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Application of Nonlinear Consolidation Theory to Investigate Slurry/Marsh Soil Consolidation Behavior at Louisiana Coast

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***Abstract* -** This study delves into the consolidation behavior of Louisiana marsh soil, employing a one-dimensional nonlinear approach and leveraging data from ongoing wetland restoration projects and a series of laboratory experiments. A precise model is developed to accurately predict slurry/marsh consolidation, guided by tactful interpretations of oedometer test results and incorporating specific parameters tailored for large strain consolidation. The research draws on Gibson's nonlinear consolidation theory, implemented in MATLAB, which accounts for essential factors such as the finite strain coefficient of consolidation (g), variable compressibility coefficients (λ), and other soil properties. The study reaffirms the model's robustness through validation against laboratory and field data. The findings, encompassing settlement-time curves, degree of consolidation-time profiles, and void ratio fluctuations along with depth, provide indispensable tools for field engineers in estimating allowable loads, predicting settlement, and designing coastal restoration projects. This contribution enhanced the understanding of self-weight consolidation in geotechnical engineering practices. This research has significant implications for advancing coastal restoration strategies and the sustainable management of wetlands.

***Keywords*:** Consolidation, Finite strain non-linear theory, Degree of Consolidation, Numerical analysis.

1. Introduction

This study focuses on examining the consolidation process of Louisiana marsh soil in coastal restoration projects. It aims to investigate the consolidation following the one-dimensional nonlinear theory, using laboratory and field data collected from ongoing projects that involve creating marshlands. To effectively predict the consolidation progress in these marsh soils, employing a precise consolidation model, correctly interpreting laboratory consolidation test results, and determining the specific consolidation parameters required for the large strain consolidation model are crucial.

Numerous models were created previously to explore the process of one-dimensional self-weight consolidation (Duncan 1993; Wang et al. 2020). Among these, the conventional model, also known as the infinitesimal strain model (Terzaghi 1924) or finite strain model (Gibson et al. 1967; Gibson et al. 1981), is most commonly employed. Gibson et al. (1967; 1981) derived the governing equation for one-dimensional finite strain consolidation using a broader set of assumptions than is typically used. Notably, they did not restrict their model to minor strains and accounted for soil compressibility and permeability changes during consolidation. However, the high nonlinearity of the Gibson finite strain consolidation equation has made finding analytical solutions challenging. Various solutions, such as analytical, semi analytical, and numerical approaches, have been developed for one-dimensional large strain consolidation, frequently employing simplified compressibility and permeability relationships for ease of problem-solving. Gibson et al. (1981) offered a solution for the linearized version of this model in the Lagrangian coordinate system. Despite its limitations, the linearized Gibson et al. model has been widely adopted in finite strain consolidation theory as a reference point. It has been the basis for numerous studies (Schiffman et al. 1994; Wang et al. 2020). In the field of geotechnical research dedicated to understanding the self-weight consolidation behavior of Louisiana marsh soil, a numerical model has been developed inspired by Gibson's nonlinear consolidation theory. This model has been explicitly tailored to predict how Louisiana marsh soil responds to self-weight loading, providing valuable insights into soil compression and settlement processes.

In this research, the core of our approach relies on applying Gibson's nonlinear consolidation theory, which is pivotal in understanding the nonlinear attributes of soil consolidation. Our methodology centers on utilizing solution charts derived from a MATLAB-based numerical solution of the dimensionless governing equation from Gibson's nonlinear finite strain consolidation theory. We employ numerical algorithms in MATLAB to solve these governing equations, facilitating the consolidation analysis over time. This enables us to make predictions regarding the settlement and degree of consolidation concerning time, forming the basis for a comprehensive assessment. We also assess variations in the void ratio (e) with depth, utilizing Gibson's equations to offer a holistic perspective on how soil responds during consolidation. To ensure the reliability of our numerical MATLAB model, we validate it by comparing its predictions with experimental data obtained from self-weight consolidation tests and modified oedometer tests. By using this theoretical framework, our model is designed to offer a dependable way of forecasting Louisiana marsh soil's consolidation behavior.

