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Evaluation of the Structural Reliability of a Fixed Offshore Structure over Time

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Abstract - In this paper, a structural reliability assessment of a structural offshore considering the effect of the cumulative damage caused by fatigue at the end of an instant of time is presented. The reliability is expressed in terms of the confidence factor. The confidence factor is obtained for three kinds of approaches: a) without damage, b) the structural capacity is reduced over time and c) the structural capacity and demand vary over time. The uncertainties related to the occurrence of operational and storms loadings as well as epistemic uncertainties are considering. A fixed offshore jacket platform located in the Gulf of Mexico is used to estimate the reliability indicators. The structural demand and capacity are evaluated by considering operational and storm conditions. The damage is defined by the crack growth in tubular members due to fatigue. Undesirable reliability levels are obtained after 9 years of the offshore construction. The evaluation of the structural reliability levels helps to make decisions about design o re-design the structural system.

***Keywords*:** Confidence factor, reliability, fatigue, fixed offshore jacket platform, cumulative damage.

1. Introduction

The assessment of structural reliability implies estimating events whose occurrence is uncertain. Moreover, it is difficult to predict when an element or structural system present an undesirable reliability level. On the other hand, if the structural elements are manufactured in the same lot and there are installed by the same engineer. Additionally, the structure is under identical operating and environmental conditions, there is a high probability that all components fail at different times. Therefore, in handling events whose occurrence is not deterministic, the solution requires applying probability concepts. Then, these events can be estimated quantitatively, and it is necessary to implement mathematical tools that allow estimating the reliability levels.

A fixed offshore platform is a structure that is structured by tubular steel sections, and it is supported by pile foundations; at the top of the structure, it can find an area for installing drilling equipment, production facilities or living areas. The offshores are exposed to different loads such as seismic, wind, sea current, waves, and others.

In order to prevent an undesirable behaviour level, several studies have been developed to estimate the structural reliability. [1] exposed a method to calculate the reliability index which considers the effect of fatigue and extreme stress. [2] proposes a reliability approach for inspection by considering fatigue and the corrosion effect. [3] proposed a method for measuring structural reliability considering fatigue. Some authors emphasize the importance of representing all the uncertainties to estimate the structural reliability [4], [5]. [6] exposed a method for assessing the reliability due to seismic loadings. The implementation of artificial neural networks has been combined with Monte Carlo simulations to optimate the structural reliability on fixed platforms located in of the Persian Gulf, Iran [7], and the Bohai Sea, China [8].

In Mexico, Hurricane Roxanne occurred in 1995 produced the need to inspect a group of more than 200 marine structures installed in the Gulf of Mexico. In accordance with the above, [9] presented a report of damage that include dents, cracks, and twisted members that required repair actions in 26 platforms. Moreover, several approaches have been proposed to calculate the structural reliability in offshore platforms located in Campeche Bay of Mexico. [10] proposed a framework to estimate the failure probability in offshore structures using Monte Carlo simulations. [11] propose an approach to estimate the structural reliability by considering the degradation of the capacity over time. [12], [13] estimate the structural reliability considering that the structural capacity and demand vary over time.

In this study, the structural reliability is estimated in an offshore platform located in the Bay of Campeche in the Gulf of Mexico. The structural reliability is expressed in terms of the expected number of failures and confidence factor over time. Three cases are considered: a) without damage, b) the structural capacity is reduced over time c) the structural capacity and demand vary over time.

