Proceedings of the 2nd World Congress on Civil, Structural, and Environmental Engineering (CSEE'17)
Barcelona, Spain – April 2 – 4, 2017
Paper No. ICGRE 155
ISSN: 2371-5294
DOI: 10.11159/icgre17.155

Correlating the Shear Wave Velocity with the Cone Penetration Test

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Abstract - The shear wave velocity V_S is an essential parameter in various geotechnical analyses. It can be determined using laboratory testing of undisturbed samples, in-situ geophysical measurements, or by using correlations of the shear wave velocity with the common in-situ penetration tests such as the standard penetration tests (SPT) and the cone penetration test (CPT). The latter approach is often preferred by engineers for many reasons including cost optimization of the geotechnical investigations and infeasibility of undisturbed sampling in some formations such as non-cohesive soils. Accordingly, many correlations were envisaged to determine the shear wave velocity using the CPT; these correlations were developed through statistical and regression analyses of compiled CPT and shear wave velocity databases. Yet, to date, substantial discrepancies between the existing CPT correlations and the measured shear wave velocities are still revealed when the CPT correlations are compared with more recent case histories and databases. As such, there is a continuous need to update these correlations. In this study, a proposed approach is presented to define the stress-dependency parameters of the shear wave velocity in terms of the CPT measurements. Hence, enhanced CPT correlations for the shear wave velocity and the small strain modulus in both cohesionless and cohesive soils are realised. Two case studies are analysed using the proposed CPT correlation for the shear wave velocity as well as the commonly applied correlations. It is shown that the proposed CPT-V_S correlation provides consistent predictions with the measured shear wave velocity; hence, it may be considered as an enhancement to the currently adopted methods.

Keywords**:** Cone penetration test, Shear wave velocity, Small strain shear modulus, Sand, Clay

1. Introduction

Determination of strength and stiffness parameters for soils using laboratory tests requires high-quality undisturbed samples. Yet, undisturbed sampling is often expensive and, sometimes, infeasible. The alternative approach is to utilize correlations with reliable in-situ testing such as the cone penetration test (CPT).

The CPT is carried out by pushing a conic penetrometer into the ground at a velocity of 20 mm/sec and obtaining the cone tip resistance q_t, sleeve friction f_s and the porewater pressure u_2 at typical intervals of 20 or 50 mm. It is commonly utilized to characterize soil strength parameters, particularly in sites where uncemented relatively weak soils prevail. It is also utilized to estimate the stiffness parameters/moduli for these soils but with a less degree of confidence than strength parameters [1], [2]. The CPT is sometimes equipped with geophones on its probe so that velocity of the acoustic shear wave V_S that is generated by a surficial source, is measured at the depths of interest; in this case the CPT is termed as the seismic cone penetration test (SCPT) [3].

The shear wave velocity V_S is utilized to determine the small strain shear modulus of soils G_0, which represents the shear stiffness of soils at shear strains less than 10^{-3} to 10^{-4}%. Many studies related the operative shear modulus (i.e., the shear modulus at operative strains, which are typically in the range of 0.1%) to the small strain shear modulus, the shear strain level and/or the shear stress level [4]–[8]; accordingly, settlements of shallow and deep foundations under operative loads are reliably determined [9]–[11]. Additionally, the shear wave velocity V_S is an essential parameter in determination of the seismic site response [12] and liquefaction susceptibility analyses [13]. The shear wave velocity can also be correlated with many other geotechnical parameters [14]–[16].

Although direct measurements of V_S are more accurate than the values obtained from correlations with CPT, there is always an essential need to reliably correlate the CPT measurements (i.e., tip resistance q_t, skin friction f_s and pore water pressure u_2) to the shear wave velocity V_S. Such correlations are considered vital especially in geotechnical investigations with little or no direct measurements of the shear wave velocity and where V_S measurements are impractical due to the surrounding noises (e.g., at congested urban areas and construction sites with heavy moving equipment) or due to the great depth of the investigations, which prevents obtaining reliable acoustic signal at the cone. Additionally, CPT is cheaper and faster than SCPT; hence, CPT is more appealing to many geotechnical engineers than SCPT in planning of geotechnical investigations [17], [18]. Therefore, the purpose of this paper is set to reappraise the commonly utilized CPT-V_S and to define new correlations that give improved estimates of V_S and G_0 based on quantifying the stress dependency parameters of the shear wave velocity in terms of the CPT measurements.

