

Comparative Study of Topologically Optimized Bridge Girders Considering Static and Cyclic Loads

Maiara G. Montaute¹, Hugo Luiz Oliveira², Thomaz E. T. Buttignol³

^{1,2,3}School of Civil Engineering, Architecture and Urban Design, University of Campinas
Saturnino de Brito 224, 13083-889, Campinas, São Paulo, Brazil

¹maiara.montaute@gmail.com, ²hluiz@unicamp.br, ³thomazb@unicamp.br

Abstract – Topology optimization is a mathematical method which spatially optimizes the distribution of material within a defined domain based on given constraints, boundary conditions and a predefined cost function. It is adopted herein to investigate simply supported concrete beams with different cross-sections and spans. The objective is to find the greatest reduction in concrete volume for RC beams subjected to static and cyclic loads. A 3D parametric beam model is constructed using the Algorithm-Aided Design (AAD) Grasshopper, which is integrated with the plugins Topos, based on the SIMP methodology and Karamba3D, based on the BESO methodology. The model is plugged to Galapagos, a genetic algorithm plugin of Grasshopper, which allows the search automation of the minimum volume of beams with spans between 5 m to 20 m, considering rectangular, T and I sections.

Keywords: concrete beams, topology optimization, parametric modelling, genetic algorithm, Algorithm-Aided Design (AAD).

1. Introduction

Stability, performance, economic and environmental requirements of structural elements are directly related to architectural, engineering and construction (AEC) design procedures and optimization techniques, aiming to guarantee structural safety and stiffness. According to Lopez and Beck [1], optimization consists of finding the minimum and maximum points of a predefined function using an iterative process. As stated by Nocedal and Wrighy [2], it allows real problems to be solved based on three basic aspects: i) creation of an algorithm that contains the conditions that must be satisfied; ii) convergence to the solution; and iii) speed of convergence. The evolution of digital computing combined with the demand for efficient projects have contributed to the development of optimization techniques that are aimed to search for efficient solutions.

According to Bendsoe and Sigmund [3], optimization procedures are subdivided in three main categories: i) dimensional optimization, where the shape of the elements is kept constant while the cross-sectional area and the Moment of Inertia are changed; ii) shape optimization, which enhances an existing geometry (height, radii, length) without changing material connectivity to distribute stresses more evenly; iii) and topology optimization, which is aimed to find the best layout of the material based on predefined boundary conditions and load cases. In all cases, the solution should satisfy structural performance and stability conditions.

In topology optimization, the cross-section of a structural element is not represented by standard parametric functions, but rather by a set of distributed functions. The functions are defined in a fixed domain, representing a parameterization of the continuous stiffness tensor. The appropriate choice of parameterization is required to lead to a coherent design formulation [4].

Said that, this paper is aimed to study the effectiveness of topology optimization to reduce the concrete volume of simply supported RC beams subjected to static and cyclic loads. A 3D parametric beam model is set up using the Algorithm-Aided Design (AAD) Grasshopper, which is native to Computer-Aided Design (CAD) software Rhinoceros. The analyses are performed by means of the plugins Topos, based on the solid isotropic material with penalization (SIMP) methodology and Karamba3D, based on the bi-directional evolutionary structural optimization (BESO) methodology. Galapagos, a genetic algorithm plugin of Grasshopper, is adopted to provide an automatic procedure to search for the minimum volume of beams with rectangular, T and I sections and spans ranging from 5 m to 20 m.

2. Problem Statement

Currently, there is a lack of studies on topology optimization for concrete beams with different cross sections. Few analyses are available combining static and cyclic loads. Hence, this research becomes relevant as a preliminary scope of analysis of concrete beams with different sections and loading categories. It is important mentioning that the optimization methodology used in this research considered the material to be isotropic, despite its non-linear anisotropic behavior. Furthermore, for the topology optimization, parameters that, in many cases, are unknown or variable are estimated [5], so that a deterministic analysis can be performed. The appropriate quantification of the parameters is essential to avoid an incorrect representation of the physical behavior of the structural element [6].

