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Analytical Approach to the Bond Strength of Plain Round Bars under Lateral Tensile Stresses

Xue Zhang[1, 2], Changming Lv[1], Zhimin Wu[1], Jinping Ou[1]
[1]State Key Laboratory of Coastal and Offshore Engineering, Dalian University of Technology
[2]State Key Laboratory of Structural Analysis for Industrial Equipment, Dalian University of Technology
No. 2 Linggong Road, Dalian, P.R. China
xuezhang@dlut.edu.cn; charminglvv@sina.com; wuzhimin@dlut.edu.cn; oujinping@dlut.edu.cn

Abstract - The bond strength of the plain round bar plays important role in the assessment of historical buildings and has been widely investigated. This paper presents an analytical approach to the bond strength of plain round bars embedded in concrete subjected to lateral tensile stresses. Based on the theory of elasticity, the contact condition of the bar/concrete interface is discussed, and an analytical solution for the ultimate bond strength is further derived. When the shrinkage of concrete, the geometric and mechanical parameters of bars and concrete are given, the analytical solution can be used to evaluate the ultimate bond strength of plain round bars subjected to uniaxial and biaxial lateral tensile stresses. 153 pull-out specimens are used to verify the analytical solution. The results show that, the analytical predictions are in good agreement with the experimental values.

Keywords**:** concrete; plain round bar; bond strength; lateral tensile stresses

1. Introduction

Recently, more attention has been paid to the rehabilitation of historical buildings reinforced with plain round bars [1]. One of the main purposes of the assessment of these old buildings is to verify whether the anchorage between concrete and plain round bars is reliable or not. Therefore, many efforts have been made to investigate the bond strength of plain bars through experimental and analytical approaches [2-4]. It should be noticed that, in present theoretical analyses, only the frictional mechanism is considered and the contribution of chemical adhesion to the pull-out resistance is neglected. As a result, when these solutions are applied to plain round bars embedded in concrete, the pull-out load or bond strength is highly dependent on the lateral stress applied on the bar/concrete interface [3,4]. To model the actual stresses in beam-column joints or bridge decks with orthogonal reinforcement, the effect of lateral compressive and tensile stresses on the bond behavior of plain bars was investigated [5-9]. Experimental results showed that, the bond strength of plain bars increases with an increase in lateral compressive stress [5-7,9], while the lateral tensile stress decreases the bond strength significantly [8].

Aforementioned literatures [5-9] explicitly revealed the positive and negative effects of lateral compressive and tensile stresses on the bond behavior of plain bar, and concluded empirical equations of the bond strength under complex lateral stress. Moreover, the analytical solution for the bond strength shown in literature [7] was beneficial to comprehend the bond mechanism of plain round bars subjected to lateral pressure. So far, however, the analytical investigation into the bond behavior of plain bars under lateral tensile stresses is still insufficient. Therefore, it is essential to further establish an analytical approach to the ultimate bond strength of plain round bars considering the influence of lateral tensile stresses. In this paper, the contact condition of the bar/concrete interface is discussed, and an analytical solution for the ultimate bond strength is derived and verified with experimental results of 153 pull-out specimens.

2. Contact Condition of the Bar/Concrete Interface

When a plain round bar is pulled from a concrete matrix, the chemical adhesion and frictional action between the bar and concrete prevent the bar from sliding. During the first stage, the bond resistance is attributed mainly to the chemical adhesion [10,11]. When a relative slip between the bar and concrete occurs, the frictional action becomes a primary factor [2]. When a lateral tensile stress is applied, however, the tensile stress exerts a negative effect on the bond behavior by weakening the compaction of the bar/concrete bond interface. With the increase of the lateral tensile stress, the bar/concrete bond interface is gradually looser and becomes instable. Once the lateral tensile stress exceeds a certain value, the bar and concrete separate from each other and the anchorage fails completely. Therefore, the key in analyzing the bond strength of plain bars under lateral tensile stresses is the contact condition of the bar/concrete interface.

