Effect of a Long-Duration Irregular Excitation on the Dynamic Behavior of a Vertically Baffled Base-Isolated Rectangular Fluid Container

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Abstract - The dynamic behavior of a partially filled rectangular fluid container isolated by nonlinear hysteretic bearing is numerically investigated. The computational fluid domain with the presence of a submerged vertical baffle is modeled based on a velocity potential-based Galerkin’s finite element method. The competency of the developed model is verified with the existing results. A long-duration irregular harmonic motion is imposed for the time domain analysis. The results indicate that the isolation system has excellent adaptability for different tank-fluid-baffle configurations and is very effective in controlling critical dynamic responses of the fluid tank such as hydrodynamic base shear and sloshing amplitudes. The peak values of the hydrodynamic responses for base-isolated and non-isolated tanks are evaluated for different heights and widths of the submerged baffle, where the effect of change in h/d ratio has a significant impact on the variation in sloshing amplitude and base shear responses as compared to the change in w/L ratio. The application of base isolation reduces the sloshing amplitudes by 12 to 34% in different configurations of the baffle except at h/d=0.5. The base shear is also reduced by 2 to 48% in base-isolated tanks depending upon different baffle configurations. The isolator displacement is invariably found to be maximum when h/d=0.5, irrespective of the width of the baffle.

Keywords: Nonlinear hysteretic bearings, Galerkin’s finite element method, submerged baffle, long-duration irregular harmonic motion.

1. Introduction

Liquid storage tanks are the important lifeline structures in various field of applications essentially in public sectors, industries and nuclear power plants. The behavior of free surface oscillation of liquid inside these applications due to external physical disturbance is commonly termed as sloshing. Generally, the sloshing phenomenon has a significant impact on the dynamic characteristics of liquid storage containers when subjected to external forces and subsequently it has a control over them. Due to the excessive hydrodynamic pressure generated by sloshing, it has been observed that tanks utilised for the storage of different chemicals, fuels, and water are susceptible to significant damage in earthquakes. Consequently, the damage to these containers poses risks including fire and contamination, which can cause many societal problems [1]. Therefore, the investigation of sloshing characteristics in partially filled containers has been a paramount interest.

In order to suppress the adverse effect of liquid sloshing phenomena, internal baffles are used within partially filled containers as anti-sloshing devices in order to mitigate the intensity of liquid sloshing. Therefore, many researchers have numerically examined the importance of comprehending the influence of submerged baffle within the containers, which has significantly affected the dynamic behavior of liquid sloshing [2-4]. Also, many studies have investigated the influence of vertical baffles with different configurations on suppressing the sloshing pressure through an experimental method [5,6]. Jiang et al. [7,8] examined the effect of internal baffles on suppressing the sway motion of the tank coupled with the internal sloshing flow under the wave action. In summary, the aforementioned literature primarily concentrated on the sloshing responses under the regular excitation or regular wave action, whereas in reality, the liquid containers may undergo irregular external excitations. In general, the hydrodynamic behaviour of the sloshing response under irregular motion is more complex than that under regular excitation. Emphasizing this point of view, a few works have focused on the effect of various irregular excitation in liquid sloshing problems in ground supported tanks [9-11].

Sometimes the mitigation of the intensity of liquid sloshing through internal components alone may not be sufficient while enhancing the seismic performance of liquid containers. Therefore, base isolation technique is one of the most effective
way to improve the dynamic behavior under different major events. The efficacy of base isolation was examined by many researchers [12-14]. Shrimali and Jangid examined the behavior of base-isolated liquid storage tanks using friction type isolator systems under real earthquake excitations [15]. In addition, they analyzed the efficacy of a New Zealand (NZ) bearing system to control the seismic parameters [16]. They observed that the base shear experiences an increase as the isolation damping surpasses a specific optimal value while it effectively mitigates the occurrence of sloshing and minimizes bearing displacements. Jadhav and Jangid examined the behavior of base-isolated liquid-filled tanks using sliding type and elastomeric type bearings systems [17]. In the summary, the above studies are mainly based on the lumped mass approach where the liquid is divided into three nodal masses such as impulsive, convective, and rigid. However, analysis of liquid containers using numerical finite element model considering the entire liquid domain could be more efficient.

