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Finite Element Simulation of Reinforced Concrete Beams under Fatigue Loading

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Abstract **-** Extensive attention has been directed towards the fatigue phenomenon since repeated loadings usually occur in highly important concrete infrastructures. Appreciable amount of experimental and numerical researches have been carried out to examine the RC beams behaviour subjected to monotonic loading. Conversely, limited studies have been conducted to investigate the performance of RC beams under fatigue loading. Up to this end, the principal objective of this study is to create an accurate two-dimensional finite element model to improve understanding the fatigue performance of RC beams. This is achieved by utilizing appropriate structural elements such that the concrete and supporting plates are modelled as 2D plane membrane elements, whereas truss elements are used to simulate the steel reinforcements. Regarding constitutive laws, suitable models are selected to accurately simulate the behaviour of concrete and reinforcing steel. For simplicity, full bond is assumed between the concrete and steel reinforcement. Additionally, the load-control method is used for fatigue load application. The cyclic load is characterized as a sinusoidal load with the maximum and minimum amplitude changed with a fixed frequency. The accuracy of the model is verified by comparing the numerical results to the experimental ones in terms of load-deflection relationship, load-carrying capacity, stress profiles, crack patterns and failure modes for various number of load cycles. Eventually, it can be concluded that the numerical model is capable of precisely simulating the real RC beam behaviour with acceptable degree of convergence when compared to the experimental results.

Keywords**:** Finite element simulation; Reinforced concrete beams; Fatigue loading; Fatigue Performance; Load cycles; Material Modelling; Structural Modelling; Load carrying capacity; Load-deflection relationship; Steel stress; Crack pattern

1. Introduction

Fatigue represents a phenomenon that occurs when the material is subjected to repetitive loading, which induces inner structural cumulative alteration. Consequently, this may result in internal cracking and lead to irreparable damage of the material properties including stiffness, load-carrying capacity, deflection, durability, etc. In some cases, fatigue loading causes complete breakage and failure of the material at large repetitive loadings [1]. Concrete bridges are type of infrastructures that have massive concern and must be deeply investigated since it is persistently vulnerable to fatigue loading cycles [2].

Concrete and reinforcing steel show diverse responses under fatigue loading. The latter experiences micro-cracks that grow through the fatigue life into discoverable sizes yielding to cumulative irreversible energy deformation by means of cracks and creep, and ultimately fracture. In addition, it has been proclaimed that the reinforcing steel represents the primary component affecting the fatigue behaviour of the reinforced concrete beams owing to the fact that redistribution of stresses occurs due to the evolution of the cracks and load cycles stresses; thus, the fatigue failure usually varies with the change in the stress level [2, 3, 4]. Therefore, it has been suggested by the ACI guideline [1] to maintain the maximum stress range as 161 MPa for a reinforcing steel exposed to fatigue loading, as given in equation (1). The fatigue loading causes notable strains and cracks in the concrete behaviour compared to its behaviour under static loading. This has limited the fatigue strength of certain structural members to 10 million load cycles and the fatigue life to 55% of its static strength, as stated by the ACI [1].

$$\Delta\sigma = 161 - 0.33\sigma_{min} \geq 136 \text{ MPa} \quad (1)$$

where $\Delta\sigma$ is the maximum allowable stress range (MPa); σ_{min} represents the minimum stress (MPa).

One of the most economical methods to investigate the behaviour of reinforced concrete (RC) beams subjected to repeated loadings as well as to explore the various parameters affecting the fatigue behaviour is through the development of a finite element (FE) model for preventing wastage of time, materials, resources and effort. Benjara and Ramanjaneyulu [6] developed a finite element model using the ATENA software. The model was capable of simulating the non-linear fatigue behaviour of RC beams such that the numerical predictions were in good consistency with the experimental results. On the other hand, the finite element model conducted by Huang *et al.* [7] studied the deterioration and degradation occurs on RC beams subjected to fatigue loading at both the constant and variable amplitudes. The numerical results for both constant and variable amplitudes presented were in good agreement with the experimental ones. Additionally, Benjara and Ramanjaneyulu [8] developed a finite element model to examine the fatigue life of plain concrete. The results acquired numerically showed good correlation with the ones obtained experimentally. Their model was able to propose an integrated stress-number of load cycles (S-N) curves for plain concrete.

Up to the authors' knowledge, there is a huge lack in the finite element models that have been developed to investigate the behaviour of RC beams subjected to repeated loadings. This may be attributed to the complex nature of cyclic loading, which includes many variables, such as cyclic loading conditions (upper and lower load limits, frequency and number of applied cycles), the repetitive nature of loads that may increase the rate of damage due to cumulative fatigue degradation of the beam. Therefore, the fundamental objective of this study is to investigate the complicated fatigue behaviour of RC beams accompanied by validating the accuracy of the finite element model to the experimental results.