2. Previous CPT-V_S Correlations

Numerous studies attempted to correlate the shear wave velocity V_S with the CPT measurements. Wair et al. [19] compared the different correlations in the geotechnical literature and recommended the following three correlations to be utilized to obtain shear wave velocity of cohesionless and cohesive soils [17], [20], [21]:

$$V_S = 118.8 \log f_s + 18.5 \tag{1}$$

$$V_S = 2.41\, q_t^{0.395} I_c^{0.124} \tag{2}$$

$$V_S = \sqrt{10^{(0.55\, I_c + 1.68)} \left(\frac{q_t - \sigma_v}{p_a}\right)} \tag{3}$$

Where q_t and f_s in Eqs. (1) and (2) are in kPa, p_a is the atmospheric pressure (100 kPa), σ_v and σ'_v are the total vertical stress and effective vertical stresses, respectively. The behavioural index I_c is calculated in terms of the normalized net tip resistance Q_{tn} and friction ratio F_r as follows [21]:

$$I_c = \sqrt{[3.47 - \log(Q_{tn})]^2 + [1.22 + \log(F_r)]^2} \tag{4}$$

Where

$$Q_{tn} = \left(\frac{q_t - \sigma_v}{p_a}\right) \Big/ \left(\frac{\sigma'_v}{p_a}\right)^n \tag{5}$$

$$F_R = 100 \left(\frac{f_s}{q_t - \sigma_v}\right) \tag{6}$$

$$n = 0.381\, I_c + 0.05 \left(\frac{\sigma'_v}{p_a}\right) - 0.15 \le 1.00 \tag{7}$$

Many researches demonstrated that the current CPT-V_S correlations do not perform well when compared with the measured shear wave velocity [18], [22].

3. Stress Dependency of the Shear Wave Velocity

The shear wave velocity V_S is dependent, among other factors, on the state of effective stresses in soils. Among several relationships between V_S and effective stresses affecting on soils, this study focuses on the following relationship between V_S and σ'_v [23], [24]:

$$V_S = \alpha_v \left(\frac{\sigma'_v}{1\, kPa}\right)^{\beta_v} \tag{8}$$

Where the stress-dependency parameters α_v and β_v are interrelated as follows [23]:

$$\beta_v = 1.00 - 0.18 \ln\left(\frac{\alpha_v}{1\, m/sec}\right) \tag{9}$$

Alternatively, the relationship between α_v and β_v may be stated as follows:

$$\alpha_v = 258.67\ \exp(-5.556\,\beta_v) \tag{10}$$

Hence:

$$V_S = (258.67\ m/sec)\exp(-5.556\,\beta_v)\left(\frac{\sigma'_v}{1\ kPa}\right)^{\beta_v} \tag{11}$$

Accordingly, the parameter β_v can be determined from the measured shear wave velocity V_S and the effective vertical stress σ'_v as follows:

$$\beta_v = \frac{\ln\left(\frac{V_S}{258.67\ m/sec}\right)}{\left[\ln\left(\frac{\sigma'_v}{1\ kPa}\right) - 5.556\right]} \tag{12}$$

4. Correlations for Drained CPT Penetrations (Sands)

4.1. CPT-V_S Database

The database for drained CPT (i.e., u_2 is equal to the equilibrium porewater pressure u_0) comprises the geotechnical properties and the drained CPT resistances for 15 high-quality undisturbed sand samples. This database was compiled and presented by Mayne [25]. It comprises alluvial deposits, hydraulic fills, and mine tailings from different locations around the world. It was also used by Ahmed et al. [26] and Ahmed [18] to develop the correlations for the small strain modulus and shear wave velocity for sands based on the equivalence of the ratio (G_0/σ'_v) and the behavioural index I_c. Table 1 shows the CPT measurements and relevant geotechnical properties of the database samples.

Table 1: Database for sands [25].

