

$$P_{\delta(k)}\mathbf{f}\mathbf{f}^H-\gamma_k\mathbf{R}_k=\mathbf{R}_k^{\frac{1}{2}}\left(P_{\delta(k)}\mathbf{R}_k^{\frac{-1}{2}}\mathbf{f}\mathbf{f}^H\mathbf{R}_k^{\frac{-1}{2}}-\gamma_k\mathbf{I}_L\right)\mathbf{R}_k^{\frac{1}{2}}, \tag{15}$$

where $\left(P_{\delta(k)}\mathbf{R}_k^{\frac{-1}{2}}\mathbf{f}\mathbf{f}^H\mathbf{R}_k^{\frac{-1}{2}}-\gamma_k\mathbf{I}_L\right)$, $k=1,2$, can be shown to be a non-negative definite matrix. We conclude that the optimization problem (12) is feasible if and only if for any given pair of P_1 and P_2

$$\lambda_{\max}\left(P_{\delta(k)}\mathbf{R}_k^{\frac{-1}{2}}\mathbf{f}\mathbf{f}^H\mathbf{R}_k^{\frac{-1}{2}}-\gamma_k\mathbf{I}_L\right)\geq 0,\ k=1,2. \tag{16}$$

Since $\mathbf{R}_k^{\frac{-1}{2}}\mathbf{f}\mathbf{f}^H\mathbf{R}_k^{\frac{-1}{2}}$ is a rank-one matrix, the largest eigen-value of the matrix $\left(P_{\delta(k)}\mathbf{R}_k^{\frac{-1}{2}}\mathbf{f}\mathbf{f}^H\mathbf{R}_k^{\frac{-1}{2}}-\gamma_k\mathbf{I}_L\right)$ is $P_{\delta(k)}\mathbf{f}^H\mathbf{R}_k^{-1}\mathbf{f}-\gamma_k$. Therefore, (16) can be equivalently rewritten as

$$P_{\delta(k)}\geq\frac{\gamma_k}{\mathbf{f}^H\mathbf{R}_k^{-1}\mathbf{f}}=\frac{\gamma_k}{\mathbf{f}^H\left(\sum\limits_{i=1}^{n_C}P_{\xi_{R,i}}\bar{\mathbf{f}}_{k,i}\bar{\mathbf{f}}_{k,i}^H+\sigma_\eta^2\mathbf{H}_k\mathbf{H}_k^H\right)^{-1}\mathbf{f}},\ k=1,2. \tag{17}$$

It is worth mentioning that the two constraints in (17) are necessary and sufficient conditions for (12) to be feasible.

We now convert the joint power allocation and relay beamforming problem in (12) into two nested problems: i) over P_1 and P_2, and ii) over $\mathbf{w}$. To this end, without loss of optimality, we rewrite the objective function of (12) as follows

$$\min_{P_1,P_2}\Big(P_1+P_2\Big)+\min_{\mathbf{w}}\mathbf{w}^H\Big(P_1\mathbf{D}_1+P_2\mathbf{D}_2+\sum_{i=1}^{n_C}P_{\xi_{R,i}}\mathbf{D}_{\xi_{R,i}}+\sigma_\eta^2\mathbf{I}_L\Big)\mathbf{w}. \tag{18}$$

We also add the two conditions of (17) as the additional constraints, without loss of optimality. Then we have the following problem

$$\min_{P_1,P_2} \quad P_1+P_2 \quad + \quad \min_{\mathbf{w}} \mathbf{w}^H \left(P_1\mathbf{D}_1+P_2\mathbf{D}_2+\sum_{i=1}^{n_C} P_{\xi_{R,i}}\mathbf{D}_{\xi_{R,i}}+\sigma_\eta^2\mathbf{I}_L \right)\mathbf{w} \tag{19a}$$

