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Modelling CO_2 Emissions between China and Taiwan Using Lotka-Volterra Equations

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Abstract - To our knowledge, this study explores the correlation between the CO_2 emissions of regions with close industrial interdependence by using Lyapunov functions for the first time. Analytical results indicate that China's increasing imports of semi-finished products from Taiwan have led to a decline in China's fossil fuel consumption, subsequently lowering its CO_2 emission growth rate. The industrial dependence between Taiwan and China markedly affected CO_2 emissions of the latter. The analysis results of Lyapunov function reveal that the future trajectory of CO_2 emissions from China and Taiwan will fail to reach stable equilibrium points. Their increased CO_2 emissions cannot be absorbed entirely by plants through photosynthesis, implying that the excessive CO_2 emissions from Taiwan and China will create ecological imbalances and worsen continuously. Any change in such CO_2 emissions from Taiwan and China depends on how their industries are related. In terms of forecasting accuracy, the proposed Lotka-Volterra model can accurately predict future CO_2 emissions from China and Taiwan since this model considers both the previous CO_2 emissions in one region, and the economic competition and cooperation between these two regions. Results of this study significantly contribute to the efforts of governmental authorities in promulgating environmental policies.

Keywords**:** CO2 emissions; global warming; Lotka-Volterra model; Lyapunov functions; equilibrium analysis

1. Introduction

To our knowledge, this study analyzes for the first time the relationship between the cumulative carbon dioxide (CO2) emissions of China and Taiwan by using the Lotka-Volterra model and Lyapunov functions. CO2 emissions contributed the most to global warming during the previous deglaciation period. Considerable efforts have been made to forecast trends in CO2 emissions of a region [1, 2]. However, previous studies have still not thoroughly investigated CO2 emission trends of two regions jointly. Whether a certain co-competition relationship exists between them has not been discussed as well. A priority concern is to understand how China influences the CO2 emission trends of regions with whom China is economically interdependent, because of China's profound impact on environmental protection policies worldwide. As is well known, China and Taiwan have a close industrial relationship in terms of economic development and export dependence. In particular, the economic development and history of China closely resemble those of Taiwan in certain aspects. Additionally, Taiwan's export dependence on mainland China is approximately 40.24%. As China's main export products are semi-finished products and raw materials, most of them are first manufactured in Taiwan, processed in China, and then exported worldwide. Such high interdependence and manufacturing of similar export products closely link China with Taiwan in terms of CO2 emissions. Owing to the industrial interdependence of China and Taiwan, this study explores for the first time the correlation between CO2 emissions of these two regions by using the Lotka-Volterra model and Lyapunov functions.

Capable of describing how two species interact with each other, the Lotka-Volterra model is extensively adopted to evaluate the dynamic competitive relationships between two groups in various fields. Related studies have demonstrated that the model can accurately predict how two groups compete or cooperate with each other [3, 4]. By using the Lotka-Volterra model, Tsai et al. [5] examine the market competitiveness between liquid crystal display panels in mobile and desktop computers. This study focuses mainly on how the CO2 emissions of China and Taiwan are related by observing the parameter estimates of our proposed Lotka–Volterra model. Additionally, the long-term equilibrium stability in CO2 emissions between China and Taiwan is determined using Lyapunov function analysis. Whether the CO2 emissions of China and Taiwan will reach a stable equilibrium status is also examined. If the cumulative Chinese and Taiwanese CO2 emissions fail to reach a long-term stable equilibrium, this instability clearly indicates that plants via photosynthesis cannot completely absorb the increase in CO2 from industrial development. Taiwan and China will continuously increase their CO2 emissions excessively in the future. Moreover, the relation between emissions of the two regions may vary as their future economies and trade dependency change.

The contributions of this work can be summarized in the following three aspects: model specification, equilibrium analysis, and stability analysis of long-term equilibrium. Regarding model specification, previous studies still lack a thorough investigation of CO2 emission trends of two regions jointly as well as a discussion on whether a certain co-competition relationship exists between them. This study is the first to utilize Lotka-Volterra model to investigate the CO2 emissions among industrially dependent regions. To estimate the CO2 emissions volume in China, the CO2 emission volume in Taiwan is also be considered, and vice versa. Therefore, our proposed Lotka-Volterra model can precisely predict CO2 emission in China and Taiwan. Referring to stability analysis of long-term equilibrium, this study employs Lyapunov functions to prove the instability of the long-term equilibrium in CO2 emission between China and Taiwan. Previous studies have failed to quantitatively demonstrate the long-term CO2 emission trends. Our instability result is the first to clearly indicate that plants via photosynthesis cannot completely absorb the increase in CO2 from industrial development in the cross-strait region. The stability examination of CO2 emission across different industrially dependent regions is highly promising for use in drafting worldwide ecological economics policies. As for the applicability of this approach to environmental protection, the predictive results show that China, the major suppliers of global products, have greater growth rate of CO2 emissions than Taiwan in the long run.