2. Structural reliability assessment without damage

The mean annual failure rate can be estimated as follows [14], [15], and [16]:

$$E(v_F) = \int -\frac{dv_D(d)}{dd}\big(P(C \leq d)\big)dd \tag{1}$$

where $E(v_F)$ corresponds to the annual rate of structural failure; $\frac{dv_D(d)}{dd}$ represents the derivative of the demand hazard curve; $P(C \leq d)$; represents the probability that the structural capacity, C, is less than or equal to a given value, d. In order to solve Eq. (1), [16], [17] and [18] assume the following hypotheses:

- The hazard curve, $v(y)$, is represented for the intensity of interest in the function $(y) = ky^{-r}$; where k and r are parameters that define the shape of the environmental hazard curve.
- The median of structural demand, $\widehat{D}$, presents a log-normal distribution with a variance of the natural logarithm of $\widehat{D}$, equal to $\sigma^2_{lnD|y}$ [19]. $\widehat{D}$ can be estimated as $\widehat{D} = a \cdot y^b$ where a and b are parameters that define the shape of the median of structural demand.
- The median of the structural capacity, $\hat{C}$, follows a log-normal distribution function, with a variance of the natural logarithm of $\hat{C}$, equal to σ^2_{lnC}.

Considering the hypothesis and including the epistemic uncertainties, the expected value of v_F can be estimated as follows [20]:

$$E(v_F) = k(y_{\hat{C}})^{-r} \exp\left[\frac{r^2}{2b^2}\left(\sigma^2_{lnD|y} + \sigma^2_{lnC} + \sigma^2_{UD} + \sigma^2_{UC}\right)\right] \tag{2}$$

where $y_{\hat{C}} = \left[\frac{\hat{C}}{a}\right]^{\frac{1}{b}}$ is the intensity associated with the median of the limit state of the capacity, $\hat{C}$; $\sigma^2_{lnD|y}$ and σ^2_{lnC} are the variances of the natural logarithms of structural demand, D, and structural capacity, C; σ^2_{UD} and σ^2_{UC} represent the variances of the epistemic uncertainties associated with demand and capacity, respectively. [16] assume that the mean annual failure rate, $\bar{v}_F$, is less or equal to a permissible value, v_0. Based on the above and making some algebraic arrangements, the following equation is obtained:

$$\phi\hat{C} \geq \gamma\widehat{D}^{v_0} \tag{3}$$

where ϕ is the capacity reduction factor, γ is the demand intensification factor; both expressed are as follows:

From Eq. (3), the confidence factor is deduced:

$$\lambda_{\mathrm{conf}} = \frac{\phi\hat{C}}{\gamma\widehat{D}^{v_0}} \tag{4}$$

3. Structural reliability considering the variation of the structural capacity

[21] provided simplified expressions to obtain the expected number of failures and the confidence level considering that the structural capacity is reduced over time. The authors proposed the following expression to estimate the expected number of failures as follows:

$$\eta_L(t,\Delta t) = k\left(\frac{\hat{C}(t)}{a}\right)^{-r} \exp\left[\frac{r^2}{2b^2}\left(\sigma^2_{\left(lnD|y_{\hat{C}(t)}\right)} + \sigma^2_{(lnC|\mathrm{t})} + \sigma^2_{(UD|\mathrm{t})} + \sigma^2_{(UC|\mathrm{t})}\right)\right]\Omega_L \tag{5}$$

where $\hat{C}(t) = \alpha + \beta t$; Ω_L is the correction factor, it is expressed as follows:

$$\Omega_L(t,\Delta t) = \frac{(\alpha+\beta t)}{\beta}\frac{b}{(b_t - r)}\left[\left(1+\frac{\beta\Delta t}{\alpha+\beta t}\right)^{1-\frac{t}{b}} - 1\right] \tag{6}$$

To obtain the confidence factor, $\lambda_{conf_L}(t,\Delta t)$, over time, the authors make some mathematical arrangements, and they propose the following expression:

$$\lambda_{conf_L}(t,\Delta t) = \left[\frac{\phi\hat{\mathrm{C}}}{\gamma\hat{\mathrm{D}}^{v_0}}\right]\left[\frac{\Omega_L(t,\Delta t)}{\Delta t}\right]^{-\frac{b}{r}} \tag{7}$$

