

## Nonlinear Optimal Control – New Framework and New Perspective

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### Extended Abstract

Most physical systems and real dynamics are recognized to be nonlinear and require often nonlinear control synthesis and design. In particular, the nonlinear optimal control problem has been a popular subject for a number of researchers. Simple techniques based on local linearization methods and more complex ones, including state-dependent-Riccati equation, nonlinear-matrix-inequality- and frozen-Riccati-equation-based methods, have been discussed. Their main drawbacks are related to stability properties and implementation issues. But, recent possible avenues of research investigating Lyapunov-function (LF) based control methods and Kronecker product (KP) algebra are renewing interest in the nonlinear optimal control. Indeed, the KP algebra has played an important role in recent research dealing with control analysis and design.

Based on the fact that state space polynomial models can represent most of nonlinearities, using the matrix KP and the vector power series, more (closely and fairly) than the linearized models, this modeling resembles the classical linearization, but with a difference that the order of truncation of the decomposition of the actual dynamics representation is higher and the modeling errors are much more reduced for such complex system dynamics. Moreover, various physical systems are counted to be well-posed using state-dependent polynomials; from mass-spring systems with softening/hardening springs, artificial pneumatic muscles to flight engine devices, *etc.*

This research addresses the design and the stability analysis of nonlinear optimal control framework. Through this study, we investigate new concepts for nonlinear control based on, not only, the LF and KP tools, but also, using the strong framework of linear matrix inequality (LMI). The controller is developed using the well-known optimality conditions by converting the nonlinear Hamilton-Jacobi-Bellman equations into a set of algebraic equations using the KP algebra. The new proposed design is also taking into account the stability conditions of the optimal state-feedback. This work represents a great opportunity to highlight global and local asymptotic stability of polynomial systems, and also, it turns out to estimate the domain of attraction (DA) for such nonlinear dynamics, extending existing studies where the stability and DA estimate (*i.e.*, computation of the largest estimate of the domain of attraction) features can be cast as convex feasibility and optimization problems (*e.g.*, eigenvalue problem) solved using the LMI formalism. New mathematical background is now rendered, combining the existing complete square matrix representation with a new formulation of the so-called complete rectangular matrix representation, to make the computation of the control gains and the stabilization via optimal control for nonlinear systems possible (implementable).

Numerical results and experimental illustrations have been developed to validate the feasibility and implementation of such techniques. The extension of these results for more relevant problems such as finite-horizon optimal control and  $H_\infty$  problems are now possible using the new proposed framework.