2. Methodology

2.1. Test Material

Slurry soil specimens were created from the excavated soil mass obtained at the No-Name Bayou Swamp production site in Louisiana, representing the in-situ characteristics of the dredged soils. Prior to preparing these slurry samples for consolidation testing, we conducted preliminary investigations. According to the Unified Soil Classification System ASTM D2487 (Howard 1984), the dredged soil samples were categorized as Low Plastic Clay (CL). We determined the specific gravity (G_S) following ASTM D854 (ASTM 2016) and assessed the Atterberg limits in accordance with the ASTM D4318 (ASTM 2018) standard. The procedure involved initial oven-drying of the soil samples for 24 hours, followed by pulverization using a grinder. The dried soil mass served as the base material for generating the slurry samples. Water was then added to the crushed dry soil to create a uniform slurry with the desired moisture content. The wet soil mixture was subsequently placed in a mixing drum and agitated for approximately one hour until it achieved homogeneity. The resulting slurry samples exhibited moisture contents of 210.1%, 180.3%, and 150.2%, respectively. To emulate natural conditions and reduce initial moisture content before conducting oedometer consolidation tests, we introduced the slurry samples into acrylic settling column cylinders with a height of 10 inches. Under the influence of gravity, the soil particles settled, causing a decrease in the void ratio as pore water migrated to the top of the settled soil. Each test took nearly four weeks to complete, enabling self-weight consolidation. For the slurry samples with moisture contents of 210.1%, 180.3%, and 150.2%, following the settling column cylinder tests, the average moisture content at the top of the settled soils decreased from 150% to 60% at the bottom of the settling column cylinder. After completing each settling column test, we first removed water from the top of the cylinder using a hand pump. Subsequently, soil specimens were extracted from various depths to perform oedometer consolidation tests. We enhanced a conventional oedometer using 3D printing technology to conduct consolidation tests on soil samples from the settling column tests. The results from both the settling column and modified oedometer tests were presented in the MS thesis (Apu 2022).

Table 1: Fundamental geotechnical properties of test soil slurries.

Specific Gravity (G_S)	Unite weight of solids, γ_s (KN/m^3)	Liquid Limit ω_l %	Plastic limit ω_p %	Clay content %(<5 µm)	Initial water content %
2.6	25.9	36 - 42	17 - 28	36.8	150.2 - 210.1

The research encompasses a comprehensive analysis of void ratio and effective stress profiles, as well as a wide range of the coefficient of variation (λ) vs. effective stress profile derived from laboratory experiments. The experiment results and soil parameters determined in the MS thesis (Apu 2022) and in papers authored by Apu et al. 2021, Sarker et al. 2021, Apu and Wang 2022; 2023, Sarker et al. 2023 were utilized in this study. The essential physical properties of the slurry samples can be found in Table 1.

2.2. Application of Gibson's Theory

In this study, we've employed Gibson's non-linear consolidation theory to analyze the consolidation data of Louisiana Louisiana mash soil. Gibson et al. (1981) introduced Equation 1 as the governing equation and provided several non-dimensional variables outlined in Equations 2 through 7. Where σ is effective stress, l is the equivalent height of solids for for the dredge soil layer, g is the finite strain-based coefficient of consolidation, γ_s is unit weight of solids and γ_f is unit unit weight of water. This approach considers the non-linear behavior exhibited by marsh soils during the consolidation process.

$$\frac{\delta^2 e}{\delta z^2} \pm \left(\gamma_s - \gamma_f\right) \frac{d}{de}\left(\frac{de}{d\sigma}\right) \frac{\delta e}{\delta z} = \frac{1}{g} \cdot \frac{\delta e}{\delta t} \tag{1}$$

To simplify the numerical work, Gibson et al. (1981) introduced various non-dimensional variables E, Z, T, N, B and R. Where, λ is coefficient of variation, which has an exponential relationship with the void ratio and the effective stress.

$$E(z,t) = e(z,t)\ /\ e(0,0) \tag{2}$$
$$Z = z/l \tag{3}$$
$$T = \ gt/l^2 \tag{4}$$
$$N = \lambda l\left(\gamma_s - \gamma_f\right) \tag{5}$$
$$B = e_\infty\ /\ e(0,0) \tag{6}$$
$$R = e(0,t)\ /\ e(0,0) \tag{7}$$