4. Structural reliability considering the variation of the structural capacity and demand

[12] propose simplified expressions that consider the variation of the structural capacity and demand over time. The authors proposed the following expressions:

$$\overline{\eta}_{LL}(t,t+\varDelta t) = k\left(\frac{\alpha+\beta t}{e+ft}\right)^{-\frac{r}{b}} \exp\left[\frac{r^2}{2b^2}\left(\sigma^2_{lnD|y} + \sigma^2_{lnC} + \sigma^2_{UD} + \sigma^2_{UC}\right)\right]\Omega_{LL}(0,\varDelta t) \tag{8}$$

where Ω_{LL} is the correction factor, it is expressed as follows:

$$\Omega_{LL}(0,\varDelta t) = \frac{b\alpha}{\beta(b-r)}\left(\frac{a\beta}{-\alpha f+\beta(e+ft)}\right)^{-\frac{r}{b}}\Bigg[-F(A,B;C;t) + \left(1+\frac{f\beta\Delta t}{\beta(e+ft)}\right)^{-\frac{r}{b}}\left(1+\frac{\beta\Delta t}{\alpha}\right)\cdot\left(\frac{\alpha+\beta\Delta t}{\alpha+f\Delta t}\right)^{-\frac{r}{b}}\cdot \left(\frac{\alpha}{e+ft}\right)^{-\frac{r}{b}}F(A,B;C;t+\Delta t)\Bigg] \tag{9}$$

The hypergeometric function $F(A,B;C;t)$ in Eq. (9) is solved by using the hypergeometric series [22], [23]:

$$F(A,B;C;x) = 1 + \frac{AB}{1!\,C}x + \frac{A(A+1)B(B+1)}{2!\,C(C+1)}x^2 + \frac{A(A+1)\ldots(A+n-1)B(B+1)\ldots(n+1)}{C(C+1)\ldots(C+n-1)n!}x^n \quad (10)$$

where:

$$A = 1 - \frac{r}{b}; B = -\frac{r}{b}; C = 2 - \frac{r}{b}; x(t) = \frac{f(\beta' t + \alpha)}{f\alpha - a\beta'}; x(t+\Delta t) = \frac{f(\beta'(t+\Delta t) + \alpha)}{f\alpha - a\beta'}$$

To obtain the expression of the confidence factor, λ_{confLL}, [12] obtained the confidence factor, λ_{confLL}, as follows:

$$\lambda_{conf_{LL}}(t,\Delta t) = \left[\frac{\phi\hat{\mathrm{C}}}{\gamma\hat{\mathrm{D}}^{v_0}}\right]\left[\frac{\Omega_{LL}(t,\Delta t)}{\Delta t}\right]^{-\frac{b}{r}} \quad (11)$$

4. Illustrative example

The reliability indicators are estimated in an offshore jacket platform located in Campeche Bay in the Gulf of Mexico. Three different approaches are considering: a) without damage, b) the structural capacity is reduced over time, and c) both the demand and capacity vary over time. The structure is 72.5 m high, and the water depth is 66 m (see Fig. 1); mean mechanical properties are considered for tubular A-36 structural steel members. The NRF-003-PEMEX-2000 code [24] facilitates the environmental conditions of waves, wind, and marine currents for the site. Figs. 2, 3, and 4 show the environmental hazard curves; those curves are fitted employing the cumulative probability function of Gumbel for Ku site.