2. Sample and Data

The research sample in this work focuses on the CO_2 emissions in China and Taiwan, since China is the leading region of CO_2 emissions worldwide over the past decade, and Taiwan is industrially related to China. This research collects CO_2 emissions of China and Taiwan from 1980 to 2010 through the International Energy Agency. This work divides the sample period into two parts, training and test periods. Training period ranges from 1980 to 2007 and test period is from 2008 to 2010. This work constructs the Lotka-Volterra model by using the data of a training period sample. Moreover, in addition to estimating the model parameters and analyzing the model goodness, this work utilizes the test period sample to forecast the predicted values in the test period and examine the prediction accuracy of our proposed Lotka-Volterra model.

3. Methodology

3.1. The Lotka-Volterra Model

The Lotka-Volterra model, also known as the predator–prey ecological model, is composed of two first-order, non-linear, differential equations [6, 7]. The Lotka-Volterra is written as:

$$\frac{dX}{dt} = (a_1 - b_1 X - c_1 Y)X = a_1 X - b_1 X^2 - c_1 XY \tag{1}$$

$$\frac{dY}{dt} = (a_2 - b_2 Y - c_2 X)Y = a_2 Y - b_2 Y^2 - c_2 YX \tag{2}$$

where $\frac{dX}{dt}$ and $\frac{dY}{dt}$ are the CO_2 emissions of China and Taiwan at year t. X and Y are cumulative CO_2 emissions in China and Taiwan from the starting point to year t. Three parameters are used to explain the growth relations of CO_2 emission between these two regions. Negative b_i in Eqs. (1) and (2) suggest that the future growth rate of CO_2 emission increases with cumulative CO_2 emitted in the past. XY and YX show the interaction terms between China and Taiwan. The coefficient c_i represents the impact of CO_2 emission interactions between China and Taiwan on the growth of CO_2 emissions. The use of time-series data in our sample involves converting the Lotka-Volterra model, which is continuous, into a discrete time version. Leslie [9] proved that Eqs. (1) and (2) could be converted into discrete time equations as Eqs. (3) and (4).

$$X(t+1) = \frac{\alpha_1 X(t)}{1 + \beta_1 X(t) + \gamma_1 Y(t)}, \tag{3}$$

$$Y(t+1)=\frac{\alpha_2 Y(t)}{1+\beta_2 Y(t)+\gamma_2 X(t)}. \tag{4}$$

In Eqs. (3) and (4), α and β are self-growing and self-restraining parameters, respectively for each region's CO_2 emissions, and γ represents the magnitude of the effects of the CO_2 emissions of one region on those of the other. The relationships between parameters α_i, β_i, γ_i in discrete-time Lotka-Volterra (Eqs. (3) and (4)) and continuous Lotka-Volterra equations a_i, b_i, and c_i (Eqs. (1) and (2)) can be written as equations (5) - (7):

$$a_i = \ln \alpha_i \tag{5}$$

$$b_i = \frac{\beta_i a_i}{\alpha_i - 1} = \frac{\beta_i \ln \alpha_i}{\alpha_i - 1} \tag{6}$$

$$c_i = \gamma_i \frac{b_i}{\beta_i} = \frac{\gamma_i}{\beta_i}\frac{\beta_i \ln \alpha_i}{\alpha_i - 1} = \frac{\gamma_i \ln \alpha_i}{\alpha_i - 1} \tag{7}$$

3.2. Equilibrium Analysis

The equilibrium point of Lotka-Volterra model can be expressed as Eq. (8):

$$\frac{dX}{dt}=0 \text{ and } \frac{dY}{dt}=0 \tag{8}$$

Since CO2 emissions of China and Taiwan cannot be zero, that is, or. The equilibrium solution of Eqs. (1) and (2) can be obtained as shown in Eq. (9).