In this study, the behaviour of liquid sloshing in the baffled base isolated rectangular tank under irregular excitations is investigated numerically using a finite element model. As per the above discussion, it can be noted that the previous work mainly focused on the impact of a submerged baffle on sloshing characteristics of ground supported liquid tanks under irregular excitations. However, the study of examining the impact of internal baffle in a partially filled base-isolated container is relatively new and yet to be addressed effectively under irregular excitations, which is a vital aspect of describing the irregular liquid sloshing motion in these structures. Moreover, the available studies based on the tank-baffle-isolation systems mainly focused on a specific dimension of a baffle and the analysis are limited to the real seismic excitations. Therefore, the present numerical analysis includes different configurations of a baffle by varying the height and width. New understandings of the sloshing behavior under the irregular excitation in rectangular base-isolated and non-isolated tanks are established and the probable reason behind the sloshing response is revealed, which is the prime objective of this research.

2. Numerical modeling

Fig. 1 illustrates a vertically baffled base-isolated rectangular tank model, which defines the geometry of the problem.

![Fig.1: Vertically baffled base-isolated tank model.](image)

The liquid domain is represented by Ω and the boundaries such as Y_s, YTw, and Y_b represent the free surface, tank walls and base of the tank, respectively. Whereas, the liquid surface area in contact with the top surface and walls of the baffle are represented by Y_Bt and Y_Bw, respectively. The dimension of the baffle is altered by changing the width (w) and height (h). The liquid density is denoted by ρ_f. Liquid motion is governed by the well-known Laplace equation by considering the liquid to be incompressible, inviscid with the assumption of irrotational flow.

2.1. Boundary conditions

There are two boundary conditions required to complete the mathematical description at the free surface. The kinematic boundary condition states that the liquid particle must stay on the free surface. When the nonlinear components and atmospheric pressure are neglected, the unsteady form of Bernoulli's equation is used to produce the dynamic boundary condition, which is,
\[ \frac{\partial^2 \Phi}{\partial t^2} + \mu \frac{\partial \Phi}{\partial t} + g \frac{\partial \Phi}{\partial y} = 0 \text{ on } \mathcal{Y}_s \]  

(1)

In the above expression \( \Phi \) is the velocity potential function whereas \( \mu \) and \( g \) are the liquid viscosity and gravitational acceleration respectively. To define the boundary condition along the tank and baffle walls, the velocity of liquid is equated to the instantaneous wall velocity \( (V_v) \) in the perpendicular direction \( (v) \). This condition generates the following expression,

\[ \frac{\partial \Phi}{\partial v} = V_v \text{ on } \mathcal{Y}_w = \mathcal{Y}_{Tw} \cup \mathcal{Y}_{Bw} \]

(2)

The tank bottom has an impervious surface and hence it should have a no-flux condition. Therefore, it can be stated as,

\[ \frac{\partial \Phi}{\partial v} = 0 \text{ on } \mathcal{Y}_B = \mathcal{Y}_b \cup \mathcal{Y}_{Bt} \]

(3)

2.2. Mathematical formulation

The liquid domain is discretized with four noded isoperimetric quadrilateral element. Each node is associated with a single degree of freedom i.e. time dependent potential \( \Phi(x, y, t) \). It can be defined as,

\[ \Phi = \sum_{k=1}^{n} \Phi_k(t) N_k(x, y) \]

(4)

\( N_k \) represents the shape function that is expressed through the natural coordinates. The Galerkin weighted-residual method is applied to the Laplace equation, which yields,

\[ \int_{\mathcal{O}} N_i \left( \frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} \right) d\mathcal{O} = 0 \]

(5)

The dynamic equilibrium equation can be derived by utilizing the integration by parts method based on Stoke’s and Green’s theorem to the above equation and it is expressed as follows,

\[ \frac{1}{g} \int_{\mathcal{Y}_s} N_i \sum_{k=1}^{n} N_k \Phi \, dY_s + \frac{\mu}{g} \int_{\mathcal{Y}_s} N_i \sum_{k=1}^{n} N_k \Phi \, dY_s + \int_{\mathcal{O}} \left[ \frac{\partial N_i}{\partial x} \sum_{k=1}^{n} \frac{\partial N_k}{\partial x} + \frac{\partial N_i}{\partial y} \sum_{k=1}^{n} \frac{\partial N_k}{\partial y} \right] \Phi_k \, d\mathcal{O} = \int_{\mathcal{Y}_w} N_i V_v \, dY_w \]