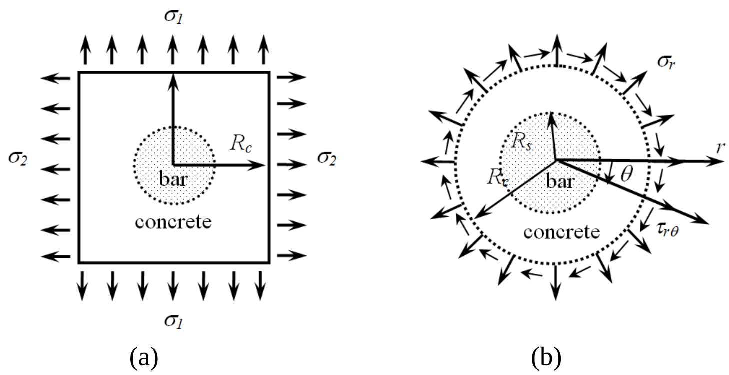


Fig. 1: Planar stress state of the plain round bar embedded in concrete subjected to lateral tensile stresses: (a) under principle stress axes; (b) under polar axes.

When analyzing the mechanical behavior of reinforced concrete members, it is easy to determine the planar stress state around the quadrate concrete element shown in Fig. 1(a) by transforming the arbitrary boundary stresses into the principle axes of stresses. When the polar coordinates are adopted, as shown in Fig. 1(b), the stress and displacement of the concrete element can be expressed by functions of σ_1, σ_2, R_c, R_s and θ. Considering the bond mechanism, the displacement of concrete at the bar/concrete interface is contributed by the effect of concrete shrinkage and lateral tensile stresses, which are discussed respectively as follows.

According to the theory of elasticity [12], when concrete is subjected to a shrinkage strain ε_0, the radial displacement of concrete at the bar/concrete bond interface, u_{rc1}, can be expressed as

$$u_{rc1} = \frac{1}{E_c'}\left[-(1+\nu_c')\frac{A_1}{R_s} + 2B_1(1-\nu_c')R_s\right] \tag{1}$$

Where, the coefficients A_1 and B_1 are given by

$$A_1 = -\frac{R_c^2 R_s^2 E_c' E_s'}{E_c'(R_c^2 - R_s^2)(1-\nu_s') + E_s'(R_c^2 + R_s^2 - R_s^2\nu_c' + R_c^2\nu_c')}\varepsilon_0 \tag{2a}$$

$$B_1 = \frac{R_s^2 E_c' E_s'}{2[E_c'(R_c^2 - R_s^2)(1-\nu_s') + E_s'(R_c^2 + R_s^2 - R_s^2\nu_c' + R_c^2\nu_c')]}\varepsilon_0 \tag{2b}$$

$$E_c' = E_c/(1-\nu_c^2) \tag{2c}$$

$$E_s' = E_s/(1-\nu_s^2) \tag{2d}$$

$$\nu_c' = \nu_c/(1-\nu_c) \tag{2e}$$

$$\nu_s' = \nu_s/(1-\nu_s) \tag{2f}$$

With E_c, E_s, ν_c, and ν_s being the Young's moduli and Poisson's ratios of concrete and the plain round bar, and R_s and R_c are the radii of the plain round bar and concrete element, respectively. When a combination of lateral tensile stresses (σ_1, σ_2) is applied, the radical displacement of concrete at the bar/concrete bond interface, u_{rc2}, can be expressed as

$$u_{rc2} = \frac{1}{E_c'}\left[-(1+\nu_c')\frac{A_2}{R_s} + 2B_2(1-\nu_c')R_s\right] + \frac{2\cos 2\theta}{E}[-2\nu_c' C_2 R_s^3 - (1+\nu_c')D_2 R_s + 2F_2\frac{1}{R_s} + (1+\nu_c')G_2\frac{1}{R_s^3}] \tag{3}$$