$$X=\frac{a_1 - c_1 Y}{b_1} \text{ and } Y=\frac{a_2 - c_2 X}{b_2} \tag{9}$$

The two lines, $dX/dt = 0$ and $dY/dt = 0$, cross each other, implying an equilibrium point. When the two straight lines in Eq. (9) intersect in the first quadrant, the two orbits of CO_2 emission are possible to comply with the stability conditions and reach equilibrium; otherwise, no equilibrium exists. This equilibrium point indicates that two CO_2 emissions can coexist without dynamic changes. Additionally, the stability of the equilibrium point depends on parameter values in the proposed Lotka-Volterra model. By using those estimated parameters, this work employs the following two approaches to examine whether the final equilibrium point of both China's and Taiwan's CO_2 emission are stable under the fluctuating environment: Eigenvalues of Jacobian matrix at equilibrium point and Lyapunov function.

3.3. Prediction Accuracy Analysis

This research divides the data sample of CO_2 emissions of China and Taiwan from 1980 to 2010 into the training period (from 1980 to 2007) and the test period (from 2008 to 2010) so as to estimate the model parameter with data of training period and predict the CO_2 emissions in the test period. With regard to the prediction accuracy of the model, previous studies use the method of "mean absolute percentage errors" (MAPEs). This work also examines the accuracy of our proposed Lotka-Volterra model with MAPEs. The smaller the MAPE is, the greater ability this Lotka-Volterra model contains to explain the data. The calculation formula of MAPE is shown Eq. (10),

$$MAPE=\frac{1}{n}\sum_{k=1}^{n}\frac{\left|x(k)-\hat{x}(k)\right|}{x(k)}\times 100\,\% \tag{10}$$

where n, $x(k)$ and $\hat{x}(k)$ represent the number of observations in the test period, the actual value at the kth time and the predicted value based on the Lotka-Volterra model at the kth time, respectively.

4. Empirical Results and Analysis

4.1. Empirical Results of the Lotka-Volterra Model

Table 1 summarizes the estimation results of CO_2 emissions of China and Taiwan obtained by using the Lotka–Volterra model. This table reveals that although the coefficient γ_2, which describes the effect of China's CO_2 emission on Taiwan, is negatively yet insignificant, the coefficient γ_1, which illustrates the effect of Taiwan's CO_2 emissions on China, is positive and significant. Thus, although Taiwan has reduced the CO_2 emissions of China, Taiwan's CO_2 emissions have not been obviously affected by China. This phenomenon may be a result of China's increasing imports of semi-finished products from Taiwan that have reduced the growth rate of fossil fuel consumption in China. Therefore, China's CO_2 emissions have obviously reduced. Above results suggest that the interdependence of the two regions' overseas trade has markedly affected China's CO_2 emissions. The possible reason is that Taiwan's tax regulations provide Taiwanese enterprises with a preferential tariff rate if they install environmentally friendly equipment. Although increased exports from Taiwan to China have strengthened industrial production in the former, the Taiwanese equipment with environment-protecting functions allows for mass production without significant consumption of fossil fuels and, ultimately, no significant increase in CO_2 emissions in Taiwan. Thus, no evidence suggests that Taiwan's CO_2 emissions growth rate is improved by the industrial interdependence between Taiwan and China. Notably, the coefficient of self-interaction terms in China is negative and significant. A greater amount of CO_2 emitted by China in the past implies a larger amount of CO_2 emitted in the future. From 1980 to 2010, as China's industrial infrastructures developed and living standards rose, China relied more on industrial production powered by electricity, generated mainly by burning fossil fuels. Moreover, Chinese energy consuming industries, including petrochemical and petroleum refinery industries, accelerate their CO_2 emissions. Thus, the fossil fuels consumption of China grew substantially and elevated the region's CO_2 emissions.

Table 1: summarizes the estimation results of CO_2 emissions of China and Taiwan obtained by using the Lotka–Volterra model.

