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Tailings Dams Numerical Models: A Review

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Abstract - A significant number of tailings dam failures have occurred around the world in the last few decades resulting in fatalities, damage to infrastructure and environmental harm. Among these, many have been caused by the liquefaction phenomenon that can suddenly transform an earthen dam into a liquid river of mud. To date, many general aspects related to tailings dam failures and tailing management have been dealt with in the literature. However, the materials used to build the dams, mainly consisting of underconsolidated silts, are still poorly studied and adequate modeling of their behavior is still an open challenge. This paper presents the state of existing knowledge on this latter topic. The problem related to the storage of mining residues in tailings dams is first described. For this purpose, fifteen scientific articles, in which numerical modeling is carried out on this type of structures, are analyzed. Aspects relating to the type of structure investigated and connected to numerical modeling such as software and constitutive models used are reported and commented. A summary of the main geotechnical parameters used in the modeling is presented and analyzed. Finally, the most salient aspects of the results obtained from the various analysis are exposed.

***Keywords*:** Tailings dams, numerical analysis, constitutive models, liquefaction

1. Introduction

The mining industry produces thousands of tons of waste every year of which the main part is represented by tailings [1]. In general tailings are maiifde of a mix of soil, crushed rock, water, little residues of metals and chemicals used for the processing. Tailings dams are storage facilities consisting of one or more embankments that retain the finest fraction of the resulting material called 'slurry'. The rupture of these structures often involves the release of material in the downstream territories with catastrophic consequences for the populations, environment and the economy. The frequency with which these events happen is far from negligible with a probability of occurrence of 1.2% in 100 years [2]. The origins of these failures can be traced back to problems concerning the design, the type of construction and maintenance during both the operational and after closure phases. The failure triggering causes of tailings dams are numerous and can be summarized as: defects of the structure, foundation issues, unusual climatic events, seismic events, overtopping, subsidence and others [2] [3], [4]. It has been observed that the failure mechanisms that occur more often are linked to static or seismic liquefaction [5]. In the last decades, numerical models have been increasingly used to predict the behavior of these structures. These methods have the advantage of being able to determine also the deformations that the structure will experience, unlike the more classical limit equilibrium methods. In addition, these methods, if coupled with constitutive models capable of carrying out analyzes in terms of effective stresses, can determine the increase in pore pressure and the consequent possibility of soil liquefaction. On the other hand, they often need many soil parameters which are often difficult to determine and can require a high computational cost. In this work the aspects that characterize these structures will be briefly described. Then the characteristics and causes of collapse of these types of structures will be briefly analyzed with reference to publications and databases. Then various scientific papers will be examined where the stability of tailings dams in relation to static and seismic liquefaction is evaluated through numerical methods.

2. Tailings Dams Features

There are estimated to be more than 3500 tailings dams worldwide. These structures have the function of containing the waste material produced by the mining industry. Generally, the construction of tailings dams begins with the creation of an embankment called 'starter dike' formed with available material behind which the slurries are place. Once the slurries have reached a certain safety boundary with respect to the crest of the starter dike a new embankment is then built to raise the height of the containment structure. At this stage the embankments are generally made with the coarser fraction of the worthless materials extracted during the mining process. The raising of the structure takes place in phases that follow the filling trend of the settling basins. There are three main methods used for the construction of the embankments which are: upstream, downstream and centerline. In the upstream procedure the embankments are arranged in a regressive manner by placing part of the base on the underlying embankment and part directly on the slurry. In the downstream method, the new embankment is built externally to the structure, partially incorporating the previous embankment. In the centerline method, embankments are built centrally above the crest of the levee below (Fig. 1).