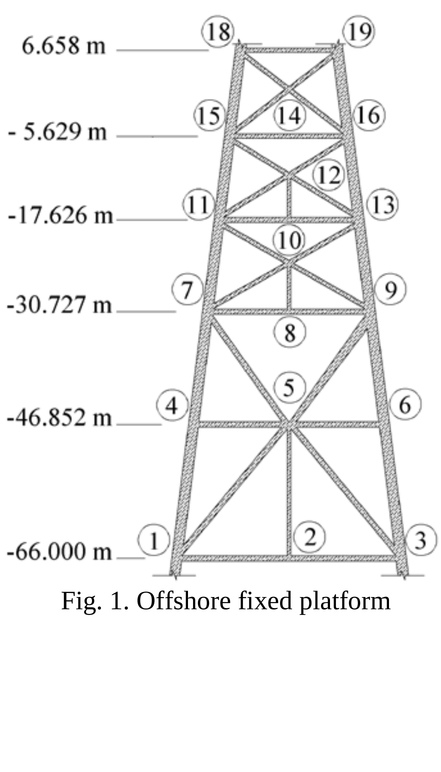

Fig. 1. Offshore fixed platform

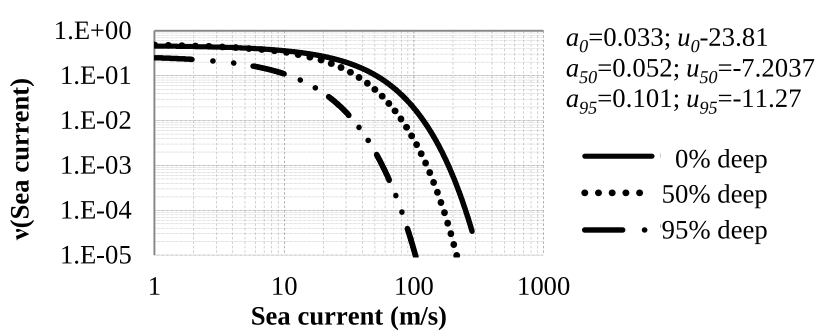

Fig. 2. Sea current hazard curve

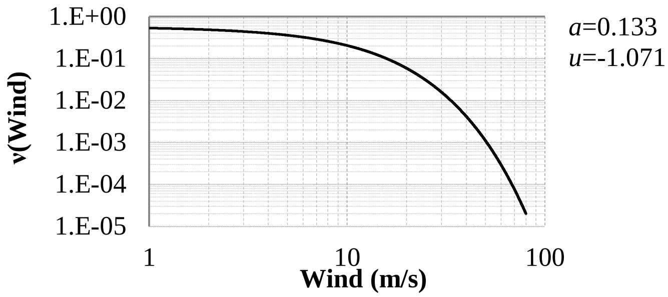

Fig. 3. Wind speed hazard curve

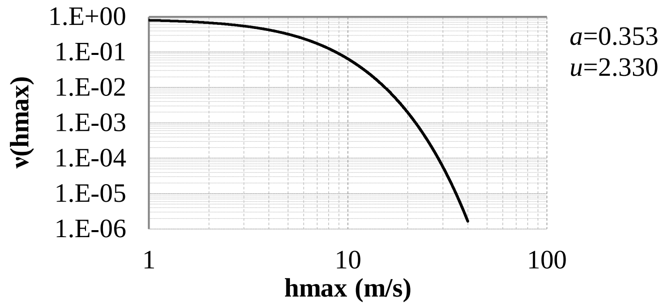

Fig. 4. Wave hazard curve

4.1 Fatigue analysis

The fatigue affects the behaviour of the offshore structures due to there are exposed to environmental loads such as operational and storm conditions. The effect of fatigue can lead to the failure of the structural elements and then the failure of the structural system. The statistical data used for the simulation of crack growth are taken from [10]. The estimation of crack growth under random loadings are obtained by using the modified differential equation [25]:

$$\frac{da'}{dt} = C(\Delta K_{mr})^m v' \tag{12}$$

$$\Delta K_{mr} = Y S_{mr}\sqrt{\pi a'} \tag{13}$$

where C and m are parameters that depend on the material properties, ΔK_{mr} is the mean stress intensity interval, v' is the rate of positive crosses by zero, Y is the geometric correction factor [26], S_{mr} is the mean stress interval of the random response of the elements [27] and a' is the crack size. In this equation, the random load is replaced by an equivalent cyclic load whose amplitude and frequency are expressed as a function of the mean properties of the random process. Substituting Eq. (13) in Eq. (12) and making some algebraic arrangements, the following expression results as follows:

$$\int_{a_0}^{a_f} \frac{da}{\left(Y\sqrt{\pi a'}\right)^m} = C S_{mr}^m v' t \tag{14}$$

where a_0 is the initial crack size and a_f, is the final crack size after N cycles. The crack simulation was obtained by the Eq. (14). The probabilistic estimation of the crack growth due to fatigue is estimated by using the Monte Carlo simulation technique.