	China			Taiwan	
	Parameter	*t*- statistics		Parameter	*t*- statistics
α_1	1.1473	76.92[c]	α_2	1.1722	83.94[c]
β_1	-0.1657×10^{-4}	1.75[a]	β_2	1.0649×10^{-4}	2.09[b]
γ_1	1.1487×10^{-4}	2.18[b]	γ_2	-0.1055×10^{-4}	1.15
R^2	0.9995		R^2	0.9995	
Adjusted R^2	0.9994		Adjusted R^2	0.9995	
	a_i	b_i	c_i	Equilibrium point (million metric tons)	
Taiwan	0.1589	0.9830×10^{-4}	-0.0970×10^{-4}	2,351.8804	
China	0.1374	-0.1150×10^{-4}	1.0720×10^{-4}	7,417.3059	

[a]p <0.1, [b]p <0.05, [c]p<0.01

4.2. Equilibrium Analysis Using Eigenvalues of Jacobian Matrix and Lyapunov Function

Equilibrium analysis is performed by selecting the average values of parameters a_i, b_i, and c_i to estimate the stability of the equilibrium point for the CO_2 emissions of Taiwan and China. The equilibrium point is calculated at 2,351 and 7,417 million metric tons for Taiwan and China, respectively. The Jacobian matrix is calculated as $\boldsymbol{A} = \begin{bmatrix} -0.2311 & 0.0229 \\ -0.7948 & 0.1146 \end{bmatrix}$. Eigenvalues of the Jacobian matrix at the equilibrium point are -0.1673 and 0.0510. Analytical results of the eigenvalues indicate that the orbits computed using our estimated Lotka–Volterra equations fail to satisfy the stability conditions, indicating the instability of the equilibrium point for Chinese and Taiwanese CO_2 emissions.

As for the Lyapunov stability equilibrium analysis, the equilibrium point is transformed into the new origin on the coordinate axis. Via coordinate transformation, the points calculated on the trajectory of CO_2 emissions, 5,824.2344 and 13,568.5415 from Taiwan and China, in the new coordinate system are transformed as $z = (u, v)$ = (3,472.3541, 6,151.2356). Incorporating the Jacobian into the Lyapunov function, we further calculate Lyapunov function $V(z^{'}) = -2.8464\times10^{8} < 0$, and the first-order differential of the Lyapunov function $\dot{V}(z^{'}) = -2.1803\times10^{8} < 0$, demonstrating that the trajectory fails to satisfy the stable conditions of its equilibrium point. The future trajectory will not converge to the equilibrium points 2,351 and 7,417 million metric tons in Taiwan and China, respectively. Since the equilibrium status is unstable, the CO_2 emissions from both China and Taiwan increase infinitely. The increase of CO_2 emissions from industries cannot be completely absorbed by plants through photosynthesis, implying that the excessive CO_2 emissions from Taiwan and China will cause ecological imbalances and worsen continuously. Both China and Taiwan increasingly raise their CO_2 emissions over time, explaining why CO_2 emissions from these regions will grow continuously in the future. This continuous growth could be attributed to the gradual transfer in the energy-consuming industries in China from the coastal to the inland areas owing to sustained economic activity, ultimately leading to a continuous rise in the CO_2 emissions.

4.3. Prediction Accuracy of Lotka–Volterra Model

This study considers CO_2 emissions from 2008 to 2010 in our sample for predictions. Figures 1 and 2 compare the actual and predicted CO_2 emissions in China and Taiwan, respectively. The predicted and actual values in the test period are also compared to obtain the MAPE in order to evaluate the prediction accuracy of the proposed Lotka–Volterra model. A smaller MAPE implies a closer proximity of the predicted values to the actual values; otherwise, the prediction accuracy is irrational. Table 2 lists the cumulative CO_2 emissions predicted by using the Lotka–Volterra model. The MAPEs of Chinese and Taiwanese CO_2 emissions predicted by using the Lotka–Volterra model are 4.77% and 3.81% for the training sample and 2.8% and 0.86% for the test sample, respectively, both of which are less than 5%. This finding suggests that both of the predictions are excellent according to the criteria of Martin and Witt [10]. The proposed Lotka–Volterra model can thus accurately predict the trend of CO_2 emissions.

Table 2. MAPEs of the Predicted Cumulative CO2 Emissions for China and Taiwan.

	Training Period	Test Period
China	4.77％	2.80％
Taiwan	3.81％	0.86％

This study analyzed the CO_2 emissions generated from fossil fuels expended in China and Taiwan by using the Lotka–Volterra model. Analytical results indicate that the growth rate of China's CO_2 emissions reduced with an increase in the CO_2 emissions in Taiwan. This phenomenon may be owing to that China's imports of semi-finished products and raw materials from Taiwan cause a decline in manufacturing in China, thus decreasing the growth rate of fossil fuels expended in China. Conversely, no evidence suggests that China's CO_2 emissions significantly affect the growth rate of Taiwan's CO_2 emissions. Since Taiwanese enterprises have equipment with environmental protection features, the region's mass production of semi-finished products, which are exported to China, does not lead to a substantial increase in the growth rate of Taiwan's CO_2 emissions.