$$\text{s. t.} \quad \frac{\mathbf{w}^H\mathbf{f}\mathbf{f}^H\mathbf{w}}{\mathbf{w}^H\left(\sum_{i=1}^{n_C} P_{\xi_{R,i}}\bar{\mathbf{f}}_{1,i}\bar{\mathbf{f}}_{1,i}^H+\sigma_\eta^2\mathbf{H}_1\mathbf{H}_1^H\right)\mathbf{w}+\mathbf{g}_1^H\mathbf{P}_{\xi_1}\mathbf{g}_1+\sigma_{\nu_1}^2} \geq \frac{\gamma_1}{P_2} \tag{19b}$$

$$\frac{\mathbf{w}^H\mathbf{f}\mathbf{f}^H\mathbf{w}}{\mathbf{w}^H\left(\sum_{i=1}^{n_C} P_{\xi_{R,i}}\bar{\mathbf{f}}_{2,i}\bar{\mathbf{f}}_{2,i}^H+\sigma_\eta^2\mathbf{H}_2\mathbf{H}_2^H\right)\mathbf{w}+\mathbf{g}_2^H\mathbf{P}_{\xi_2}\mathbf{g}_2+\sigma_{\nu_2}^2} \geq \frac{\gamma_2}{P_1} \tag{19c}$$

$$P_1 \geq \frac{\gamma_2}{\mathbf{f}^H\left(\sum_{i=1}^{n_C} P_{\xi_{R,i}}\bar{\mathbf{f}}_{2,i}\bar{\mathbf{f}}_{2,i}^H+\sigma_\eta^2\mathbf{H}_2\mathbf{H}_2^H\right)^{-1}\mathbf{f}} \tag{19d}$$

$$P_2 \geq \frac{\gamma_1}{\mathbf{f}^H\left(\sum_{i=1}^{n_C} P_{\xi_{R,i}}\bar{\mathbf{f}}_{1,i}\bar{\mathbf{f}}_{1,i}^H+\sigma_\eta^2\mathbf{H}_1\mathbf{H}_1^H\right)^{-1}\mathbf{f}}. \tag{19e}$$

Specifically, for any fixed pair of P_1 and P_2 satisfying (19d) and (19e), the inner minimization can be rewritten as

$$\min_{\mathbf{w}} \quad \mathbf{w}^H \left(P_1\mathbf{D}_1+P_2\mathbf{D}_2+\sum_{i=1}^{n_C} P_{\xi_{R,i}}\mathbf{D}_{\xi_{R,i}}+\sigma_\eta^2\mathbf{I}_L \right)\mathbf{w} \tag{20a}$$

$$\text{s. t.} \quad \left|\mathbf{w}^H\mathbf{f}\right| \geq \sqrt{\frac{\gamma_1}{P_2}\left(\mathbf{w}^H\left(\sum_{i=1}^{n_C} P_{\xi_{R,i}}\bar{\mathbf{f}}_{1,i}\bar{\mathbf{f}}_{1,i}^H+\sigma_\eta^2\mathbf{H}_1\mathbf{H}_1^H\right)\mathbf{w}+\mathbf{g}_1^H\mathbf{P}_{\xi_1}\mathbf{g}_1+\sigma_{\nu_1}^2\right)} \tag{20b}$$

$$\left|\mathbf{w}^H\mathbf{f}\right| \geq \sqrt{\frac{\gamma_2}{P_1}\left(\mathbf{w}^H\left(\sum_{i=1}^{n_C} P_{\xi_{R,i}}\bar{\mathbf{f}}_{2,i}\bar{\mathbf{f}}_{2,i}^H+\sigma_\eta^2\mathbf{H}_2\mathbf{H}_2^H\right)\mathbf{w}+\mathbf{g}_2^H\mathbf{P}_{\xi_2}\mathbf{g}_2+\sigma_{\nu_2}^2\right)}. \tag{20c}$$