2. Finite Element Model

In this study, the finite element (FE) model is developed through the utilization of VecTor2 finite element software. Owing to the fact the software modelling library includes comprised of strength, post-peak behaviour, failure mode, crack patterns, and deflections. Besides, the software is characterized by its facility in changing the diverse materials properties, their complicated loadings and geometries. Furthermore, it has the ability to perform the analysis under three different types of loads: monotonic, cyclic, and reverse cyclic loadings [9]. It can be mentioned that the software can simulate a non-linear analysis since it has been generated based on the modified compression field theory (MCFT). In this theory, the concrete is modelled as an orthotropic material that accounts for various applied loadings. The reinforced concrete membrane elements can represent the behaviour in combined shear and normal stresses, or in pure shear on the load-deformation behaviour, as shown in Fig. 1. This theory has showed accurate theoretical prediction of the shear strength for reinforced concrete members [9, 10]. In addition, the finite element model under fatigue loading has been developed according to certain assumptions [11]:

1. Cracks are uniformly distributed;
2. Reinforcement is uniformly distributed;
3. Loading in combined axial stresses and shear or loading in pure shear;
4. Cumulative strain history is neglected;
5. Full perfect bond between steel and concrete.

The library of VecTor2 contains several models to simulate the concrete and the reinforcing steel. Whereas, the authors recommend specific models upon a number of verifications. In this study, three materials are used to exemplify the concrete, reinforcing steel (longitudinal and transverse), and loading and support steel plates as follows:

2.1. Concrete

Concrete type is the paramount factor affecting the selection of appropriate FE model. In this study, the experimental tests selected were having a concrete compressive strength less than 40 MPa. Regarding the ascending branch of the stress-strain response of concrete strength in compression (compression pre-peak), Hognestad's parabola model [12] was utilized to simulate concrete behaviour, as shown in Fig. 2. In this model, the stress-strain relationship initiates linearly until reaching, approximately, 70% of the concrete compressive strength. Then the concrete behaviour becomes non-linear stress-strain up to failure [9]. The Hognestad's parabola is given with the following equation:

$$f_{ci} = -f_p \left\{ 2\left(\frac{\varepsilon_{ci}}{\varepsilon_p}\right) - \left(\frac{\varepsilon_{ci}}{\varepsilon_p}\right)^2 \right\} < 0 \; for \; \varepsilon_{ci} < 0 \tag{2}$$

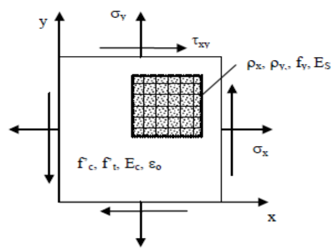
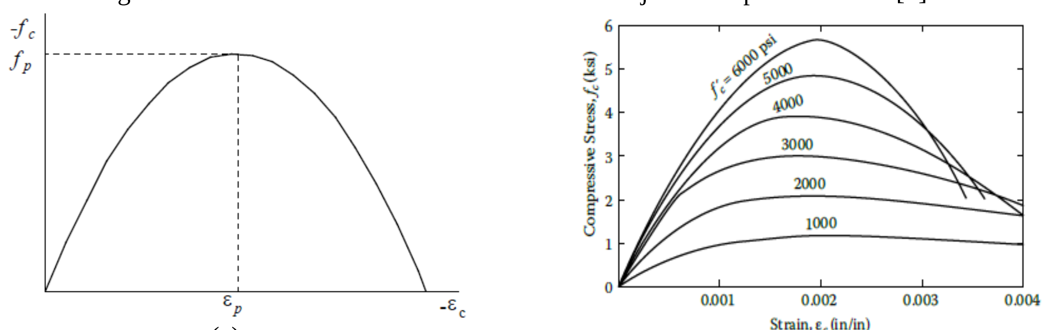