Where, the coefficients A_2 to G_2 are given by

$$A_2 = -\frac{R_c^{\,2} R_s^{\,2}}{R_c^{\,2} - R_s^{\,2}} \left(\frac{\sigma_1 + \sigma_2}{2} \right) \tag{4a}$$

$$B_2 = \frac{1}{2} \frac{R_c^{\,2}}{R_c^{\,2} - R_s^{\,2}} \left(\frac{\sigma_1 + \sigma_2}{2} \right) \tag{4b}$$

$$C_2 = -\left(\frac{\sigma_2 - \sigma_1}{2}\right) \frac{1}{R_c^2} \frac{K_1 n^2 (n^2 - 1)}{K_0 - 4K_1 n^2 + 6K_1 n^4 - 4K_2 n^6 + K_3 n^8} \tag{4c}$$

$$D_2 = -\frac{1}{2}\left(\frac{\sigma_2 - \sigma_1}{2}\right) \frac{K_0 + 3K_1 n^4 - 4K_2 n^6}{K_0 - 4K_1 n^2 + 6K_1 n^4 - 4K_2 n^6 + K_3 n^8} \tag{4d}$$

$$F_2 = -\left(\frac{\sigma_2 - \sigma_1}{2}\right) R_s^2 \frac{-K_1 + K_3 n^6}{K_0 - 4K_1 n^2 + 6K_1 n^4 - 4K_2 n^6 + K_3 n^8} \tag{4e}$$

$$G_2 = \frac{1}{2}\left(\frac{\sigma_2 - \sigma_1}{2}\right) R_s^4 \frac{-K_1 + K_3 n^4}{K_0 - 4K_1 n^2 + 6K_1 n^4 - 4K_2 n^6 + K_3 n^8} \tag{4f}$$

$$K_0 = (\nu_c^2 - 2\nu_c - 3)m^2 + 2(\nu_s + \nu_c - \nu_s \nu_c - 5)m + (\nu_s^2 - 2\nu_s - 3) \tag{4g}$$

$$K_1 = (\nu_c^2 + 2\nu_c - 1)m^2 - 2(\nu_s - \nu_c + \nu_s \nu_c - 1)m + (\nu_s^2 - 2\nu_s - 3) \tag{4h}$$

$$K_2 = (\nu_c^2 + 3)m^2 - 2(\nu_s \nu_c - \nu_c)m + (\nu_s^2 - 2\nu_s - 3) \tag{4i}$$

$$K_3 = (\nu_c^2 - 2\nu_c - 3)m^2 + 2(\nu_s + \nu_c - \nu_s \nu_c - 3)m + (\nu_s^2 - 2\nu_s - 3) \tag{4j}$$

Where $m=E_s'/E_c'$, n= R_s/R_c, and specifying that $\sigma_2 \geq \sigma_1 \geq 0$. It is obvious that, when the sum of u_{rc1} and u_{rc2} is equal to zero, i.e.

$$u_{rc1} + u_{rc2} = 0 \tag{5}$$

The bar/concrete bond interface is separated from each other. Thus, the combination of critical lateral tensile stresses $(\sigma_{1cr}, \sigma_{2cr})$ can be determined from Eq. (5).

3. Prediction of the ultimate bond strength

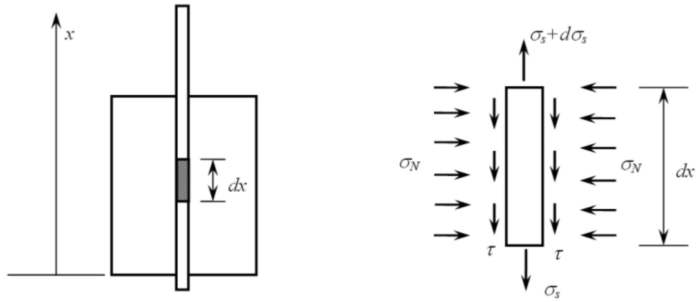
When chemical adhesion is neglected, the frictional bond resistance is dependent on the pressure applied to the plain round bar, σ_N, and the coefficient of friction μ. The normal compressive stress σ_N is induced by the shrinkage strain of concrete, lateral tensile stresses, and radial contraction of the steel bar due to the tension σ_s. Therefore, the normal compressive stress σ_N can be expressed as