It is easy to see from (20a)–(20c), if the optimal vector $\mathbf{w}_{opt}$ is replaced with $e^{j\theta}\mathbf{w}_{opt}$, for any value of phase rotation θ, the optimization problem (20a)–(20c) will not change. This means that the new vector $e^{j\theta}\mathbf{w}_{opt}$ is also a solution to (20a)–(20c). Hence, without loss of optimality, we can rotate the phase of $\mathbf{w}$ such that $\mathbf{w}^H\mathbf{f}$ is a real and positive number. Doing so, we equivalently arrive at the following optimization problem

$$\min_{\mathbf{w}} \quad \mathbf{w}^H \left(P_1\mathbf{D}_1+P_2\mathbf{D}_2+\sum_{i=1}^{n_C} P_{\xi_{R,i}}\mathbf{D}_{\xi_{R,i}}+\sigma_\eta^2\mathbf{I}_L \right)\mathbf{w} \tag{21a}$$

$$\text{s. t.} \quad \Re e\left(\mathbf{w}^H\mathbf{f}\right) \geq \sqrt{\frac{\gamma_1}{P_2}\left(\mathbf{w}^H\left(\sum_{i=1}^{n_C} P_{\xi_{R,i}}\bar{\mathbf{f}}_{1,i}\bar{\mathbf{f}}_{1,i}^H+\sigma_\eta^2\mathbf{H}_1\mathbf{H}_1^H\right)\mathbf{w}+\mathbf{g}_1^H\mathbf{P}_{\xi_1}\mathbf{g}_1+\sigma_{\nu_1}^2\right)} \tag{21b}$$

$$\Re e\left(\mathbf{w}^H\mathbf{f}\right) \geq \sqrt{\frac{\gamma_2}{P_1}\left(\mathbf{w}^H\left(\sum_{i=1}^{n_C} P_{\xi_{R,i}}\bar{\mathbf{f}}_{2,i}\bar{\mathbf{f}}_{2,i}^H+\sigma_\eta^2\mathbf{H}_2\mathbf{H}_2^H\right)\mathbf{w}+\mathbf{g}_2^H\mathbf{P}_{\xi_2}\mathbf{g}_2+\sigma_{\nu_2}^2\right)} \tag{21c}$$

$$\Im m\left(\mathbf{w}^H\mathbf{f}\right) = 0. \tag{21d}$$

One can easily show that for each feasible pair of (P_1,P_2), the optimization problem (21a)–(21d) is an SOCP problem [9] which can be efficiently solved using interior point methods [10]. To find the solution to (19a)–(19e), adopting a similar approach to the one employed in [7], we first discretize the feasibility set of (19a)–(19e) into a fine grid of (P_1,P_2) space. Then, for each vertex of the obtained grid, the SOCP problem (21a)–(21d) is solved and the corresponding total transmit power P_T is calculated. The optimal solution to (19a)–(19e) corresponds to the pair of (P_1,P_2) that results in the smallest P_T value.

4. Simulation Results

The goal of this section is to demonstrate the performance of the proposed beamformer and analyze its properties through a set of Monte Carlo simulations. Throughout all simulations, we have considered a bidirectional network consisting of $L = 10$ relay nodes. The results that we show are obtained by averaging the corresponding quantity over 1000 independent simulation runs. In each simulation run, all the channel vectors $\mathbf{h}_1$ and $\mathbf{h}_2$ as well as the interferers CSI vectors $\hat{\mathbf{g}}_1$, $\hat{\mathbf{g}}_2$, and $\hat{\mathbf{g}}_{R,i}$ $(i = 1, ..., n_C)$, are generated as complex zero-mean Gaussian random vectors with unit variances. Also, the noise power at the relays and at the two users is assumed to be one. For convenience, it is assumed that the users have the same minimum required SINR, i.e. $\gamma \triangleq \gamma_1 = \gamma_2$.