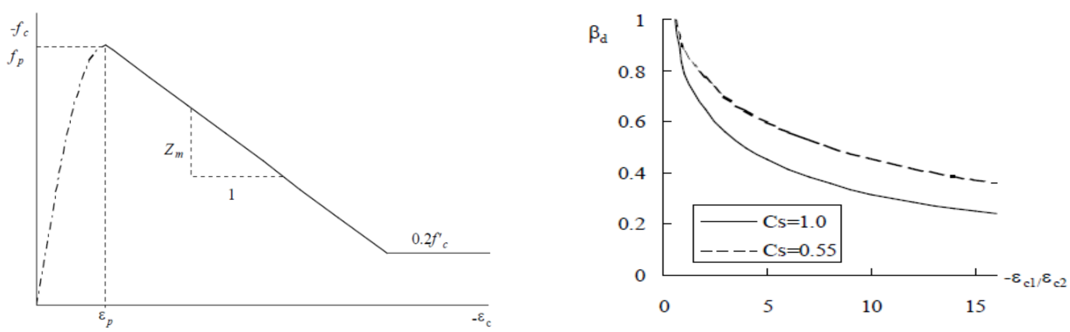
Fig. 1: Reinforced concrete membrane element subject to in-plane stresses [9]



(a) (b)

Fig. 2: Hognestad's parabola pre-peak model for concrete in compression [11, 12]

For modelling the descending branch of the stress-strain concrete curve in compression, the Modified Park-Kent model [13] was implemented (Fig. 3a). The model accounts for the confinement effect to improve the concrete ductility and strength. Furthermore, fatigue loading changes the slope of the stress-strain curve with the increase of load cycles. To account for this effect, Vecchio 1992-A model (Fig. 3 b) was utilized to precisely predict the compression softening effect by limiting the ratio of the principal tensile strain to principal compressive stress to 400. Additionally, the model provides high flexibility in considering the shear slip [9].



(a) (b)

Fig. 3: (a) Modified Park and Kent post –peak model for concrete in compression; (b) concrete compression softening Model by Vecchio 1992-A (e1/e2-Form) model [11]

The concrete tensile behaviour was simulated by employing the Modified Benz [9] model since it has the ability to simulate the tension stiffening, which is a significant feature to capture the load-deformation relationship. When concrete cracking occurs, the tensile stress diminishes instantly to zero without being redistributed to the reinforcement and the tension stiffening is ignored. As suggested by Hordijk [14], the softening branch of the concrete tension stiffening is represented as a non-linear behaviour (Fig. 4).

2.2. Steel Reinforcement

Steel reinforcement similarly behaves in both tension and compression such that for modelling purposes. The ductile stress-strain behaviour that connects the steel reinforcement bars to the concrete at certain mesh nodes is applied. It initially shows a linear response, which stops at the yielding stress; then, it offers a yielding stage. This is followed by a linear or non-linear strain hardening based on the type of steel. However, the ACI guideline [1] omitted the strain hardening effect by assuming linear level after yield stage, as illustrated in Fig. 5.

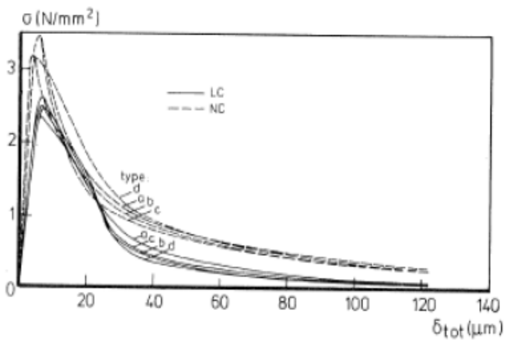


Fig. 4: Concrete Tension Softening Curves obtained by Hordijk [11]

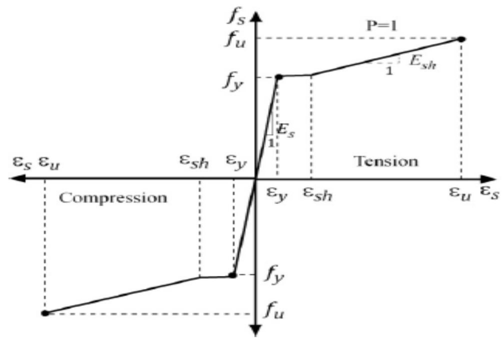
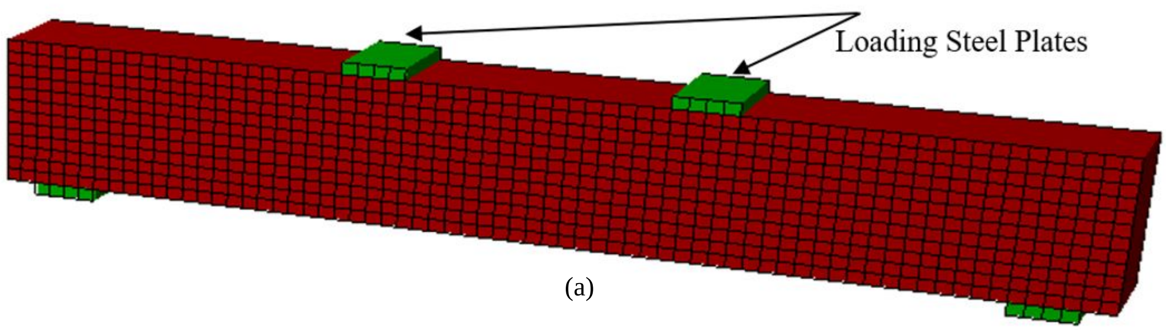