The Pierson-Moskowitz spectrum [28] is used to express the sea energy due to the transfer of forces from the wind. For the wave simulation, the sea surface, h, is represented as a stationary, homogeneous, Gaussian process, which can be expressed as a linear superposition of regular waves with random generation at different phase angles, f, with a uniform distribution between 0 and 2π. From the normalized derivative of the wave hazard curve, it is considered that the maximum wave height corresponding to a storm condition presents a Gumbel-type distribution and that the arrival times between storms follow an exponential distribution.

According to the above, in order to estimate the crack evolution in the tubular joints, it is necessary to develop the following procedure to obtain the modified Paris and Erdogan equation as follows:

1. Identification of the joints that contribute more in the global capacity by means of non-linear static analysis.
2. The critical hot spots are determined.
3. Non-linear dynamic analysis "step by step" in the structure using a set of simulated waves, wind speeds and sea currents for the return period is applied.
4. The stories of the mechanical elements in the critical joints are obtained. Then, the effective stress, σ_{ef}, are estimated [29].
5. The number of cycles with rate of zero positive are obtained.
6. Steps 3 through 5 are executed for each return period.
7. The mean values and its standard deviations of the mean stress interval, S_{mr} , and the rate of positive zero v', are obtained for each return period.
8. The mean values of stress interval, S_{mr}, and the rate of zero positive, v', are estimated by $a(h_{máx})^b$.

After obtained the variables required by the modified Paris and Erdogan equation, the crack size is estimated as follows

1. The initial crack size is set over time, $t = 0$.
2. Simulate a timeout between storms.
3. Simulate a maximum wave height for a storm, considering the derivative of the wave hazard curve.
4. Estimate the mean, μ, and standard deviations, σ, of S_{mr} and v'; given $h_{máx}$.
5. Calculate the crack size, a , for operation waves.
6. Calculate the crack size for storm swell. The final crack size in service condition is considered as the initial condition for calculating the storm crack.
7. Steps 2 through 6 are performed while a is smaller than the section thickness.
8. Steps 1 through 7 are repeat
9. The size of the crack, a, its μ, and σ is calculated for each time, assuming a lognormal distribution.

Once obtained the size crack in time, there is consider that the structural deterioration in the platform is presented by the appearance of cracks in the tubular joints. [30] propose that the capacity of the intact joint, P_k, is modified by a linear reduction factor as follows:

$$P_c = P_k\left(1 - \frac{A_{crack}}{A_{join}}\right) \tag{15}$$

where P_c is the remaining capacity of the cracked joint, P_k is obtained based on [29], A_{crack} is the area of the crack and A_{join} corresponds to the cross-section area.

4.2 Evaluation of structural capacity over time

The structural capacity is obtained by means of non-linear static analysis "push-over", using fifty simulated lateral loadings profiles. The load profiles are used to describe the acting forces when the simulated waves produce the maximum base shear. The capacity reduction is given by the appearance of a crack in the tubular joints. According to the above, Fig. 5 a. shows the median of the capacity, $\hat{C}$, expressed in terms of the global displacement.

4.3 Evaluation of structural demand over time

The structural demand is estimated by subjecting the offshore platform to a series of dynamic "step-by-step" analyses. A set of fifty simulated waves associated with different maximum wave heights was used; the simulation of crack growth using in the preceding section is also used in this section. Fig. 5 b shows the median of the structural demand for a given wave height, the median of the structural demand is fitted as $\widehat{D} = (6\mathrm{E}-08)(h_{máx})^{b}$; for the case of without damage and for the case where the reduction of the structural capacity is considered. For the case that the structural capacity and demand vary over time, the following expression is as $\widehat{D}(\Delta t) = (6\mathrm{E}-08+4.1\mathrm{E}-09\Delta t)\cdot {h_{máx}}^{4.6}$.