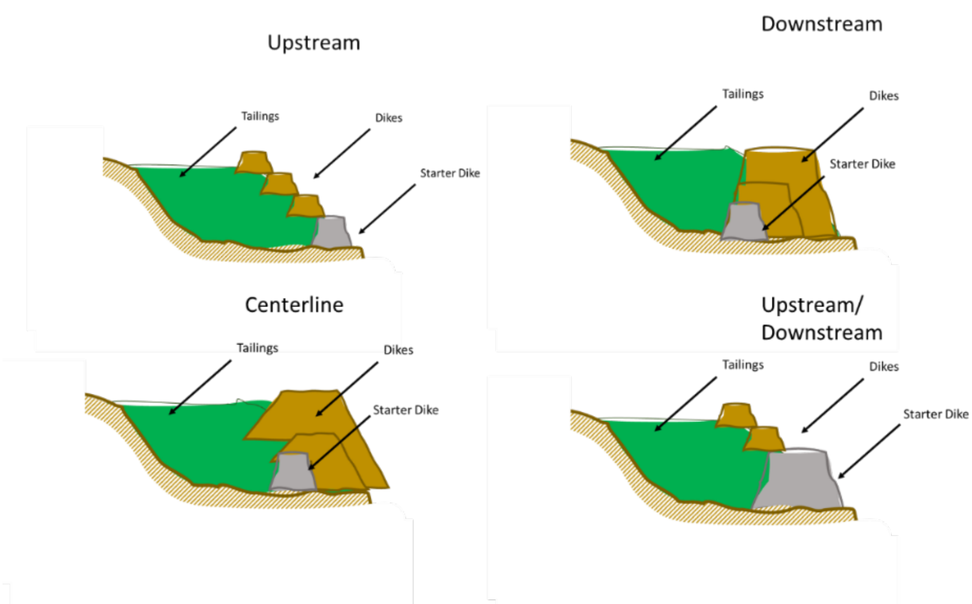

Fig. 1: Characteristic cross-sections of tailings dams made with upstream, downstream and centerline method

The upstream method is the most used because it is easier to implement and therefore cheaper but, it is also the most susceptible to failure. One of the works that will be presented below [6] proposes a tailing dam made with a hybrid upstream / downstream method (Fig. 1) in which a large starter dike is created on which the embankments are then built with the upstream technique. This method is called hybrid as a big part of the stability is guaranteed by the large starter dike. All the material extracted from the mine is taken to processing plants where the part that has an economic value is separated from that which has none. Since the transport of these materials is very expensive, tailings dams are usually located near the processing plants. Generally, the tailings are carried to the basins with a tape if dry or inside large pipes if saturated. There are different deposition techniques the most common is by means of hydro-cyclone. This device exploits the centrifugal force to separate the different pieces of material that is placed either on the embankment or in the basins. There are also a series of installation that serve the correct functioning of the facility such as hydraulic regulation systems, deep drainages, instrumentation for monitoring (piezometers, inclinometers, strain gauges, etc.), infrastructures for maintenance and more.

3. Tailings Dams Failures

As mentioned in the introduction, tailings dams are often subject to failure. There is a numerous literature that reviews the collapse mechanisms of these structures [2], [3], [4], [7], [8], and there are databases that are updated daily with information on events from all over the world [9]. However, still today there is no information about some events. This lack of knowledge is due to many factors including poor or total absence of regulations regarding mining accidents in some countries, underestimation of events considered of minor importance and in some cases concealment by management companies for fear of legal proceedings or discrediting of the public opinion. However, important conclusions can be drawn from the analysis of the existing literature. Worldwide 66% of collapses occurred in tailings dams constructed with the upstream method. The United States is the country with the highest number of known accidents followed by Europe [3]. The frequency with which these events have occurred on average in the last 20 years is 20 events / decade and the predominant causes of collapse are due to unusual rainfall and poor maintenance [2]. The percentage of accidents that occurred due to seepage is 21.6% while due to seismic events 17% [4].

4. Tailings Dams Numerical Analysis Review

In this paragraph, 15 scientific papers that deal with numerical modeling of tailings dams will be analyzed. These analyzes made it possible to determine and compare the typical characteristics of these structures and the mathematical models used to study their behavior. The studied cases refer to facilities located in most parts of the world, namely in: Tasmania, China, Australia, India, Japan, Poland, Sweden, Iran, Mexico. In cases where it has been specified, the method of construction of the dam is indicated, distinguishing between upstream, downstream and centerline. Then it is specified whether the behavior of the structure has been analyzed in static or seismic conditions and the dimension of the analysis, which can be in one or two dimensions. Since tailings are very susceptible to liquefaction phenomena, this possibility is addressed in all the articles examined. Tailings can be described as uniformly graded non-cohesive materials with contracting behavior. These materials, if saturated and subjected to a relatively rapid stress (which can be static: seepage, rapid dam raise etc. or dynamic: seismic event, vibrating machines, etc. [10]) are not able to instantly allow the flow of water contained in the pores. This leads, for a certain period of time, to an increase in interstitial pressures and to a consequent reduction or total elimination of the effective stresses. To study these phenomena correctly, it is necessary to recourse to specific non-linear constitutive models that utilize coupled analyzes in terms of effective stresses. In all the examined cases, there are areas with materials that can be subjected to liquefaction (generally the settling basins) and areas made with materials that are not susceptible to liquefaction (generally the embankments and foundations). A summary of the main general information about the case studies described by the analyzed papers is presented in Table 1, along with the software's used for the numerical analyzes and the constitutive models for soils susceptible or not susceptible to liquefaction.