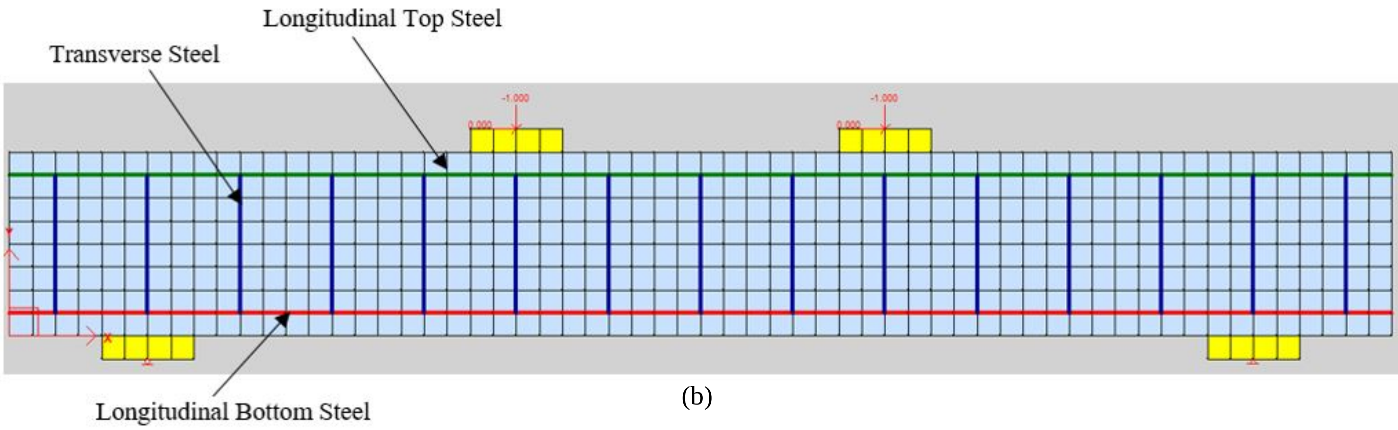
Fig. 5: Ductile Steel Model in Tension and Compression [11]

2.3. Loading Conditions, Mesh Size and Steel Plates

In this study, the performance of full scale beams of different dimensions have been simulated under fatigue loading to capture the entire and complete effect. The loading was applied using load-control for more precise observation of the load-deformation relationship. In the initial phase of analysis, the beams were modelled under monotonic loading to ensure accuracy of the numerical predictions. Accordingly, the fatigue loading was applied. With regard to the meshing type, hybrid rectangle regions were formed to simulate the concrete; whereas, truss elements were employed to model the steel reinforcement. Various element sizes were attempted; however, square mesh size of 25mm showed the accurate numerical results compared to the experimental ones. Moreover, structural steel plates were placed on the locations of applied loading and support points, identical to the real experimental tests. Figures 6 (a and b) illustrates typical beam models of this study.



(a)



(b)

Fig. 6: (a) Typical 3D Finite Element Model (VecTor2); (b) Typical 2D Finite Element Model (VecTor2)

3. Investigated Specimens

Different specimens have been considered from previous studies to investigate the fatigue behaviour of RC beams. Three different specimens tested experimentally by Dong *et al.* [15] and Murthy *et al.* [16] are numerically examined. To clarify, typical elevation view of the tested specimens is presented in Fig. 7. All of the studied specimens were of R-beam cross section type with a height ranges from 200 to 300 mm, and span length ranges from 1500 to 1700 mm. The Dong *et al.* specimen was tested under a fatigue loading of 2 Hz [15], while the Murthy *et al.* specimen was tested at 5 Hz [16] using a sinusoidal wave load-time history with stress range of between 15 and 87 percent of the ultimate failure load of the control beam. Note that the control specimen is only tested under monotonic loading, whereas the control fatigue is the one tested only under fatigue loading. All of the previously mentioned characteristics of the specimens are listed in Table 1. Noteworthy, for more specified information about any of the specimens, their corresponding references are placed at the end of the paper and can be reviewed.