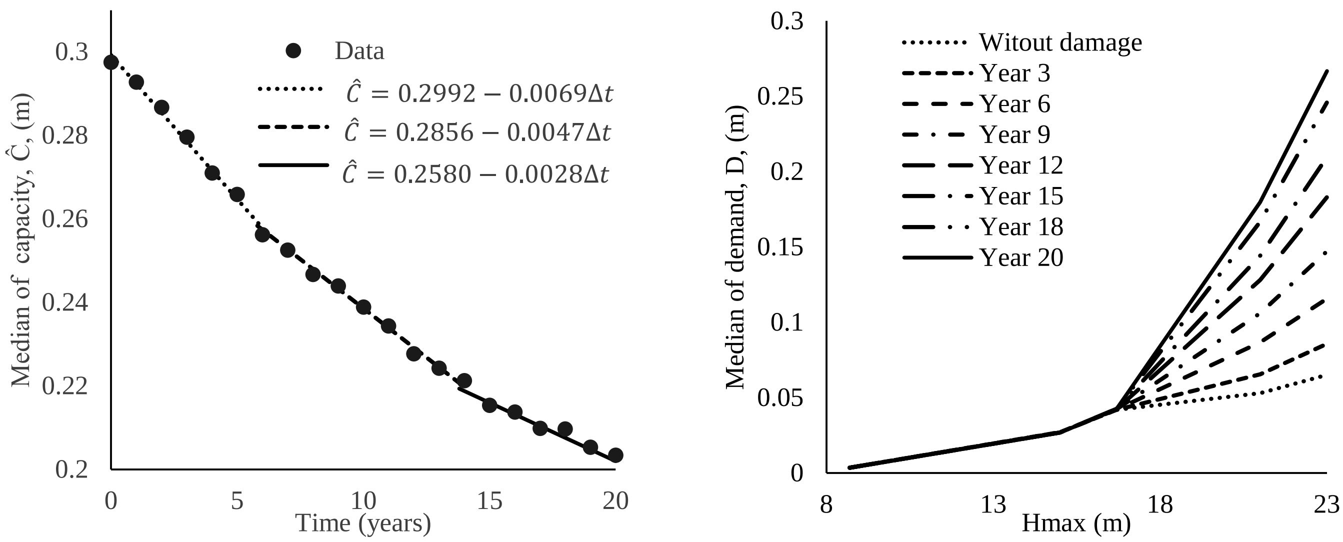

Fig. 5. a) Capacity and b) demand over time

4.4 Expected number of failures

In order to calculate the expected number of failures, it is considering a value of $\sigma_{UD}^2 = \sigma_{UC}^2 = 0.15$ [31] for epistemic uncertainties. The parameters k and r are equal to $k = 8114.47$ and $r = 5.2$. Fig. 6a shows the expected number of failures evaluated for three cases: a) without damage, b) the structural capacity is reduced over time c) the structural capacity and demand varies over time. It is observed that for instants of time less than 5 years, the expected number of failures present similar values between the three cases. Once the 5 years have been exceeded, it can be seen how the expected number of failures increases when the variation in capacity and demand over time is considered. The expected number of failures increases up to 124% when the variation of capacity over time is considered. On the other hand, the expected number of failures increases up to 212% between 0 years (without damage) and 20 years when the variation in capacity and demand over time is considered. The above implies that the structure experiments an increment in the expected number of failures about 88% when the variation of the demand over time is considered compared with the case b.

4.5 Confidence factor

The confidence factor over time is calculated by considering the cases a) without damage, b) the structural capacity is reduced over time c) the structural capacity and demand vary over time by using the Eqs. (4), (7), and (11), respectively. Fig. 6b shows the confidence factor for the three cases. In the approach without damage condition, it can be seen that there is no variation in the confidence factor over time. Undesirable structural behaviour is found when the confidence factor is lower than the unit, for cases a and b the confidence factor is greater than 1. In contrast, the marine platform presents an undesirable performance level after nine years of its construction for the case that takes into account the variation of capacity and demand over time. The confidence factor in the instant of 20 years is equal to 2.85, 1.52 and 0.37 for the cases a, b and c, respectively. The above implies a decrease of the confidence factor equal to 87% between without damage and when the structural capacity and demand vary over time.