$$\sigma_N = \frac{(1 - \mathrm{n}^2)E_s'\varepsilon_0 - m(\sigma_1 + \sigma_2) - (1 - n^2)\nu_s'\sigma_s}{(1 - n^2)(1 - \nu_s') + m[(1 + \nu_c') - n^2(1 - \nu_c')]} + \frac{2(\sigma_2 - \sigma_1)(K_5 - 3K_4 n^4 + 2K_6 n^6)}{K_0 - 4K_1 n^2 + 6K_1 n^4 - 4K_2 n^6 + K_3 n^8} \cos 2\theta \tag{6a}$$

$$K_4 = (1 + \nu_c)m^2 - (1 + \nu_s)m \tag{6b}$$

$$K_5 = (1 + \nu_c)m^2 + (3 - \nu_s)m \tag{6c}$$

$$K_6 = (3 + \nu_c)m^2 - (3 + \nu_s)m \tag{6d}$$



(a) (b)

Fig. 2: Schematic of stress analysis for pull-out process: (a) the pull-out specimen; (b) the element of the bar.

As shown in Fig. 2, considering the balance between the axial tensile stress in the steel bar and the bond stress caused by the normal compressive stress σ_N, and applying the boundary conditions of σ_s=0 for x=0 and $\sigma_s=P_u/\pi R_s^2$ for $x=l_d$ gives

$$\tau_{uf}=\frac{[(1-n^2)E_s'\varepsilon_0-m(\sigma_1+\sigma_2)]R_s}{2(1-n^2)\nu_s'l_d}\left\{1-\exp\left[\frac{-2(1-n^2)\nu_s'\mu l_d}{(1-n^2)(1-\nu_s')R_s+m[(1+\nu_c')-n^2(1-\nu_c')]R_s}\right]\right\} \tag{7}$$

Where, τ_{uf}, μ and l_d are the ultimate bond strength due to the frictional effect, the coefficient of friction and embedded length of the plain round bar, respectively. It should be noted that Eq. (7) is valid only for $\sigma_1<\sigma_{1cr}$ and $\sigma_2<\sigma_{2cr}$.

Abrams [2] concluded that the chemical adhesion accounts for approximately 50-60% of the maximum bond capacity of plain bars. Therefore, chemical adhesion should be taken into account. For simplification, the ultimate bond strength τ_u is regarded as the sum of the bond strength due to the frictional effect, τ_{uf}, and the chemical adhesion, τ_a [13], yields:

$$\tau_u=\tau_a+\tau_{uf} \tag{8}$$

4. Experimental verification

To verify the derived analytical solution for the ultimate bond strength, 153 pull-out specimens were tested under uniaxial and biaxial lateral tensile stresses, which were included in the experimental program reported in literature [8]. Plain round bars with different diameters and concrete with different compressive strengths were used. An embedded length l_d equaling to five times the nominal diameter of the bar was adopted, which ensured that the distribution of the bond stress is basically uniform along the embedded length [14]. The lateral tensile stresses p_1 and p_2 were specified by the level of the uniaxial lateral tensile stress p_2 and the stress ratio p_2/p_1. p_2 varied from 0 to 0.8f_t with an increment of 0.1f_t, where f_t is the tensile strength of concrete. The stress ratio p_2/p_1 was equal to 0, 1, 2, and 3, respectively. More detailed parameters of the experiment can be found in literature [8].

According to CEB-FIP 2010 [15], the shrinkage strain of concrete ε_0 is dependent mainly on the compressive strength of concrete, curing age, and environmental humidity, and is calculated for each specimen. With the properties of materials and dimensions of the specimen known, the critical lateral tensile stresses can be determined by Eq. (5). It should be noticed that, the displacement due to lateral tensile stresses is not uniform on the bar/concrete interface, and the maximum value of displacement appears when θ equals to zero. It is indicated that, under the lateral tensile stresses shown in Fig. 1, the bar/concrete interface parallel to σ_1 is inclined to separate with the increase of σ_2.