Table [1]: Detailed Information about the Simulated Specimens

Beam	Type of Support	Type of Cross Section	Concrete Strength (MPa)	Fatigue Load Levels (%)	Frequency (Hz)	Beam's Length (mm)	Load Cycles to Failure	Type of Failure
FB30-1 [15]	Simply Supported	R-Beam	30	15 - 40	5	1700	1 Million	Flexural
FB2 [16]	Simply Supported	R-Beam	35	8.7 - 87	2	1500	35,000	Shear

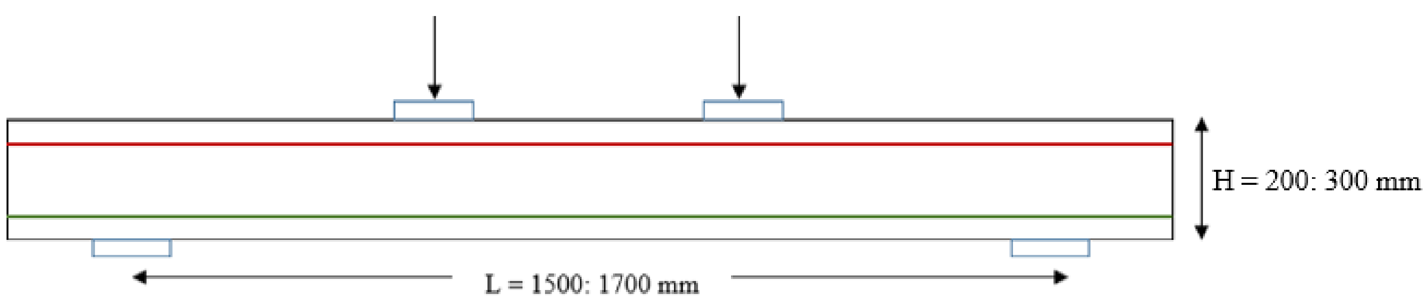


Fig. 7: Typical elevation view of the tested specimens

4. Numerical Results and Discussions

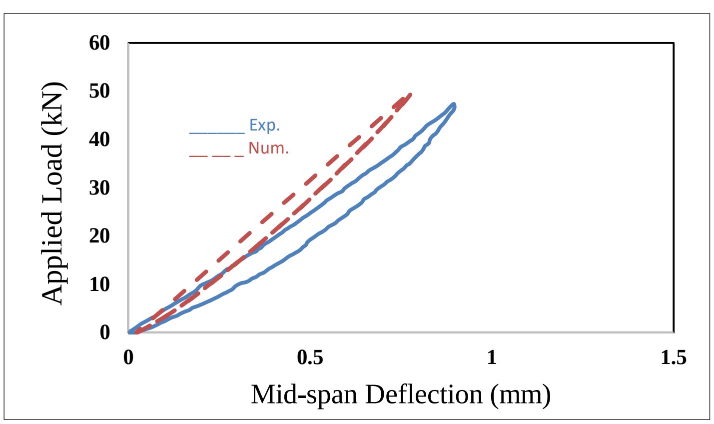
In this study, the numerical behaviour of the RC beams is initially investigated under monotonic loading to validate the accuracy of the model. The comparison of numerical results is carried out in terms of load-deflection relations, failure modes, steel stress, and load carrying capacity to those resulted experimentally.

4.1 Applied load-midspan deflection relationships

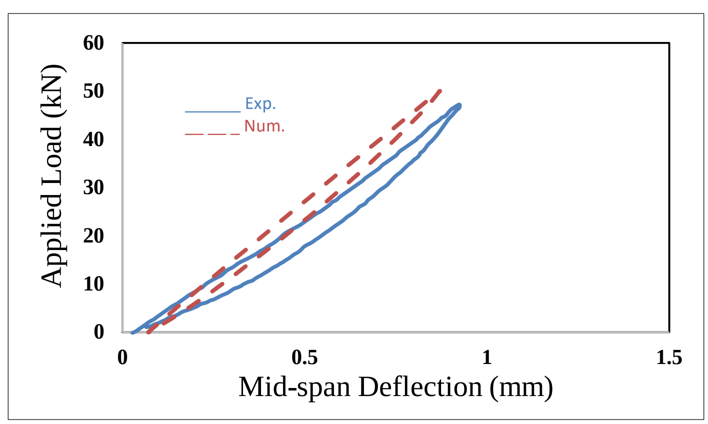
The finite element model developed in this study shows a great ability to predict the load-deflection relationship through presenting similar trend with the experimental ones. The degree of convergence between the experimental and numerical results is varied depending on the specimens considered, as illustrated in Fig. 8(a-e) and Fig. 9 also listed in Table 2. It can be concluded that the accuracy of the numerical predictions decreases as the number of load cycles increases. This is due to the applied assumptions used to develop the model such as ignoring the strain history, which considerably affects the mid-span deflection values. In addition, some assumptions were proposed by the author due to missing experimental data. It can be seen that the numerical model does not provide a failure mode identical to the experimental one under various number of load cycles.