Sample No.	Unit weight, γ (kN/m^3)	σ_v (kPa)	σ'_v (kPa)	q_t (MPa)	f_s (kPa)	F_r (%)	I_c	V_S (m/s)
1	20.53	270	180	10.2	183	1.84	2.237	175.2
2	18.65	123	102	19.9	188	0.95	1.681	183.8
3	19.11	144	123	12.8	130	1.03	1.885	204.6
4	18.74	164	143	13.9	122	0.89	1.847	198.7
5	18.53	108	87	19.7	59	0.3	1.347	205.2
6	18.27	98	84	13.1	31	0.24	1.453	180.7
7	17.88	72	51	8.0	58	0.73	1.776	137.9
8	19.02	726	516	17.2	121	0.73	2.213	227.0
9	18.18	135.4	120	6.1	24	0.4	1.932	172.0
10	18.33	175	160	8.6	31	0.37	1.852	195.7
11	18.83	60	55	1.8	15	0.87	2.345	101.5
12	18.62	121	100	4.0	16	0.41	2.059	146.6
13	18.73	178	138	5.0	18	0.38	2.044	145.2
14	19.54	120	110	3.4	14	0.42	2.162	135.4
15	20.30	57	42	11.8	30	0.26	1.400	167.0

4.2. Stress Dependency of the Shear Wave Velocity and Regression Analysis

The small strain shear modulus G_0 is related to the shear wave velocity V_S as follows [3]:

$$G_0 = \frac{\gamma}{g} V_S^2 = \left(\frac{\gamma}{\gamma_w}\right)\left(\frac{\gamma_w}{g}\right) V_S^2 = (1\,kPa)\left(\frac{\gamma}{\gamma_w}\right) V_S^2 \tag{13}$$

Where γ is the soil unit weight, γ_w is the water unit weight and g is the gravitational acceleration. Ahmed et al. [26] and Ahmed [18] expressed the small strain modulus in the following regression form:

$$G_0 = A \exp(B\, I_c)\; f(F_r)\, \sigma'_v \tag{14}$$

Where the parameters A and B, and the function $f(F_r)$ are obtained by regression analysis. By combining the different expressions for G_0, the following expression is obtained:

$$\left(\frac{\gamma}{\gamma_w}\right)(258.67\; m/sec)^2\; \exp(-11.111\,\beta_v)\left(\frac{\sigma'_v}{1\,kPa}\right)^{2\beta_v} = A\, e^{B\, I_c}\, f(F_r)\left(\frac{\sigma'_v}{1\,kPa}\right) \tag{15}$$

Assuming that $f(F_r)$ is a linear function of F_r, the following modified form

$$\beta_v = \left[C_1 + C_2\, I_c + \ln(C_3 + F_r) + \ln\left(\frac{\sigma'_v}{1\,kPa}\right) - \ln\left(\frac{\gamma}{\gamma_w}\right)\right] \Big/ \left[2\;\ln\left(\frac{\sigma'_v}{1\,kPa}\right) - 11.1111\right] \tag{16}$$

Where C_1, C_2 and C_3 are constants replacing the parameters A and B in Eq. (13). Accordingly, V_S may be expressed as following:

$$\frac{V_S}{1\; m/sec} = 258.67\;\exp\left(\left[C_1 + C_2\, I_c + \ln(C_3 + F_r) + \ln\left(\frac{\sigma'_v}{1\,kPa}\right) - \ln\left(\frac{\gamma}{\gamma_w}\right)\right]\Big/2\right) \tag{17}$$

Based on the multi-regression analysis using the database points, the parameters C_1, C_2 and C_3 were found to be: –2 .71, –1.774 and 2.257, respectively. Thus, the stress exponent β_v and the shear wave velocity V_S are as follows:

$$\beta_v = \left[-2.71 - 1.774\, I_c + \ln(2.257 + F_r) + \ln\left(\frac{\sigma'_v}{1\,kPa}\right) - \ln\left(\frac{\gamma}{\gamma_w}\right)\right] \Big/ \left[2\,\ln\left(\frac{\sigma'_v}{1\,kPa}\right) - 11.1111\right] \tag{18}$$

$$\frac{V_S}{1\; m/sec} = 1000\;\exp(-0.887\, I_c)\;\sqrt{(1 + 0.443\, F_r)\left(\frac{\sigma'_v}{p_a}\right)\left(\frac{\gamma_w}{\gamma}\right)} \tag{19}$$