First, we study the effect of the interference on the two-way relaying network. Here, we denote the power of interferers affecting U_1, U_2, and the relays by the sequences $\mathbf{p}_{\zeta_1} \triangleq \left[P_{\xi_{1,1}}, ..., P_{\xi_{1,n_A}}\right]$, $\mathbf{p}_{\zeta_2} \triangleq \left[P_{\xi_{2,1}}, ..., P_{\xi_{2,n_B}}\right]$, and $\mathbf{p}_{\zeta_R} \triangleq \left[P_{\xi_{R,1}}, ..., P_{\xi_{R,n_C}}\right]$, respectively. In Figs. 1 and 2, we consider equal-power interference case under two scenarios: (1) when interferers only affect U_1, i.e. $n_A = 2$, $n_B = 0$, and $n_C = 0$; (2) when interferers affect both U_1 and U_2, i.e. $n_A = 1$, $n_B = 1$, and $n_C = 0$. To investigate the effect of the interference power, we keep all other parameters fixed and only change the power of interferers ($P_{\xi_{1,1}}$ and $P_{\xi_{1,2}}$ in the first scenario; and $P_{\xi_{1,1}}$ and $P_{\xi_{2,1}}$ in the second scenario) from 2 to 5. Specifically, the comparison for the minimum total transmit power P_T against γ is shown in Fig. 1. It is seen that the total transmit power P_T becomes worse with increasing the interferer powers. Fig. 2 illustrates the $\frac{P_1+P_2}{P_R}$ ratio versus SINR. One can see that increasing the interferer powers from 2 to 5 leads to higher P_R consumption.

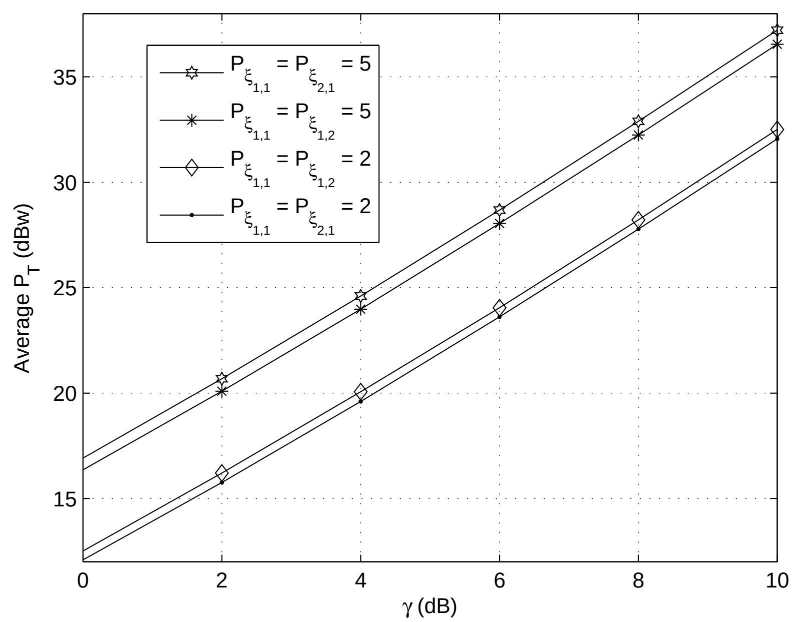

Fig. 1: The average minimum total transmit power P_T against $\gamma = \gamma_1 = \gamma_2$ of a bidirectional network under following scenarios: (A) when interferers only affect U_1; and (B) when interferers affect U_1 and U_2.