From the analysis of Table 1 it can be deduced that most of the structures analyzed are of the upstream type (9 out of 15), only 2 of the downstream type and one centerline. Only two articles present a one-dimensional analysis the others are all defined in a two-dimensional domain. The most used software's are the finite difference (FDM) analysis program FLAC (5 out of 15) and the finite element (FEM) program PLAXIS (3 out of 15) together with the SIGMA / W and QUAKE / W packages from GEOSTUDIO (3 out of 15). To model liquefiable soils, the Fynn-Byrne model (4 out of 15) was first used, which was later replaced by UBCSAND (2 out of 15). In almost all cases, the Mohr-Coulomb model of was used to model non-liquefiable soils.

Table 1: Scientific papers analyzed and main information about the reported case studies.

n.	Year	Authors	Tailings facility name/location	Construction method	Stability analysis conditions	Analysis dimensions	Software	Type of analysis	Constitutive model for liquefiable materials	Constitutive model for non liquefiable materials
1	2010	Ghahreman Nejad et al.	Bobadil - Tasmania	Upstream	Seismic	2D	Flac	FDM	Fynn-Byrne	Mohr-Coulomb
2	2010	Liang et al.	Australia	Upstream and Downstream	Seismic	2D	Plaxis	FEM		Mohr-Coulomb
3	2011	Chakraborty et al.	India	Downstream	Static and Seismic	2D	Flac	FDM	Fynn-Byrne	Mohr-Coulomb
4	2011	Meisheng et al.	China	Upstream	Seismic	2D			Fynn-Byrne	Mohr-Coulomb
5	2011	Wang te al.	Xiangyun - China	Upstream	Seismic	2D	Flac	FDM	Fynn-Byrne	Mohr-Coulomb
6	2014	Ishihara et al.	1)Takasega mori 2)Kayakari- Japan	Upstream	Seismic	2D				
7	2015	Barrero et al.			Seismic	2D	Flac	FDM	SANISAND	Mohr-Coulomb
8	2015	Xu et al.	Lingshan - China	Upstream	Seismic	2D	QUAKE/W	FEM	linear equivalent	
9	2017	Świdziński	Żelazny Most - Poland	Upstream	Seismic	1D			C/L model	
10	2017	James et al.	Canada		Seismic	1D	Flac	FDM	UBCSAND	
11	2017	Kalsnes et al.				2D				User defined Mohr-Coulomb
12	2017	Zardari et al.	Aitik - Sweden	Upstream	Seismic	2D	Plaxis	FEM	UBCSAND	Mohr-Coulomb
13	2018	Naeini et al.	Sungun - Iran	Centerline	Seismic	2D	SIGMA/W - QUAKE/W	FEM	linear equivalent	Mohr-Coulomb
14	2019	Vargas	Mexico	Upstream / Downstream	Seismic	2D	QUAKE/W	FEM	linear equivalent	
15	2020	Sottile et al.		Upstream	Satic	2D	Plaxis	FEM	(HSS)	Mohr-Coulomb

Table 2 shows the geotechnical parameters that it was possible to obtain from the analysis of the articles. The purpose of this table is to offer a useful tool for comparing the quantities used by the various authors for their models. In order to present these results in an organized manner and to allow an easy comparison, the configurations of the various structures were analyzed both from the geometrical and the mechanical point of view in order to (where possible) homogenize the layout of the table. The parameters shown in Table 2 are the effective cohesion c', the undrained shear strength c_u, the friction angle φ', the dilation angle ψ, the saturated unit weight γ, the permeability k, the Poisson's ratio ν, the initial shear modulus G_{max}, the initial damping ratio ξ, the porosity n and the elastic modulus E. The number in round brackets shown in the first column of Table 2 is the reference to the articles listed in table 1 from which each parameter was obtained. In some cases these parameters are offered as functions of other quantities, therefore the functions have been reported in Table 2 for these cases. These are some of the parameters (not all) necessary to set up a numerical model especially in the presence of complex constitutive models such as those used to determine liquefaction.

From the analysis of Table 2 it can be deduced that to the tailings are assigned effective cohesion values which are around 0 - 2 kPa, confirming the non-cohesive nature of these materials. The values assigned to the friction angle are between 30 and 36 degrees. The permeability values are those typical of sandy and silty materials.