Substituting the non-dimensional variables into Equation 1 the governing equation becomes.

$$\frac{\delta^2 E}{\delta Z} + N \frac{\delta E}{\delta Z} = \frac{\delta E}{\delta T} \tag{8}$$

Here Equations 8 to 11 present the mathematical model of the Gibson's nonlinear consolidation theory. Where Equation 9 is the initial condition and Equations 10 and 11 are the boundary conditions.

$$E(Z,0) = (1-B)\exp(-NZ) + B; 0 \le Z \le 1 \tag{9}$$
$$E(0,T) = R; T > 0 \tag{10}$$
$$E(1,T) = (R-B)\, exp(-N) + B; T > 0 \tag{11}$$

And then using Equations 12 and 13, the settlement $S(T)$ and the degree of consolidation $U(T)$ have been calculated and presented in this study.

$$S(T) = \int_0^1 [e(z,0) - e(z,t)]\, dz \tag{12}$$

$$U(T) = \frac{s(T)}{s(\infty)} = \frac{\int_0^1 [e(z,0) - e(z,t)]\, dz}{\int_0^1 [e(z,0) - e(z,\infty)]\, dz} \tag{13}$$

$$g = \ c_v/(1+e)^2 \tag{14}$$

We have applied Equation 14 to convert the finite strain coefficient of consolidation (g) from the coefficient of consolidation (c_v). The coefficient of consolidation was calculated from the experiment for varying the effective stress increments. These equations have been selected in accordance with the specific conditions under investigation, allowing us to make accurate predictions and assessments.

2.3. Numerical Modeling in MATLAB

In the phase of numerical modeling, we've created a MATLAB-based model to replicate the self-weight process. This model is rooted in the fundamental principles of Gibson's consolidation theory. Prior studies by Ames (2014), Cargill (1984) and Gibson et al. (1981) have asserted that solving Equation 8 while adhering to Equations 9 through 11 is a straightforward task. Gibson et al. (1981) employed the computer program FSCON1 for these calculations. However, in our work, we've transitioned to MATLAB for the numerical solution. Our MATLAB model takes input data, including fundamental soil properties, initial and boundary conditions, and relevant data from laboratory consolidation tests. The numerical algorithms were implemented in the MATLAB environment to solve the governing equations. A set of the numerical results provided insights into consolidation progress over time, allowing us to make predictions regarding settlement and degree of consolidation, forming the basis for a comprehensive analysis. We assessed variations in void ratio with depth using numerical simulation, offering a comprehensive view of how the soil responds to the consolidation process.

2.4. Model Validation

Validation of our numerical model represents a pivotal phase in the study. To accomplish this, we conduct a comparison between the model's predictions and the experimental data obtained from both the self-weight consolidation tests and the modified oedometer test. This validation process involves scrutinizing the void ratio vs. effective stress profiles generated by the MATLAB-based numerical model and comparing them with those obtained through laboratory experiments. This comparative analysis serves as a critical measure to evaluate the accuracy and suitability of Gibson's theory in characterizing the consolidation behavior of Louisiana marsh soil. Any disparities or congruencies between the theoretical and numerical predictions are comprehensively addressed within the results and discussion section, offering insights into the performance of the model and the underlying theory.

3. Results and Discussion

In our study, we applied the MATLAB-based numerical algorithm, as presented, to predict the consolidation properties of Louisiana marsh soils. The inputted soil parameters in the numerical model were directly derived from a series of laboratory experiments and field data. Initially, at the lower boundary, the effective stress stood at 0.02 kPa (0.002 TSM), considering the soil's saturation. After applying 29.4 kPa (3 TSM) pressure at the top of the surface during the modified oedometer consolidation test, the corresponding void ratio e_∞ reached out to 1.62 upon completion. Although λ and g are subject to change regarding effective stress and void ratios, Gibson et al. (1981) considered them constant. Following Gibson's work, we determined the average values of λ and g for each applied consolidation load-