Consequently, the small strain shear strain modulus G_0 can be estimated as follows:

$$G_0 = 10{,}000\;\exp(-1.774\, I_c)\;(1 + 0.443\, F_r)\, \sigma'_v \tag{20}$$

Fig. 1 shows the relationship between the predicted shear wave velocity and the actual shear wave velocity. The dotted red line in Fig. 1 represents the best fit between the measured and the predicted values. This figure shows that the new approach provides closer prediction to the measured velocities than the previous correlations.

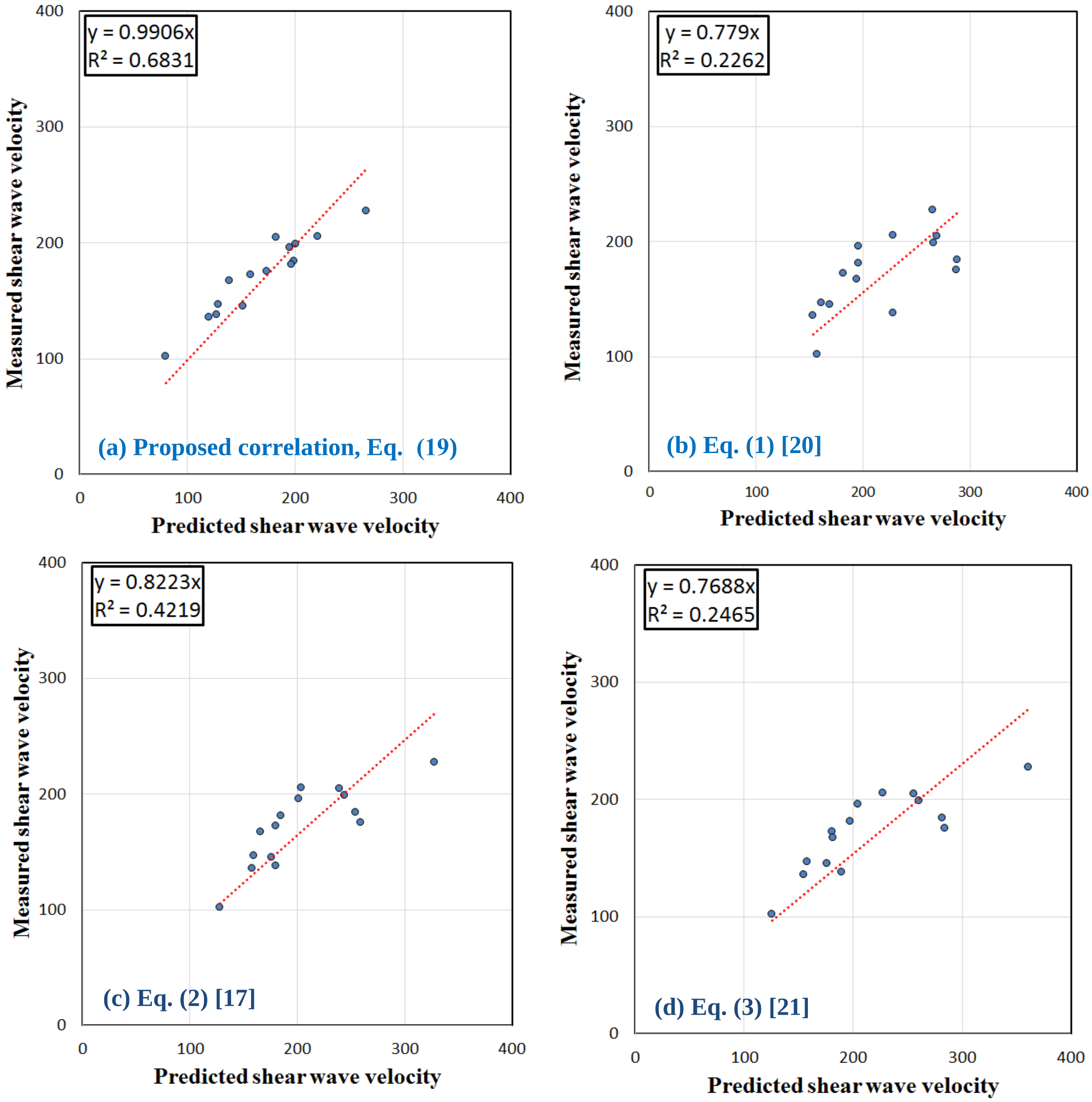

Fig. 1: Performance of the CPT-V_S correlations.