The parameters required by the complex constitutive models used were reported only in some works. These quantities are of different nature: mechanical characteristics of the material, parameters necessary for calibration, parameters necessary for numerical issues, etc. Generally, they are difficult to determine and require calibration processes in which numerical simulations of laboratory tests are generally reproduced (direct and simple shear tests, triaxial tests, monotonic or cyclic) whose results are compared with those obtained from experimental tests carried out on materials to be calibrated. It would be of fundamental importance that these values were reported in the works in which numerical modeling of geotechnical structures are carried out. Only in this way the models would be truly reproducible also by other researchers and would make it possible to make real more detailed comparisons with consequent dissemination of knowledge and development on this topic that is not addressed in the present paper.

Table 2: Geotechnical parameters of soils described in the reported case studies.

Parameter	Bedrock	Layer beetwen Bedrock and tailings - foundation	Starter Dike	Compacted tailings	Tailings - slimes	Clay -moraine dikes	Rockfill - Sand tailings	Drain layer	Alluvial	Liq. Sand	Layered sand tailings	Soft sand tailings
c'(1)		1	1									
c'(3)	300	70	0	15.2	12	80	70	0				
c'(4)			0.5	1.8-2-2.5	1-1.2	18						
c'(5)	11		20		6							
c'(7)	0	0			0		0		0	0		
$c_u(1)$				$c_u=0.38\sigma'_{v0}$	$c_u=0.2\sigma'_{v0}$	$c_u=0.35\sigma'_{v0}$						
c'(12)			1	13	10	1	1	1				6
c'(13)		10	0		0	15	0					
c'(14)			5	2	2		21					
c'(15)		1			1		5					
ϕ'(1)		40	39	0	0	0						
ϕ'(3)	35	20	42	14.7	14.7	17	20	32				
ϕ'(4)			38	32-33-35	31-35	17						
ϕ'(5)	22		28		23							
ϕ'(7)	40	40	34		33	34	34		35	28		
ϕ'(10)		33.5			36.6							
ϕ'(12)			35	26	22	37	42	32				18
ϕ'(13)		25	45		35	32	$42.2-3.4\log(\sigma'_3/p_a)$					
ϕ'(14)			42	30	30	30	23					
ϕ'(15)		40			35		33					
ψ(13)		0	10		0	0	5					
γ(3)	20	19	20	19	19	19	19	20				
γ(4)			19.13	18.37	17.02	17.65						
γ(5)	16.18		16.67		11.28							
γ(7)	20	25.6	15.8		19.9	15.8	15.8		16	14.4		
γ(9)					15.88							
γ(10)	25.49	13.72			23.24							
γ(12)			22	19	18	22	20	20			19	18
γ(13)		20.7			19.2	19.7	19.4					
γ(14)	24	22	21	f(z)	f(z)	f(z)	16.4					
k(3)	1.00E-09	1.50E-09	1.00E-02	1.00E-08	1.00E-08	1.00E-09	1.50E-09	1.00E-04				
k(4)			4.00E-03	2.00E-05	2.00E-05	1.80E-08						
k(5)	1.00E-08		2.00E-08		3.00E-07							
k(10)	2.00E-10	2.00E-07			1.20E-07							
k(12) x			1.00E-07	1.00E-06	1.00E-07	5.00E-08	1.00E-01	1.00E-03			5.00E-07	5.00E-07
k(12) y			5.00E-08	1.00E-07	1.00E-08	5.00E-08	1.00E-01	1.00E-03			5.00E-08	5.00E-07
k(13) y		6.00E-06	1.60E-03		1.4 E-7 to 8.7 E-10	6.80E-09	4.90E-06					
k(14)	5.90E-08	1.00E-07	1.00E-06	f(z)	f(z)	f(z)	1.00E-08					
ν(1)	0.4	0.33	0.33	0.45	0.45	0.45	0.26					
ν(3)	0.35	0.49	0.33	0.35	0.35	0.49	0.49	0.33				
ν(4)			0.25	0.3-0.35	0.3-0.35	0.38						
ν(7)	0.31	0.26	0.33		0.27	0.33	0.33		0.26	0.33		
ν(10)	0.2											
ν(13)		0.3	0.23		0.3	0.3	0.28					
ν(14)	0.25	0.3	0.28	0.33	0.33		0.3					
$G_{max}(1)$	1350	$22\sigma'^{0.5}_m$	$22\sigma'^{0.5}_m$	$9.7\sigma'^{0.5}_m$	$8.3\sigma'^{0.5}_m$	$0.7\sigma'^{0.5}_m$	$26.4\sigma'^{0.5}_m$					
$G_{max}(3)$	2000	15	40.5	95.4	45.6	24	15	40.5				
$G_{max}(14)$	1198.777	330.275	342.508	260.958	65.24		104.485					
ξ(1)	0.5	0.5	0.5	0.5	0.5	0.5	0.5					
ξ(3)	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1				
n(5)	0.3		0.4		0.3							
n(10)	0.02	0.45			0.39							
$N_{1,60}(10)$		14			10							
E(4)			120000	43000	16740	10000						
E(10)	2.00E+07											
E(12)			20000	8800	9300	20000	400000	20000				9800
E(13)			50000		7000	20000						
E(14)	4000000	2000000	$f(\sigma'_c)^2$	18000	5000 -10000		26760					