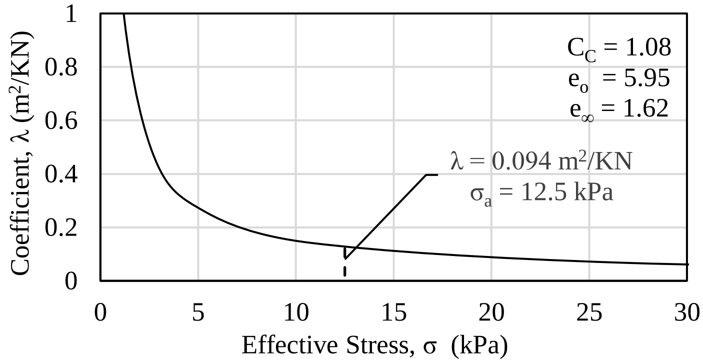


Figure 1: λ vs. effective stress relation.

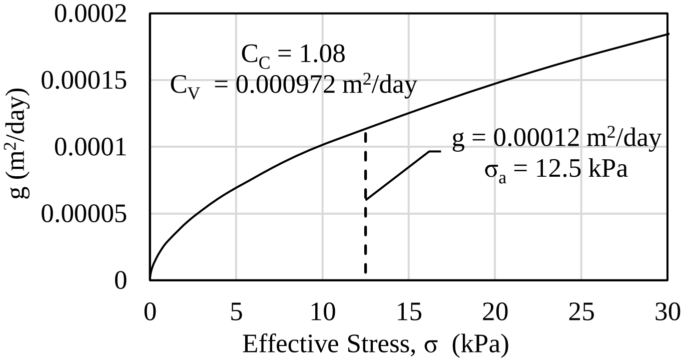
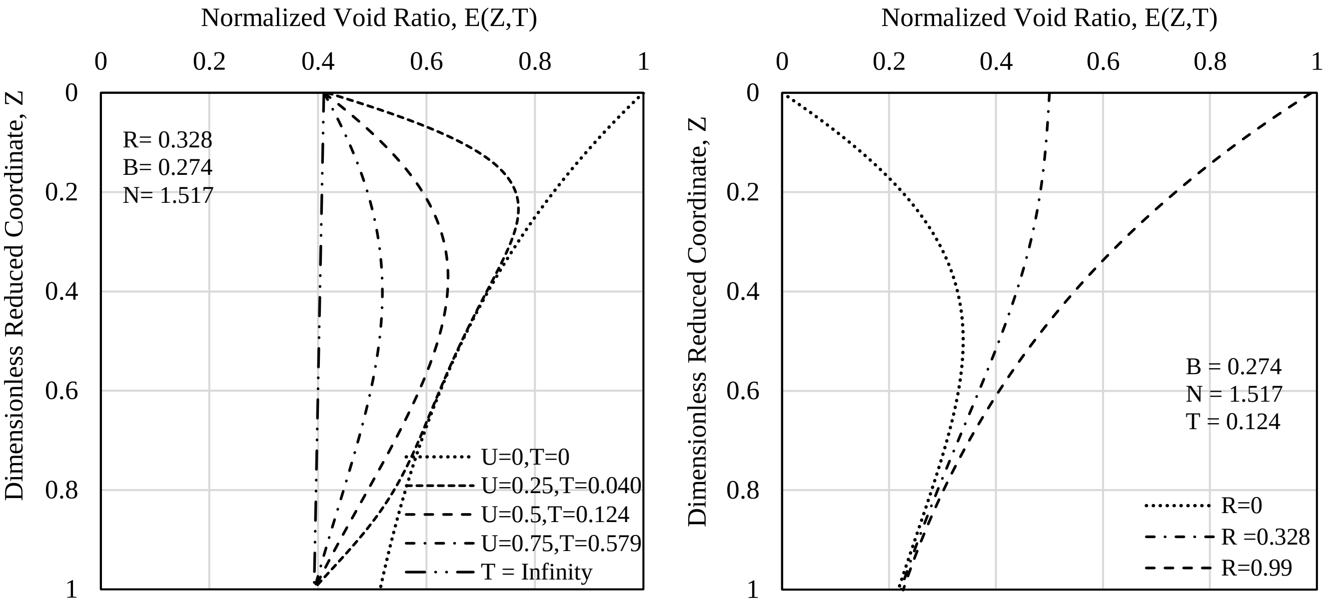
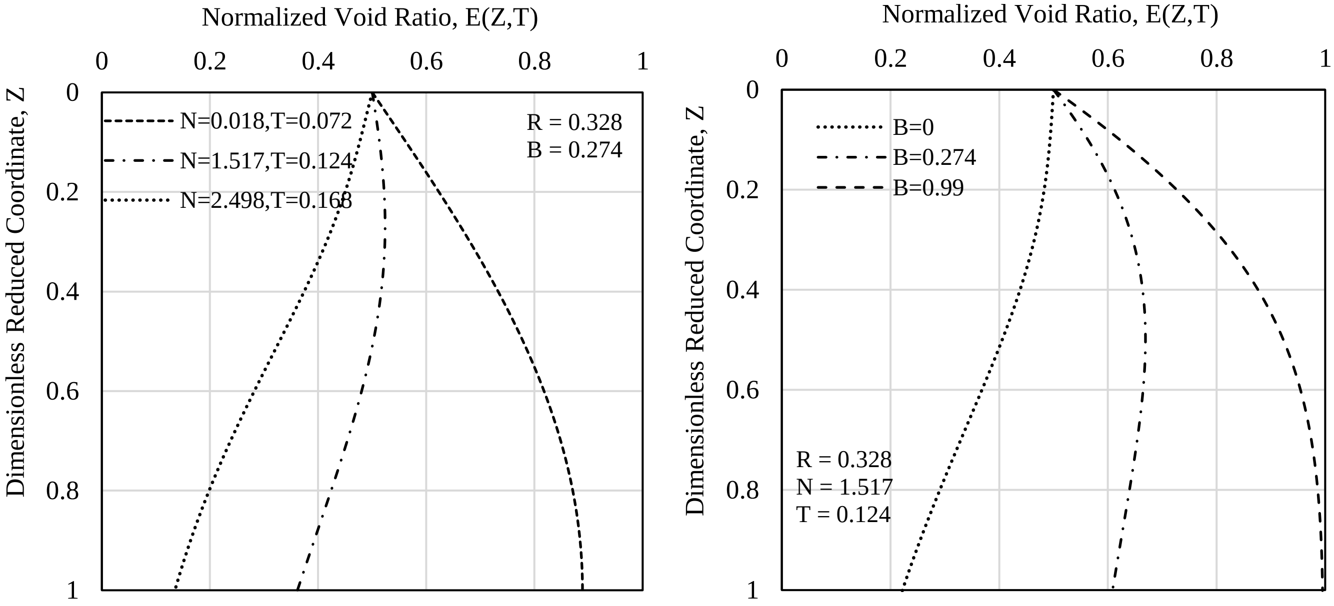