Table [2]: Comparison between Experimental and Numerical Results

Beam	Experimental Results		Numerical Results		Num./Exp. Results		Cycle No.
	Ultimate Load (kN)	Mid-span Deflection (mm)	Ultimate Load (kN)	Mid-span Deflection (mm)	% of Ultimate Load	% of Mid-span Deflection	
FB30-1 [10]	47.368	0.897	50	0.781	5.556	-0.129	0
	47.207	0.924	50	0.871	5.916	-11.580	20
	47.229	1.026	50	0.871	5.867	-15.107	200
	46.419	1.172	50	0.881	7.715	-24.829	200,000
	46.927	1.314	50	0.876	6.548	-33.333	1,000,000
FB2 [11]	34	0	34	0	0	0	0
	34	5.365	34	3.945	0	-26.468	1600
	34	6.662	34	4.054	0	-39.147	34000
	34	6.802	34	4.152	0	-38.959	35000



(a)



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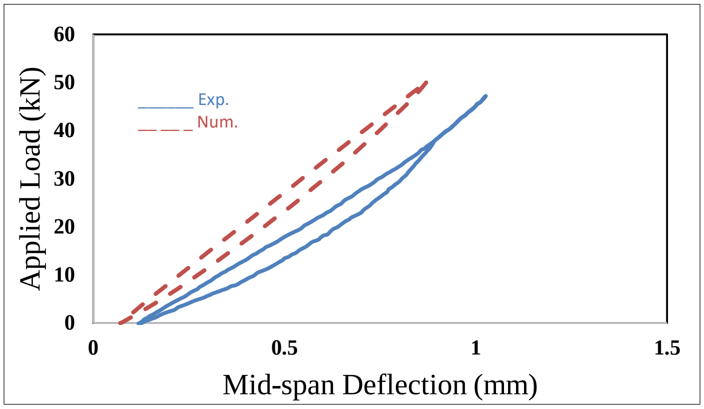

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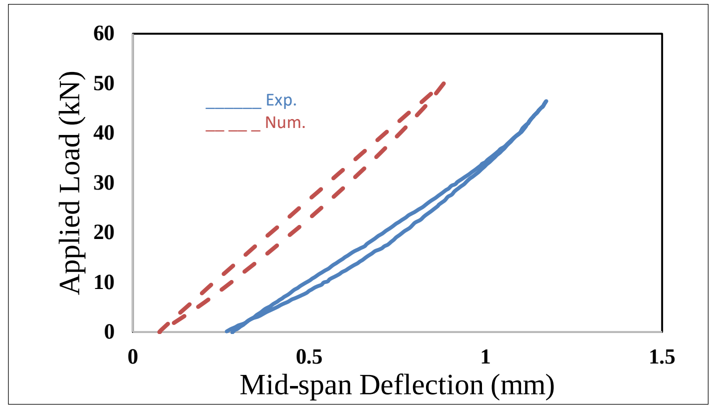

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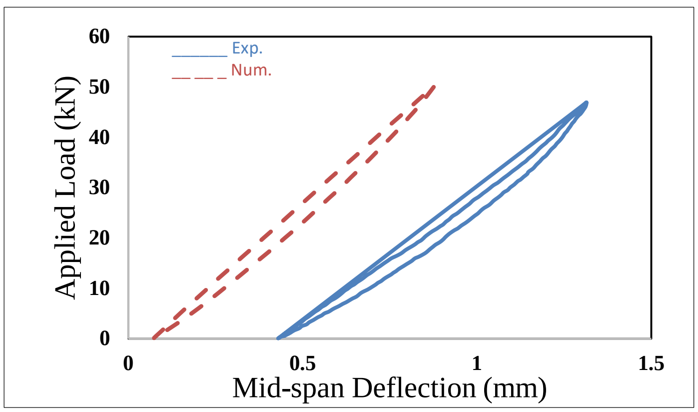


(e)

Fig. 8: Experimental versus Numerical Load-Deflection Relationship of FB30-1 at different Load Cycles: (a) Zero; (b) 20; (c) 200; (d) 200,000; (e) 1,000,000