(6)

\[ [M] \ddot{\Phi} + [C] \dot{\Phi} + [K] \Phi = \{F\} \]

(7)

The variables \([M]\) and \([C]\) represent the free surface matrix and damping matrix, respectively. The variable \([K]\) denotes the fluid coefficient matrix, while \([F]\) represents the force vector. These elements are defined as, \( M_{ij} = \sum M_{ij}^e \), \( C_{ij} = \sum C_{ij}^e \), \( K_{ij} = \sum K_{ij}^e \) and \( F_i = \sum F_i^e \) are the global matrices for different elements.

\[ M_{ij}^e = \frac{1}{g} \int_{\mathcal{Y}_s} N_i N_j \, dY_s \]

(8)

\[ C_{ij}^e = \frac{\mu}{g} \int_{\mathcal{Y}_s} N_i N_j \, dY_s \]

(9)

\[ K_{ij}^e = \int_{\mathcal{O}} \left[ \frac{\partial N_i}{\partial x} \frac{\partial N_j}{\partial x} + \frac{\partial N_i}{\partial y} \frac{\partial N_j}{\partial y} \right] d\mathcal{O} \]

(10)

\[ F_i^e = -\int_{\mathcal{Y}_w} N_i \, dY_w \]

(11)

The magnitudes of slosh amplitude \( (\Pi_s) \), total pressure \( (P_{tot}) \) and total base shear \( (B_{Stot}) \) are obtained from the following expressions.

\[ \Pi_s = -\frac{1}{g} \left( \frac{\partial \Phi}{\partial t} + \mu \Phi \right) \]

(12)

\[ P_{tot} = -\rho \left( \frac{\partial \Phi}{\partial t} + \mu \Phi \right) \]

(13)

\[ B_{Stot} = \int_{\mathcal{Y}_w} P_{tot} n_x \, dY_w \]

(14)

Here \( n_x \) denotes the unit normal vector in x direction. The impulsive component of hydrodynamic pressure can be obtained independently by neglecting the liquid sloshing.

\[ \frac{\partial \Phi}{\partial t} (x, 0, t) = 0 \]

(15)

However, the convective component can be obtained by subtracting the impulsive component from the total component.

2.3. Characteristics of nonlinear hysteretic isolator

The Wen's bilinear hysteretic model is considered in the present investigation to simulate the nonlinear characteristics of lead rubber bearings (Shrimali and Jangid 2002a). The empirical relation between force and deformation is stated as,
\[
F_b = c_b \dot{x}_b + \alpha k_b x_b + (1 - \alpha) F_y Z \\
\omega_b = \frac{2\pi}{T_b} = \sqrt{k_b/m_b} \\
c_b = 2m_b \omega_b \xi_b \\
F_y = F_0 W \\
\alpha = \omega_b^2 \frac{q m_b}{F_y} 
\] (16) (17) (18) (19) (20)

Here, \(c_b\) and \(k_b\) are the damping and pre yielding stiffness of the lead rubber isolator. Eq. (17) obtains the natural frequency \(\omega_b\) and time period \(T_b\) of the bearing based on the post yield stiffness whereas, the damping ratio \(\xi_b\) is obtained by Eq. (18). Mass \(m_b\) and weight \(W\) express the total mass and total weight of the liquid exerted on the bearing and the scaled yield strength \(F_0\) is considered to be 0.05 as obtained from literature (Shrimali and Jangid 2002a). For the present investigation, the period and damping of the bearing are considered as 3s and 0.2 respectively. \(F_y\) and \(\alpha\) represent the yield strength (Eq. 19) and the fraction of post to pre yield stiffness (Eq. 20) of the isolator. \(Z\) is the non-dimensional hysteretic displacement, which is regulated by the first order differential equation as expressed in Eq. (21).

\[
q \ddot{Z} = A \ddot{x}_b + \beta | \dot{x}_b | Z | Z | \ddot{Z} \dddot{Z} = A | \dot{x}_b | ^{n-1} - \tau \ddot{x}_b | Z | ^n
\] (21)

Here \(q\) presents the yield displacement of bearing whereas \(A, n, \beta\) and \(\tau\) are the non-dimensional bearing factors that determine the shape of the hysteresis loop. Based on the literature (Shrimali and Jangid 2002a), the bearing parameters are kept constant with \(A=1, n=2, q=2.5cm\) whereas, the values of \(\beta\) and \(\tau\) are considered as 0.5 for the present study.