Figure 2: Finite strain coefficient of consolidation g as a function of the effective stress



3(a) Isochrones of normalized void ratio at different degree of consolidation

3(b) Isochrones of normalized void ratio at 50% consolidation and different R values



3(c) Isochrones of normalized void ratio at 50% consolidation and different N values

3(d) Isochrones of normalized void ratio at 50% consolidation and different B values

Figure 3: MATLAB based numerical results for the marsh slurry soils created from original soils excavated at the No-Name Bayou marsh creation project site at coastal Louisiana.

-on the top boundary to reduce the complexity of solving the governing equation. Leveraging the values of initial void ratio (e_0) and ultimate void ratio (e_∞) which are 5.95 and 1.62, respectively, we generated a plot of λ versus effective stress, as presented in Figure 1, based on the self-weight settling column and modified oedometer consolidation tests. At an average effective stress (σ_a) of 12.5 kPa (1.248 TSM) and an average void ratio (e_a) of 1.98, we found that λ equals 0.094 m^2/KN (0.925 m^2/T). We calculated g, by considering its relationship (Equation 14) with the traditional coefficient of consolidation (c_v) as mentioned by Gibson et al. (1981) and presented in Figure 2. Consequently, based on the average effective stress, the forthcoming result of g value is 0.00012 m²/day. These soil properties were incorporated into the numerical model to simulate the consolidation process of the marsh soils.