5. Correlations for Partially Drained and Undrained CPT Penetrations (Silts and Clays)

The correlations in Eqs. are valid for drained penetrations. In the partially drained and undrained cone penetrations (i.e., in silts and clays), u_2 is different from u_0 [27]. Based on the studied case history, it is tentatively suggested that the abovementioned drained correlations are to be modified to represent undrained and partially drained cone penetrations as follows:

$$\frac{V_S}{1\ m/sec} = 1000\ \exp(-0.887\ I_c)\ \sqrt{(1+0.443\ F_r)\left(\frac{\sigma'_v}{p_a}\right)\left[1+\frac{|\Delta u|}{\sigma'_v}\left(1+2\ \bar{B}_q\right)\right]\left(\frac{\gamma_w}{\gamma}\right)} \tag{21}$$

And

$$G_0 = 10{,}000\ \exp(-1.774\ I_c)\ (1+0.443\ F_r)\ \sigma'_v \left[1+\frac{|\Delta u|}{\sigma'_v}\left(1+2\ \bar{B}_q\right)\right] \tag{22}$$

Where

$$|\Delta u| = \text{abs}\,(u_2 - u_0) \tag{23}$$

$$B_q = \frac{u_2 - u_0}{q_t - \sigma_v} \tag{24}$$

The average ratio $\bar{B}_q$ is the average value of the ratio B_q for a certain layer or layers with a similar classification.

6. Case Studies

6.1. McDonald's Farm, British Columbia, Canada

A SCPT was performed at this site; the results of the cone test and the inferred soil profile is shown in Fig. 2 [3]. It is assumed that the soil suction above the groundwater depth follows the linear hydrostatic water pressure. The soil unit weights were determined in accordance with Mayne [28]. The results of the analyses using the proposed approach and the previous correlations are shown in Fig. 3. It is shown that the proposed correlation is compared favourably with the shear wave velocity measurements. The proposed correlation yields better prediction of V_S than the previous correlations.