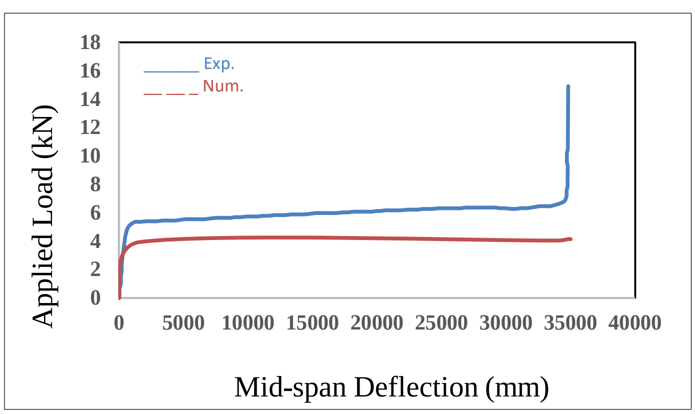
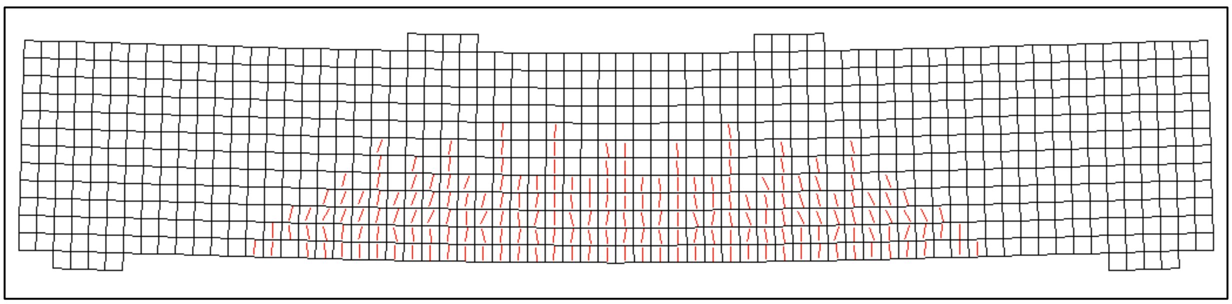


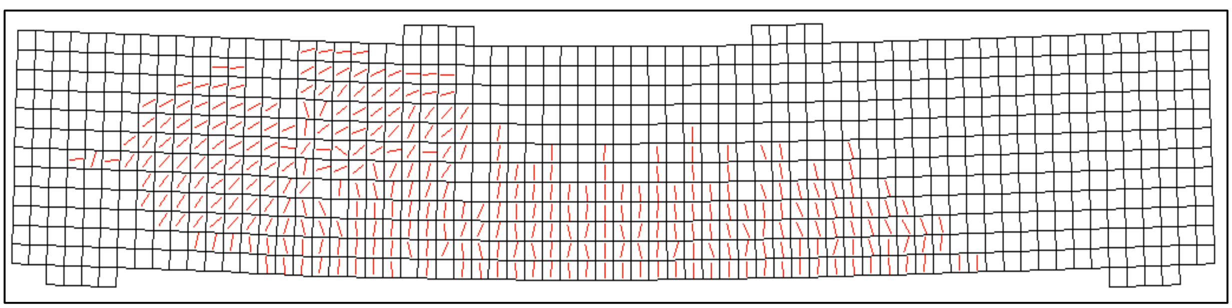
Fig. 9: Experimental vs. Numerical Load-Deflection Relationship of FB2 at different Load Cycles

4.2 Crack Pattern and Reinforcement Stresses

The crack patterns resulted from the fatigue load application under zero load cycles and 200 load cycles are in Fig. 10. It should be mentioned that no significant difference is observed with the increase of the number of load beyond 200 cycles. On the other hand, for the reinforcement stresses, it is noteworthy that the regions where the yield approximately equals the induced stress are regions of possible fracture. Besides, the reinforcement stresses increase as the number of load cycles increases, as shown in Fig. 11 (a, b). Similar to crack pattern, no difference is detected beyond 200 loading cycles.