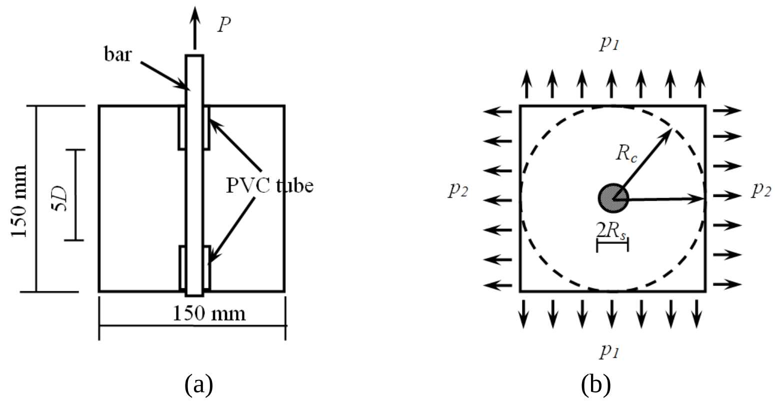


Fig. 3: Schematic of the pull-out specimen: (a) cross-section view; (b) top view and the lateral tensile stresses.

For the situations that $p_1<\sigma_{1cr}$ and $p_2< \sigma_{2cr}$, the bond strength due to the frictional effect, τ_{uf}, can be calculated from Eq. (7). According to literature [13], the coefficient of friction μ and chemical adhesion τ_a for plain round bars are suggested as 0.28 and 0.568 MPa, respectively. Thus, τ_u can be calculated from Eq. (8) and plotted against the test results in Fig. 4. It can be seen that, the predicted values agree well with the experimental data, and the coefficient of correlation is 0.743.

5. Conclusion

In this paper, an analytical approach to the ultimate bond strength of plain round bars subjected to lateral tensile stresses has been proposed and verified by the experimental results. For a given shrinkage strain of concrete, state of lateral tensile stresses, geometric and mechanical parameters of bars and concrete, the present analytical solution can be used to estimate the ultimate bond strength of plain round bars. The analytical predictions agree well with the experimental data.

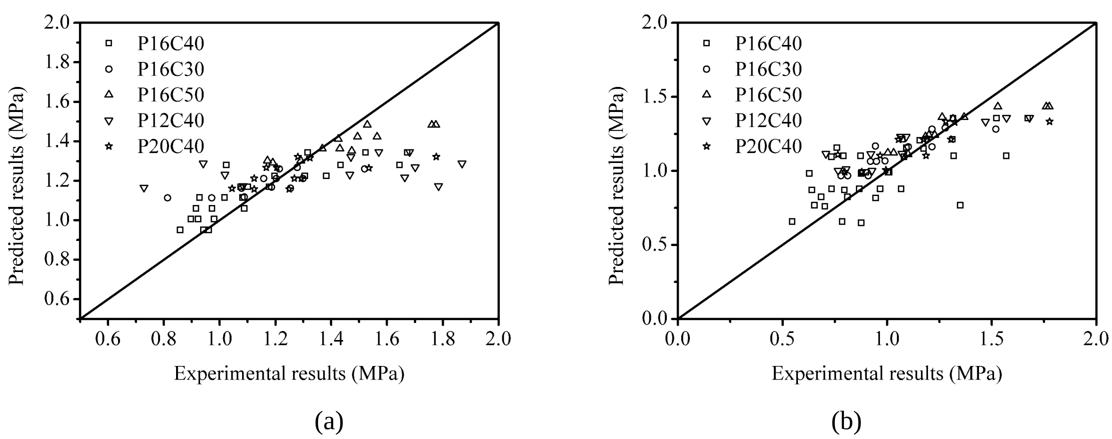


Fig. 4: Comparison between predicted and experimental ultimate bond strength under: (a) uniaxial tensile stress; (b) biaxial tensile stresses.

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