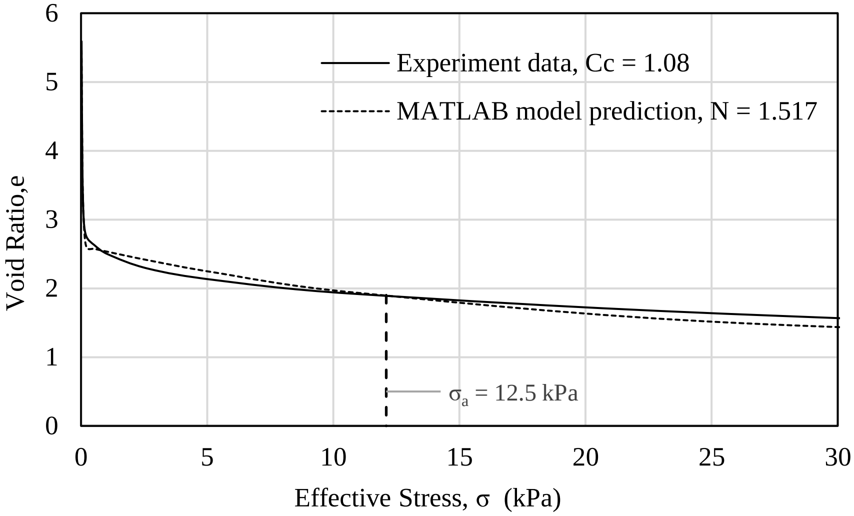


Figure 4: Void ratio - effective stress relationship.

Figure 3 represents void ratio isochrone for a layer of marsh soils with an impervious bottom and a pervious top. This example sets initial void ratio $e(0,0)$= 5.95, and after completion of the settlement test void ratio of the top layer $e(0,t)$= 1.95. Using Equations 5 to 7, the dimensionless coefficients then become N = 1.581, B = 0.274 and R = 0.328. According to Equation 5, N is related to equivalent height of solids *l*, relative density $(\gamma_s - \gamma_f)$ and λ, which means N represents the nondimensional effective stress. Similarly, according to Equations 6 and 7, B represents the normalized ultimate void ratio and R represents the normalized void ratio at the top layer after completion of the settlement. At 25, 50, and 75% consolidation (Degree of Consolidation U_{25}, U_{50} and U_{75}) , the values of the time factor are T_{25} = 0.0401, T_{60} = 0.1248, and T_{75} = 0.5785. Figures 3(a) through 3(d) provide insights into how variations in N, R, and B impact the consolidation process. MATLAB was used for the numerical analyses of the consolidation process, and Figures 3(a) to 3(d) are the visual representations of the results of that analyses.

In Figure 4, a comparison is presented between the experimental and numerical results regarding the void ratio-effective stress relationship. Notably, the numerical solution presented by MATLAB provides a reliable prediction. This observation supports that the MATLAB numerical model, grounded in Gibson's nonlinear consolidation theory, offers a trustworthy predictive capability. As to Figure 5, we delve into the non-dimensional settlement-time relationship. This figure displays settlement-time curves of the top layer for different N values. The normalized ultimate consolidation settlement varies from 0.14 to 0.48. In each case, the values of λ and g were computed based on the average ultimate effective stress and the corresponding ultimate void ratio. We consistently employed a value of 5.95 for e_0. Figure 6 illustrates the degree of the consolidation-time relationship. It's interesting to note that Gibson et al. observed that the degree of consolidation depends solely on the parameter N (Gibson et al. 1981). Our study has verified the conclusion made. According to Equation 5, the N values are contingent upon λ, a parameter directly linked to effective stress and the associated void ratio. Analysis of Figure 1 reveals a noteworthy trend wherein λ decreases as effective stress increases. As effective stress rises, the void ratio decreases, lowering the λ value and, consequently, the

N value. This signifies that a reduced N value corresponds to elevated effective stress, suggesting a more rapid completion of the consolidation process, as depicted in Figure 6.