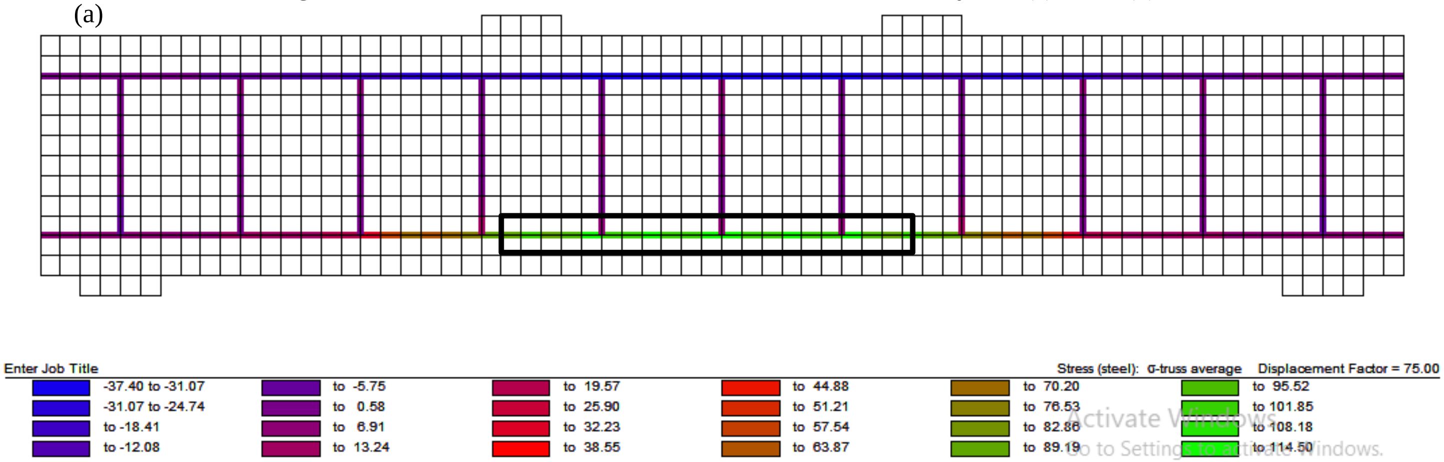
(a)



(b)

Fig. 10: Numerical Crack Pattern of FB30-1 at different Load Cycles: (a) Zero; (b) 1,000,000

(a)



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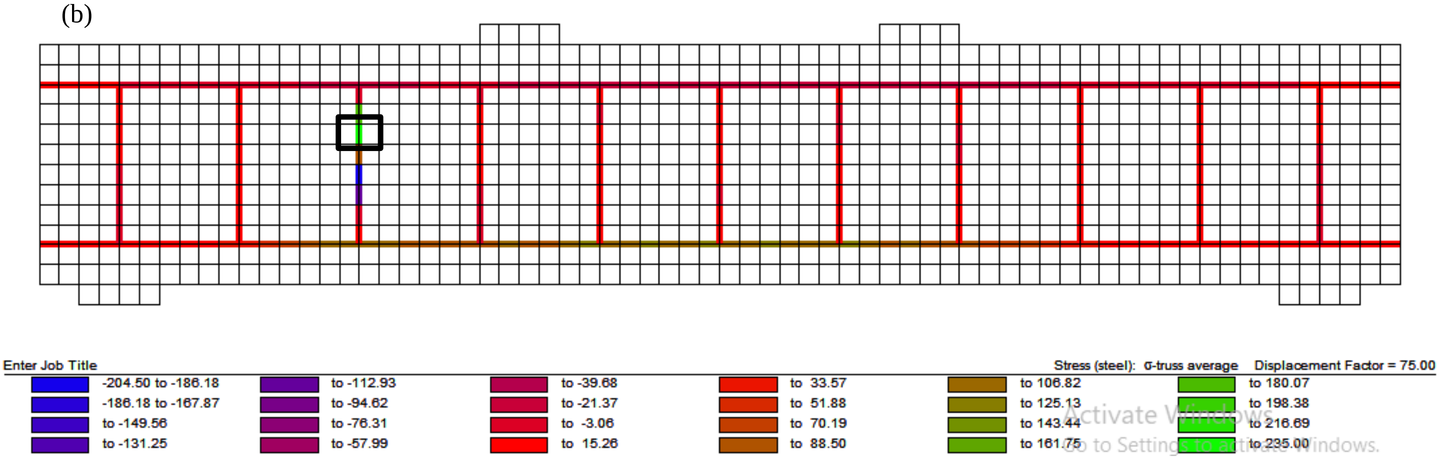

Fig. 11: Numerical Reinforcement Stresses of FB30-1 at different Load Cycles: (a) Zero; (b) 1,000,000

5. Conclusions

This study shed the light on the fatigue behaviour of RC beams by developing a finite element model that can accurately detect such behaviour and acquire clear view of the degradation of mechanical properties as the number of fatigue load cycles increases. The finite element model was developed using the VecTor2 finite element software. After the model was verified against experimental results under monotonic loading, fatigue loading was applied and the followings can be concluded:

1. The modeled specimens showed similar behavior regarding the load-deflection relationship under both monotonic and fatigue loadings. Moreover, similar results were obtained regarding the mode of failure when the monotonic load was applied;
2. The numerical model was capable to simulate the experimental load-deflection relationship with acceptable degree of correlation up to 1 million cycles;
3. The finite element model showed crack patterns and the reinforcement stresses similar to the ones observed experimentally.

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