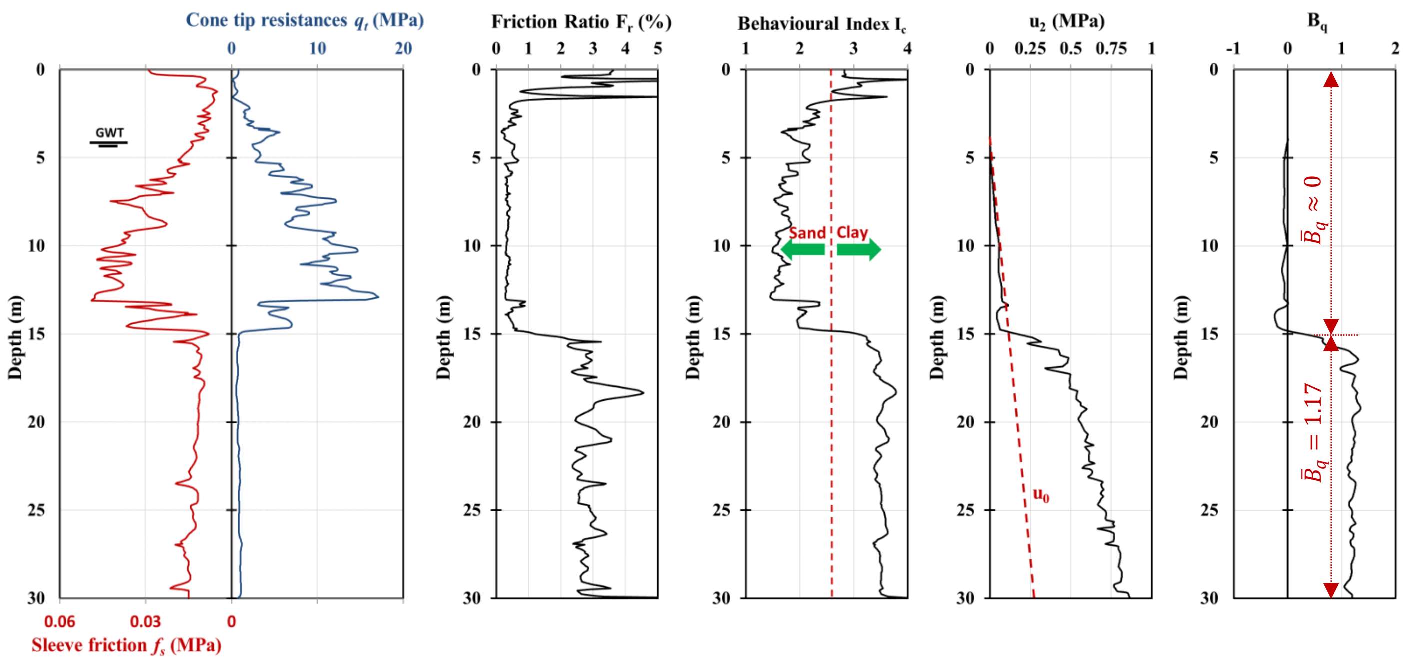

Fig. 2: Results of the SCPT in McDonald's Farm [3].

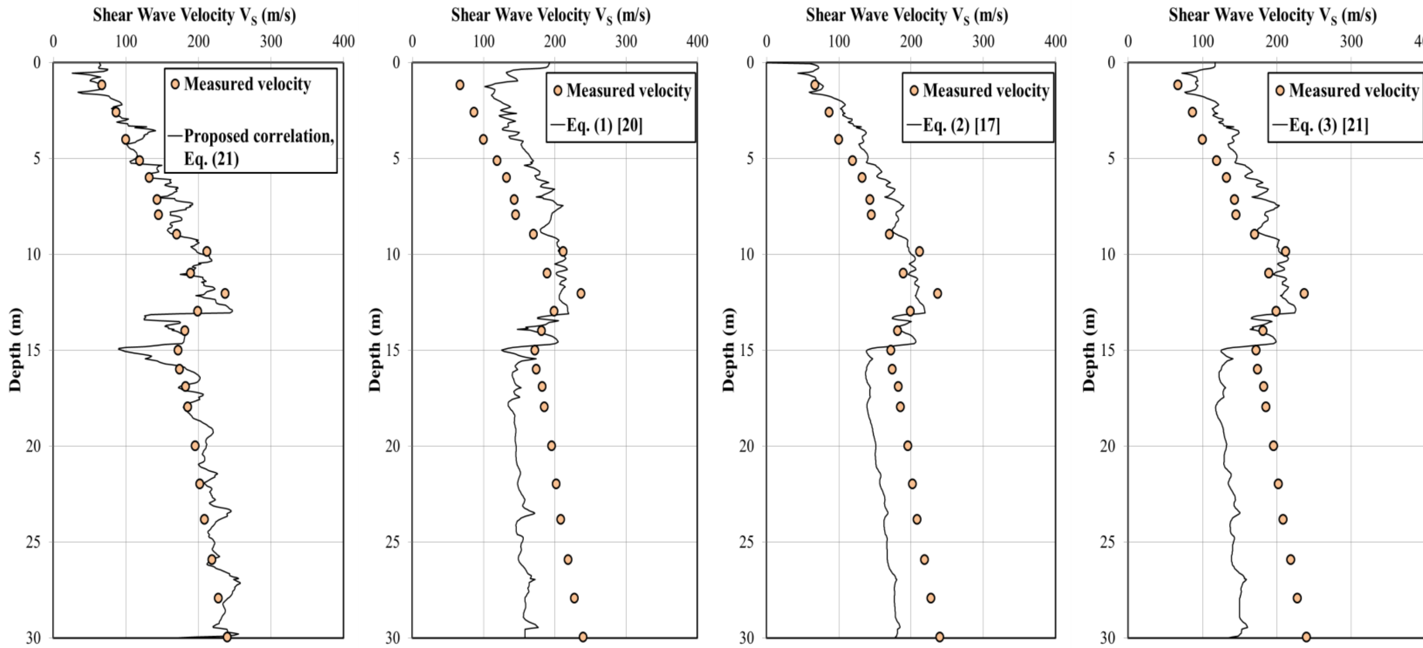

Fig. 3: Measured and predicted shear wave velocities in McDonald's Farm.