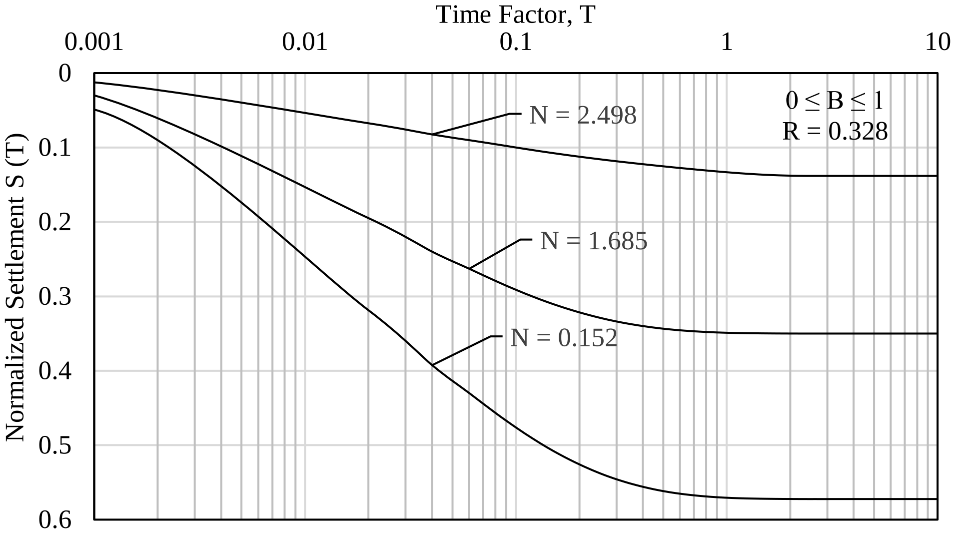


Figure 5: Non-dimensional settlement vs Time factor.

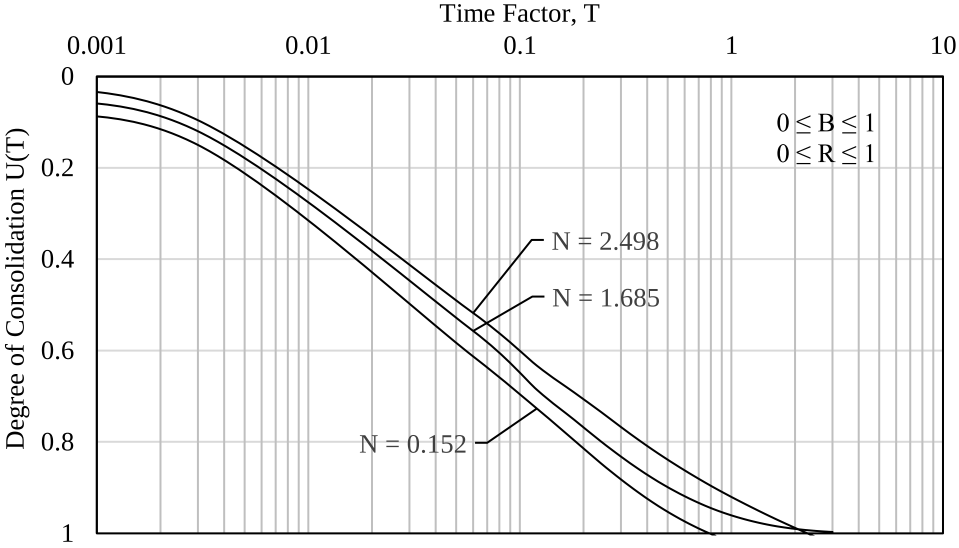


Figure 6: Degree of consolidation vs Time factor.

4. Conclusion

Following the analysis, the experimental and numerical results are comprehensively discussed, with attention emphasized on their implications within the field of geotechnical engineering and their relevance to the specific characteristics of marsh soils in coastal Louisiana. The key findings are:

- The study draws from a wide range of literature and research sources, including Gibson's theory and MATLAB-related references, establishing a robust academic foundation for the research.

- A specialized numerical model, grounded on Gibson's non-linear consolidation theory, has been developed to simulate the self-weight consolidation behavior of Louisiana marsh soil. This model offers valuable insights into soil compression and settlement dynamics.
- The study's results, including settlement-time and degree of consolidation-time curves and variations in void ratio with depth, can serve as practical tools for field engineers.
- The model's core strength lies in its numerical solution of the consolidation process, providing updates to the void ratio and effective stress at each time step. This approach facilitates a comprehensive understanding of how self-weight influences consolidation over time.

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