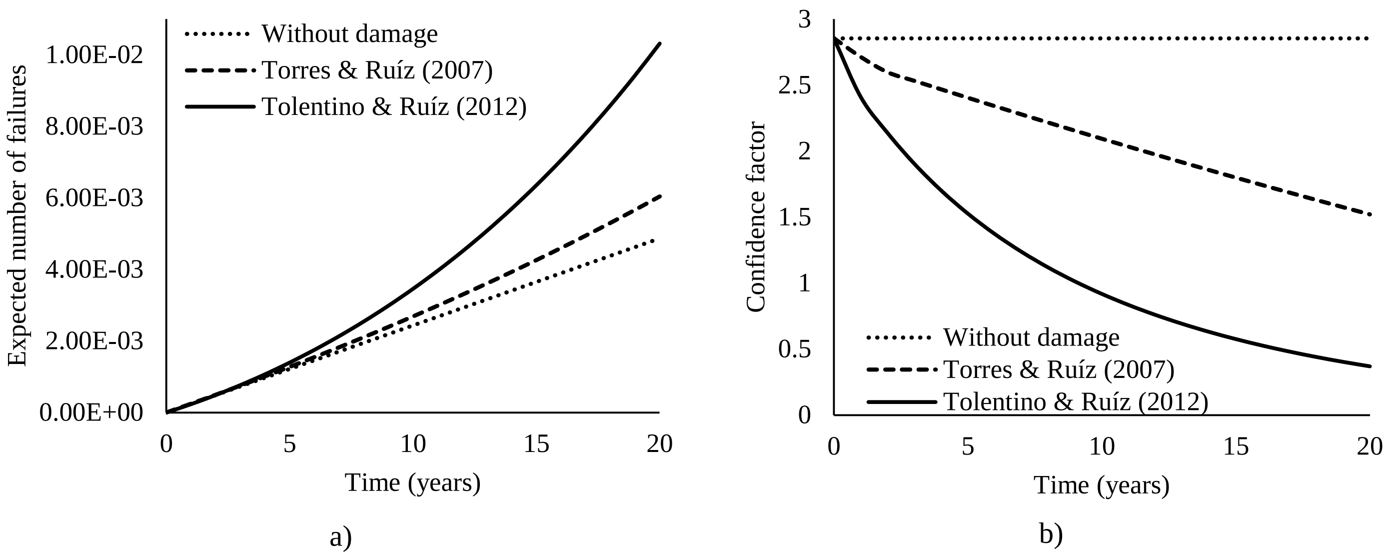

Fig. 6. a) Expected number of failures and b) Confidence factor

4. Conclusions

A comparative analysis to estimate structural reliability using three different approaches was presented. The structural reliability assessment was developed on a fixed offshore structure to define an effective approach to estimate a reliability analysis. The crack size in the tubular members due to fatigue was considered as the damage estimation parameter. Uncertainties related to environmental conditions (e.g., wind loads, waves and currents) were considered. Structural reliability was estimated in terms of the confidence factor and the expected number of failures. In the first phase, the damage is omitted to assess structural reliability, which represents a constant confidence factor of 2.85 over time. When the structural capacity is reduced over time, the effect of the cumulative damage follows a linear reduction of the confidence factor over time. Thus, the confidence factor presents a desirable reliability level with a value equal to 1.52 after 20 years. Finally, the effect of cumulative damage when it is considering the variation in demand and capacity, leads to undesirable reliability levels.

The useful life of the structure is affected when the cumulative damage is considered; however, due to the high importance of offshore fixed structures, it is important to consider all the variants to estimate the structural reliability in order to make a decision for design or repair actions, the above is represented by the approach in which the structural capacity and demand vary over time.

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