2.4. Dynamic equations for the system

The following set of characteristics equations are established (Eqs. (22) and (23)) for the coupled system of isolated tank with a vertical baffle.

\[
[M] \dddot{\Phi} + [C] \dot{\Phi} + [K] \Phi = \{F\}(\dddot{x}_g + \dddot{x}_b)
\] (22)

\[
m_b \ddot{x}_b + F_b + BS = -m_b \dddot{x}_g
\] (23)

\(F_b\) and \(BS\) denote the force subjected by the isolator and the base shear force. In addition, \(\dddot{x}_b\) and \(\dddot{x}_g\) represent velocity and acceleration of the isolator. The excitation velocity and ground acceleration are represented by \(\dddot{x}_b\) and \(\dddot{x}_g\) respectively.

3. Results and discussion

A shallow rectangular liquid tank of 10mx5m size is adopted for the present analysis where a vertical baffle is mounted at the tank floor. The mass density of confining liquid is assumed to be 1000 kg/m³. A finite element algorithm is established in MATLAB 2020b platform in which the effect of varying depth of the baffle is also analyzed for a base-isolated tank. The optimum mesh size is determined through a free vibration study considering the fundamental period of liquid sloshing and the results are presented in Fig. 2. However, a typical discretization with 400 number of elements (20x20-mesh dimension) is suitably adopted for the present study.

3.1. Model validation

A square liquid tank having a filling depth of 1m and a width of 1m is subjected to a base excitation force of \(\dddot{x}_g(t) = 0.002 \omega_f^2 \sin \omega_f t\), where \(\omega_f\) is the forced frequency of the excitation that is taken as 5.55 rad/s. The time variation of slosh amplitude
is obtained at the extreme corner of the right wall, as illustrated in Fig. 3. The obtained result has shown a good agreement with the existing solution available in the literature where a 2-D meshless local Petrov-Galerkin method is employed [18].

![Graph showing comparison between Pal (2012) and Present FEM results](image)

Fig. 3: Variation in time of the surface wave at the extreme corner of the right wall

3.2. Definition of the irregular excitation

In order to examine the sloshing response under the irregular excitation, a time history of the irregular external oscillation needs to be generated, firstly. The irregular oscillation can be modelled in a stochastic way by the sum of a large number of independent linear harmonic excitations as follows,

\[ x_g(t) = \sum_{i=1}^{n} a_i \sin(2\pi f_i t) \]

Here the \( a_i \) is the amplitude of ground acceleration, which is varied between 0.002 to 0.05 m/s². Whereas \( f_i \) is the excitation frequency that is considered over a range between 0.1 to 1 Hz with an increment of 0.1 Hz. Furthermore, \( n \) is the number of all linear sine excitations, which is considered as \( n=10 \). However, the influence of long duration irregular excitation on the sloshing amplitude, hydrodynamic pressure, and base shear parameters in a vertically baffled rectangular tank is evaluated for different depth and width of the baffle.

3.3. Behavior of sloshing amplitude

The vertical baffled tank with \( h/d = 0.2, 0.5, 0.8 \) and \( w/L=0.1, 0.2 \) are adopted in this section. Figs. 4 and 5 illustrate the time history responses of slosh amplitude for non-isolated and base-isolated tank respectively for different width ratios. It can be seen that the slosh response for the non-isolated tank is dominant at \( h/d=0.8 \) whereas for the base-isolated tank, it is dominant at \( h/d=0.5 \) among all of the configuration considered when \( w/L=0.1 \). While considering the width ratio \( w/L \) as 0.2, the sloshing response in the non-isolated tank is dominant at \( h/d=0 \) as seen from Fig. 4(b). However, the base-isolated responses remained unchanged due to the increase in the width ratio, as noticed from Fig. 5. The maximum sloshing amplitudes are presented in Table 1 for different tank-baffle configurations. It is noticed that the magnitudes are initially decreases and then increases for the non-isolated tank system whereas converse effect is shown for the base-isolated system in each cases of width ratio considered. However, the amplitudes are decreased by about 12 to 34% due to the application of base isolation in different configurations of the baffle except at \( h/d=0.5 \). It demonstrates that the liquid sloshing has shown an adverse effect in the base-isolated tank specifically at \( h/d=0.5 \) under the application of the considered irregular excitation.