5. Tailings dams numerical analysis observations

The structures investigated in paper [11] affected or not affected by the liquefaction phenomenon and in both cases use two sets of values of the stiffness parameters. From the analyzes, the authors conclude that the materials most by liquefaction are those in the settling basin. Moreover, in the case where liquefaction occurred, the horizontal on the crest of the dam were greater than in case where liquefaction did not occur. In the article [12] are compared two structures made of the same materials and subject to the same actions, but one built with the upstream method and one the downstream method. The results of their analyzes pointed out that the displacements determined for the dam built with the upstream method are about double that the ones of the dam built with the downstream method. Moreover, the latter has no stability problems with regard to liquefaction while the other does. In the article [13] is studied a tailings dam built with the downstream method both in static and seismic conditions for different positions of the groundwater level. They concluded that the zone with the highest liquefaction risk is near the surface of the settling basin. Also from the research carried out by [14] it can be concluded that liquefaction occurs in the most superficial part of the settling basin. On the other hand, from [15] conclusions, it is possible to deduce that liquefaction occurs on the surface of the dam ridge. In [16] are studied the behavior of the Kayakari and Takasega-mori dams that suffered the effects of two earthquakes that struck Japan in 2003 and 2011, concluding that the results obtained from the numerical modeling are comparable with the observed effects. In [17] the authors conclude that in the investigated point the maximum pore pressure ratio reached is 0.6. In the article by [18] an upstream tailings dam subjected to seismic action is studied; the main conclusions are that the areas in which liquefaction occurs are on the surface of the dam bank and that the horizontal displacements can reach 0.8 m. From the results reported in [19] it can be observed that liquefaction is achieved in a small area of the settling basin behind the embankment and that the displacements determined following an earthquake of magnitude 5.8 are acceptable. Also from the article [20] can be deduced that liquefaction occurs mainly in the superficial areas of the basin and that the displacements at the level of the ridge in the presence of certain earthquakes can reach 2 m. The analysis conducted by [6] allows to conclude that in the case studied no liquefaction occurs, however important values of pore water pressure are reached at a depth of 5 m in the tailings settling basin and that the greatest displacements occur on the surface of the crest and are of the order of 13 cm. Finally, from article [21] work it can be deduced that horizontal displacements of the order of 6 cm are determined on the crest of the starter dike and that the largest values of the pore water pressure ratio are generated on the rupture band of the landslide mass.

6. Conclusions

In this work, fifteen articles dealing with the numerical modeling of tailings dams are analyzed. First of all, the type of structure analyzed is examined, concluding that the predominantly modeled one is of the upstream type. Then the number of analysis dimensions, the software and the constitutive models used are considered, ensuring that: i) most of the domains are two-dimensional; ii) the most commonly used software are FLAC, PLAXIS and SIGMA / W - QUAKE / W; iii) the constitutive models used are of different types for liquefiable soils while for non-liquefiable soils the Mohr-Coulomb model is used. The comparison of the geotechnical parameters reported in the various papers allows to point out that: i) the tailings have very low effective cohesion values; ii) the friction angle stands between 30 and 36 degrees; iii) permeability values are those typical of sands and silts. Finally, from the works analyzed it is possible to conclude that the areas mostly subjected to liquefaction are the superficial parts of the settling basins and the banks of the dams. The main aspect revealed from the analysis of the reviewed papers is that many crucial aspects of the characterization and modeling of tailings are still unsolved due to the complexity and uncertainties in their behavior and that the approaches developed for clayey or sandy soils are not suitable to be applied to such "non-standard geomaterials”.

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