6.2. Treporti Test Site, Venice, Italy

This site comprises alternating layers of silty sand, sandy silt, clayey silt and silty clay. It was extensively investigated by means of several geotechnical tests including SCPTs. Fig. 4 show the results of SCPT14 [29]. The pore water u_2 and the coefficient B_q are averaged every 250 mm intervals to minimize the high variability of u_2. The unit weights of soils were determined in accordance with Mayne [28].The predicted and the shear wave velocities are shown in Fig. 5. It is noted that the proposed method is in closer agreement with the measurements than the previous correlations.

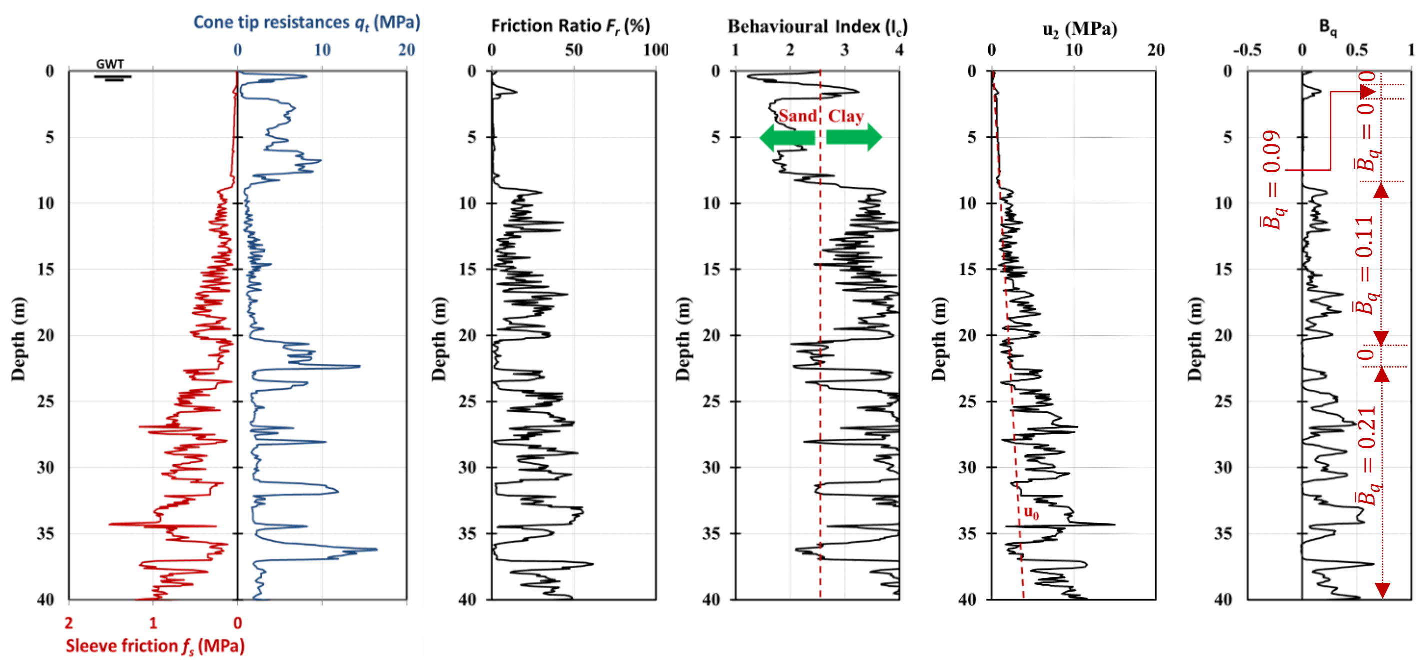

Fig. 4: The results of the SCPT14 in Treporti Site [29].