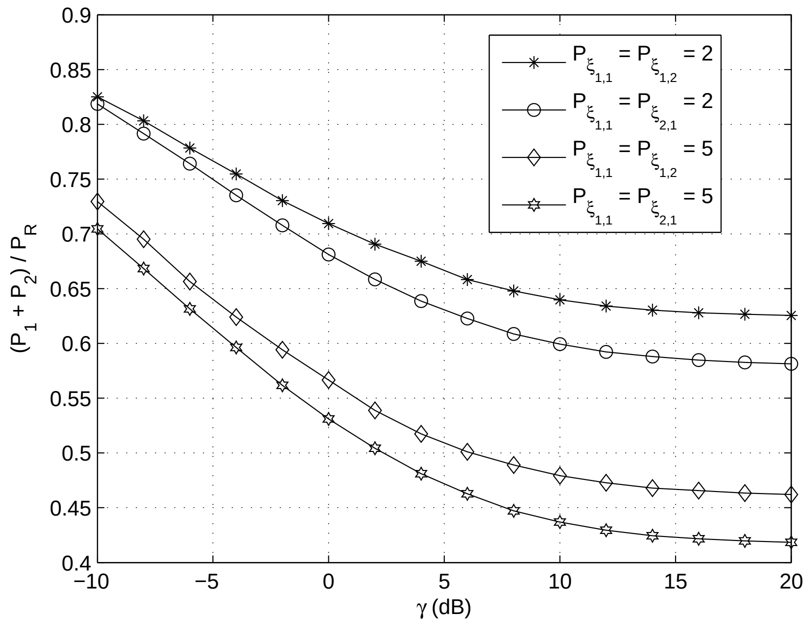

Fig. 2: $\frac{P_1+P_2}{P_R}$ ratio versus $\gamma = \gamma_1 = \gamma_2$ for a bidirectional network under following scenarios: (A) when interferers only affect U_1; and (B) when interferers affect U_1 and U_2.

Next, we compare the performance of our proposed method with that of the proposed scheme in [4]; see Fig. 3. In the related simulations, we set $n_A = 0$, $n_B = 0$, $\mathbf{p}_{\zeta_R} = [10, 10]$. As can be seen from Fig. 3, in the presence of interference, our scheme significantly outperforms the proposed method in [4], which proves the efficiency of our proposed method.

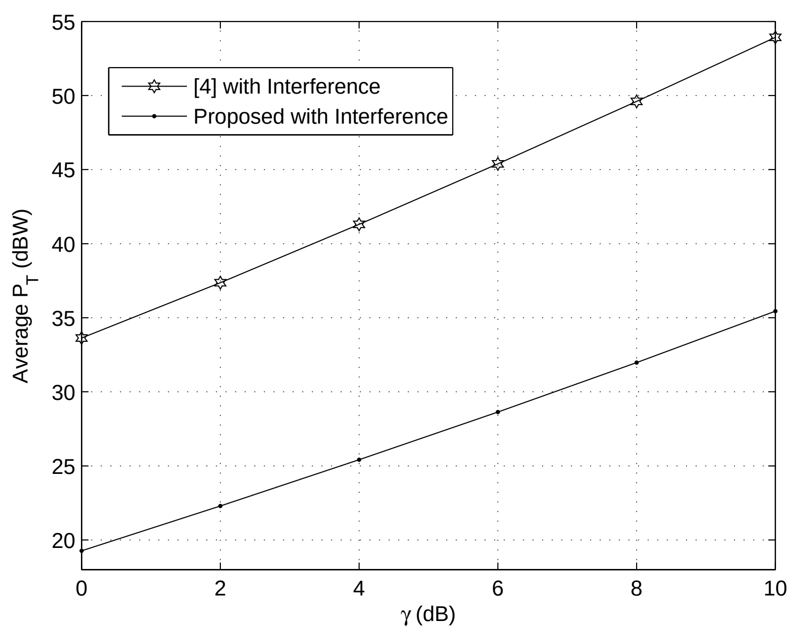

Fig. 3: Performance comparison of the proposed algorithm with that of proposed in [4].

5. Conclusion

In this paper, we considered an AF-based bidirectional relaying network in which two users intend to exchange information with the help of relays. Assuming that both users and all relays are equipped with a single antenna, we studied the problem of optimal beamforming and power allocation in the presence of interference. In contrast to previously reported works, which are limited to the idealistic assumption of interference-free environment, we developed an optimal beamforming and power allocation scheme for bidirectional relay networks in the presence of interference. Specifically, we calculated the relay beamforming vector as well as user transmit powers via minimizing the total transmit power consumed in the whole network subject to two constraints on the users received SINRs.

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