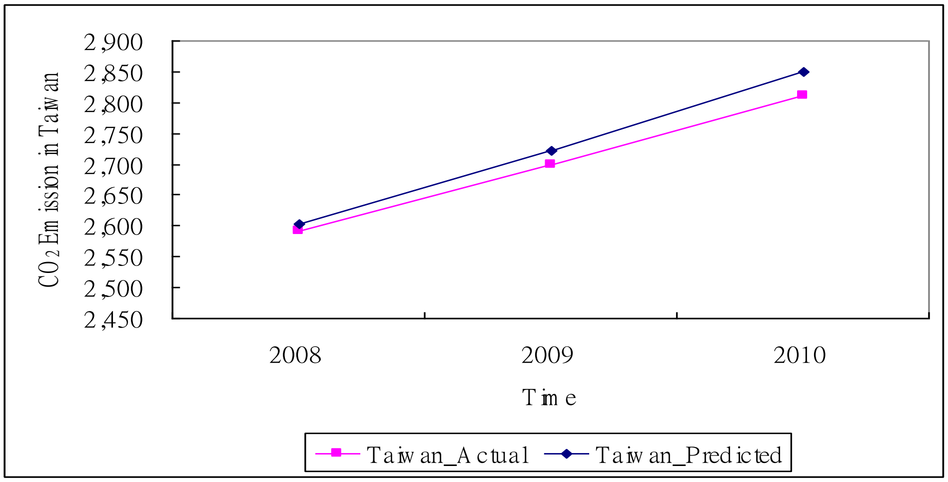

Fig. 1: Comparison of actual and predicted CO2 emissions for Taiwan (Unit: million metric tons).

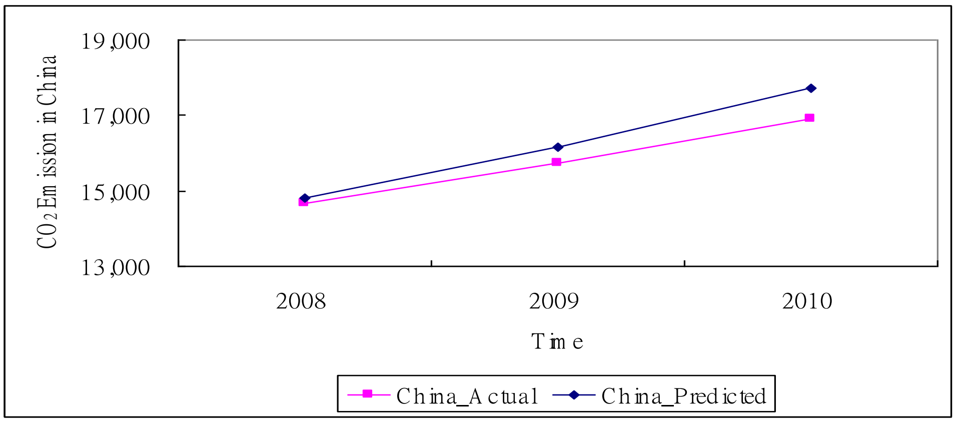

Fig. 2: Comparison of actual and predicted CO2 emissions for China (Unit: million metric tons).

5. Conclusions

Moreover, the sustained growth of domestic energy consumption has substantially increased CO_2 emissions in China. From 1980 to 2010, the Chinese government focused on raising individual living standards, as evidenced by the installation of air-conditioners, heaters, and other appliances in many households. Those households relied more on electricity, which was generated mainly by burning fossil fuels, ultimately leading to substantial growth in the consumption of fossil fuels and accelerated CO_2 emissions. The future trend of CO_2 emissions in China, the world's largest CO_2 emitter, is attracting global attention. This study demonstrates an obvious relationship between the trends of CO_2 emissions of China and Taiwan, implying many interrelated factors between the trends of CO_2 emissions of different regions. Capable of accurately identifying the transnational factors of CO_2 emissions between different regions and regions, the proposed Lotka–Volterra model is a highly effective means of forecasting transnational CO_2 emissions. Our proposed equilibrium and forecasting analysis provides a highly effective means of predicting the long-term CO_2 emissions in other production regions, including America, Europe, Africa, Asia and Oceania areas. Governments can extend our approach to devise environmental protections.

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Conflicts of Interest

The authors declare no conflict of interest.

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