![Graphs showing sloshing amplitude for different tank configurations](image)

Fig. 4: Behavior of the sloshing amplitude for the non-isolated tank; (a) \( w/L=0.1 \), (b) \( w/L=0.2 \)
### Table 1 Absolute dynamic peak responses

<table>
<thead>
<tr>
<th>w/L ratio</th>
<th>h/d ratio</th>
<th>Total base shear (kN/m)</th>
<th>Sloshing amplitude (cm)</th>
<th>Isolator displacement (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>4.43</td>
<td>2.57</td>
<td>5.92</td>
</tr>
<tr>
<td>0.1</td>
<td>0.2</td>
<td>4.11</td>
<td>3.87</td>
<td>5.71</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>4.41</td>
<td>3.65</td>
<td>4.85</td>
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<td></td>
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<td>5.37</td>
<td>2.98</td>
<td>6.82</td>
</tr>
<tr>
<td>0.2</td>
<td>0.2</td>
<td>4.02</td>
<td>3.57</td>
<td>5.60</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>4.07</td>
<td>3.98</td>
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</tr>
<tr>
<td></td>
<td>0.8</td>
<td>4.79</td>
<td>2.52</td>
<td>5.05</td>
</tr>
</tbody>
</table>

### 3.4. Behavior of base shear

Figs. 6 and 7 show the time history responses of base shear for non-isolated and base-isolated tank respectively for different width ratios. The local and global peak values of the response for the non-isolated tank are observed higher when w/L=0.1 as compared to w/L=0.2. Contrastingly, the peaks are invariably higher for the slender configuration of the baffle (w/L=0.1 and h/d=0.8), as noticed from Fig. 6(a). While for the base-isolated tank, the global peaks are invariably observed higher at h/d=0.5 irrespective of the width ratios, as noticed from Fig. 7. However, the occurrence of the local peaks at h/d=0.5 has occurred at different time instances for different baffle widths, which is clearly visible from the figure.
Furthermore, the global peak values are specified in Table 1. The non-isolated base shear values are increased and conversely, the base-isolated base shear values are decreased with the increase in the h/d ratios specifically when w/L=0.1. Subsequently, similar tendency is also noticed for the non-isolated tank at w/L=0.2 whereas for the base-isolated tank, the base shear is increased initially and then decreased significantly. In general, the implementation of hysteretic bearing has reduced the base shear in the range of 2 to 48% depending upon the configurations.

3.5. Behavior of isolator displacement

Fig. 8 shows the time history responses of the isolator displacement for different baffle configurations. It is noticed that the behavior of the isolator displacement is almost similar in both the width ratios. The maximum values are presented in Table 1 for various tank-baffle configurations. It is noticed that the isolator undergoes maximum deformation specifically when h/d=0.5 for different w/L ratios. Overall, the increase in the h/d ratio imparts an increase in the isolator displacement initially and then decreased subsequently.

4. Conclusion

The influence a long-duration irregular excitation on the dynamic behavior of a vertically baffled base-isolated rectangular fluid container is examined for different tank-baffle configurations using a nonlinear hysteretic bearing isolator. Galerkin’s finite element method is employed in the study to model the liquid domain. However, the following conclusions are made:

- The effect of change in h/d ratio has a significant impact on the variation in sloshing amplitude and base shear components as compared to the change in w/L ratio.
- The application of base isolation reduces the sloshing amplitudes by 12 to 34% in different configurations of the baffle except at h/d=0.5.
- The base shear is also reduced by 2 to 48% in base-isolated tanks depending upon different baffle configurations.
- The isolator displacement is invariably found to be maximum when h/d=0.5, irrespective of the width of the baffle.

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