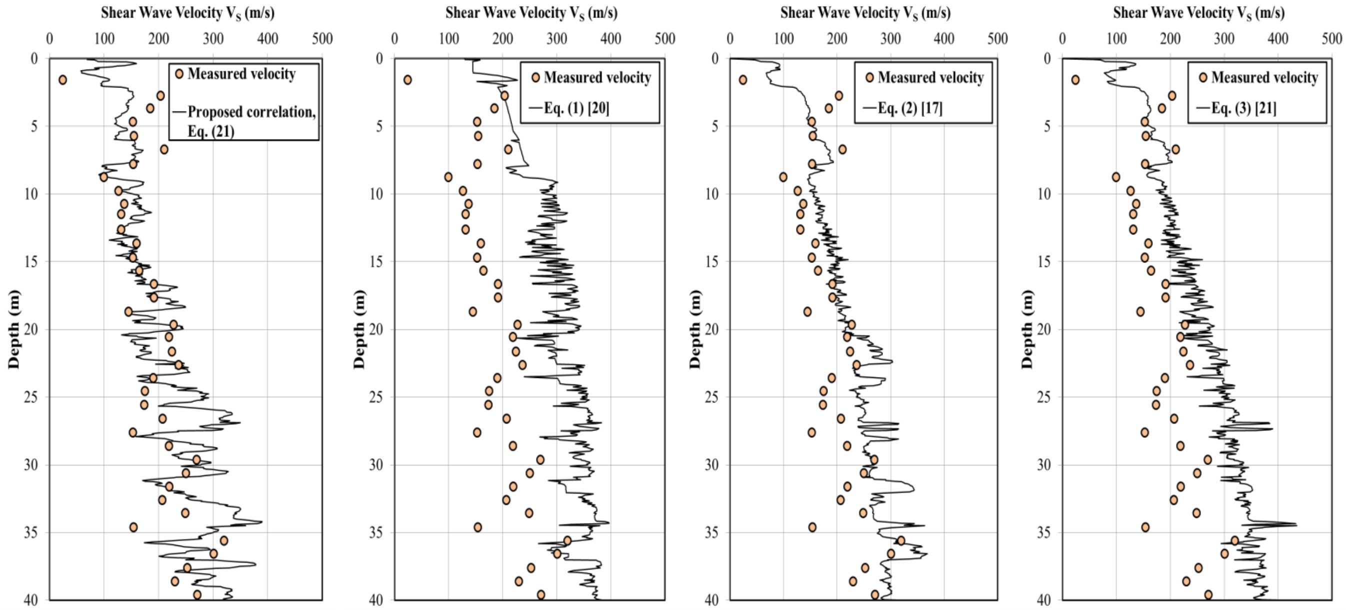

Fig. 5: Measured and predicted shear wave velocities in Treporti Site.

7. Validity and Limitations of the Proposed Correlations

Although the CPT-V_S correlation shows good agreement with the studied case histories, it should be further validated using more case studies. Moreover, the presented correlations may be utilized where the effective vertical stress profile is determinable. In case the effective vertical stress cannot be well identified (e.g., in unsaturated soils with unknown suction profiles or underconsolidated soft clays), the presented correlations may yield unreliable results.

8. Summary and Conclusions

In this article, a new approach is presented to correlate CPT measurements with the shear wave velocity V_S and the small strain modulus G_0. This approach is formulated based on stress-dependency of the shear wave velocity presented by Santamaria et al. [24], Ku et al. [23] and others. The CPT-V_S and CPT-G_0 correlations for drained penetration were developed using a sand database that was compiled by Mayne [25]. The correlations were further amended to account for the partially drained and undrained penetrations in silts and clays. These modifications were envisaged based on the studied case histories. The concluding CPT-V_S and CPT-G_0 correlations are given by Eqs. (21) and (22), respectively. Two case histories were analysed using the proposed formulations, namely: McDonald's Farm, British Columbia, Canada [3] and Treporti Test Site, Venice, Italy [29]. The results of the proposed correlations compared favourably with the measurements of the shear wave velocity. The proposed approach may be considered as an enhancement to the currently adopted correlations provided that the effective vertical stress profile is well determinable.

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