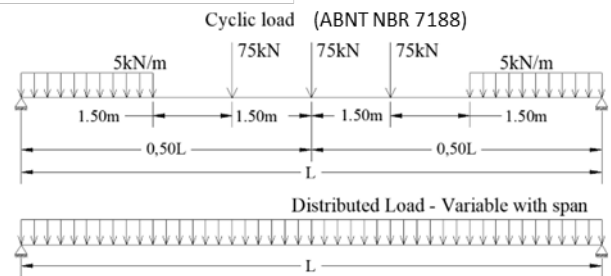
2.1. Input data

In all cases investigated, it was established that the concrete beam cross sections have the same area and the same height. The Young's modulus of each simply supported concrete beam is 30GPa and the Poisson's ratio is 0.20. The spans are ranging from 5 m to 20 m. Rectangular, I and T cross-sections (design domain) were investigated, with heights varying from 30 cm to 70 cm. For each section, the area and initial height are kept constant and the static load is applied according to Table 1. The static loading included the self-weight and the dead and live loads. The cyclic load was adopted according to the standard traffic load for bridge class 45, as defined in ABNT, NBR 7188 [7] and shown in Figure 1. In this particular case, the concentrated and distributed loads were distributed generate the maximum bending moment.

Table1: Static loading (in kN/m) applied according to the cross-section area and span.

| Span (m) | 5 | 10 | 15 | 20 | |
|--------------------------------------|-------|-------|--------|--------|--------|
| cross-section area (m ²) | 0.045 | 6.335 | 8.855 | 9.695 | 10.115 |
| | 0.06 | 6.86 | 9.38 | 10.22 | 10.64 |
| | 0.075 | 7.385 | 9.905 | 10.745 | 11.165 |
| | 0.09 | 7.91 | 10.43 | 11.27 | 11.69 |
| | 0.105 | 8.435 | 10.955 | 11.795 | 12.215 |

Fig. 1: Concrete beam with the applied loading conditions.



2.2. 3D digital parametric modelling

The workflow started with the definition of the input data (Table 1), followed by the modelling of the 3D digital parametric concrete beam in Rhinoceros/Grasshopper. The model included the material properties definition, the loading scheme and the boundary conditions, as shown in Figure 2. Grasshopper allows a flexible design process due to parametrization and quick adjustments / adaptations in the model. The script is based on visual programming, allowing algorithmically generation of geometries by composing diagrams that link data to functions (see Figure 2, on the left).

The plugins Topos and Karamba3D are plugged to the model to perform the topology optimization. The results were automated using the Galapagos plugin, which is a generic algorithm native to Grasshopper, aiming to minimize the concrete volume.

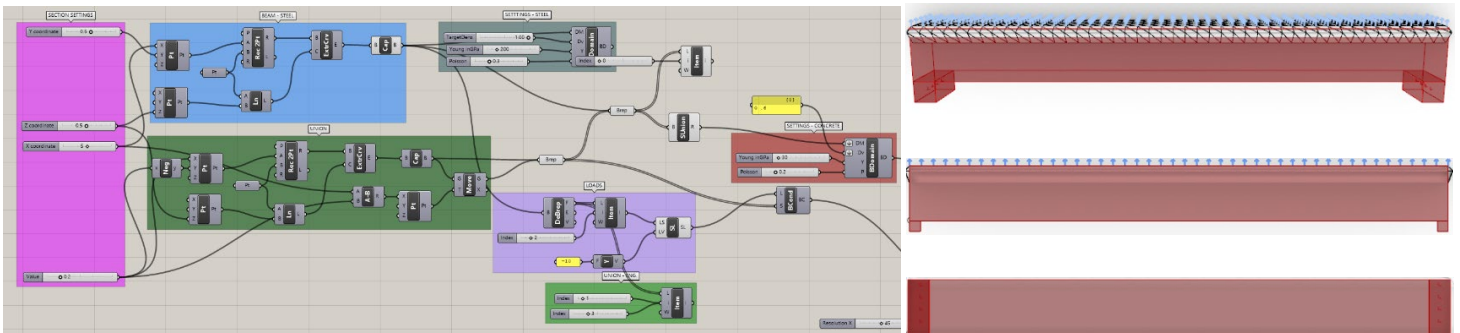


Fig. 2: Grasshopper interface with the input data on the left; 3D digital model on the right (isometric, side and top views) including the boundary conditions and loading scheme.

2.3. Optimization procedures

The topology optimization was carried out initially using the Topos plugin (SIMP-based approach), as shown in Figure 3. The SIMP method was developed in 1988 by Bendsøe and Kikuchi [4] and is based on the material density for topology optimization and parameterizes material distribution using density functions to indicate the pointwise stiffness integrity. The one-to-one relationship between geometric layout and constitutive material occurs smoothly by assigning 0 to regions that should not be considered for calculating the cost function (mechanical compliance) and 1 to fully contributing regions. Topos plugin allows a 3D geometry optimization.

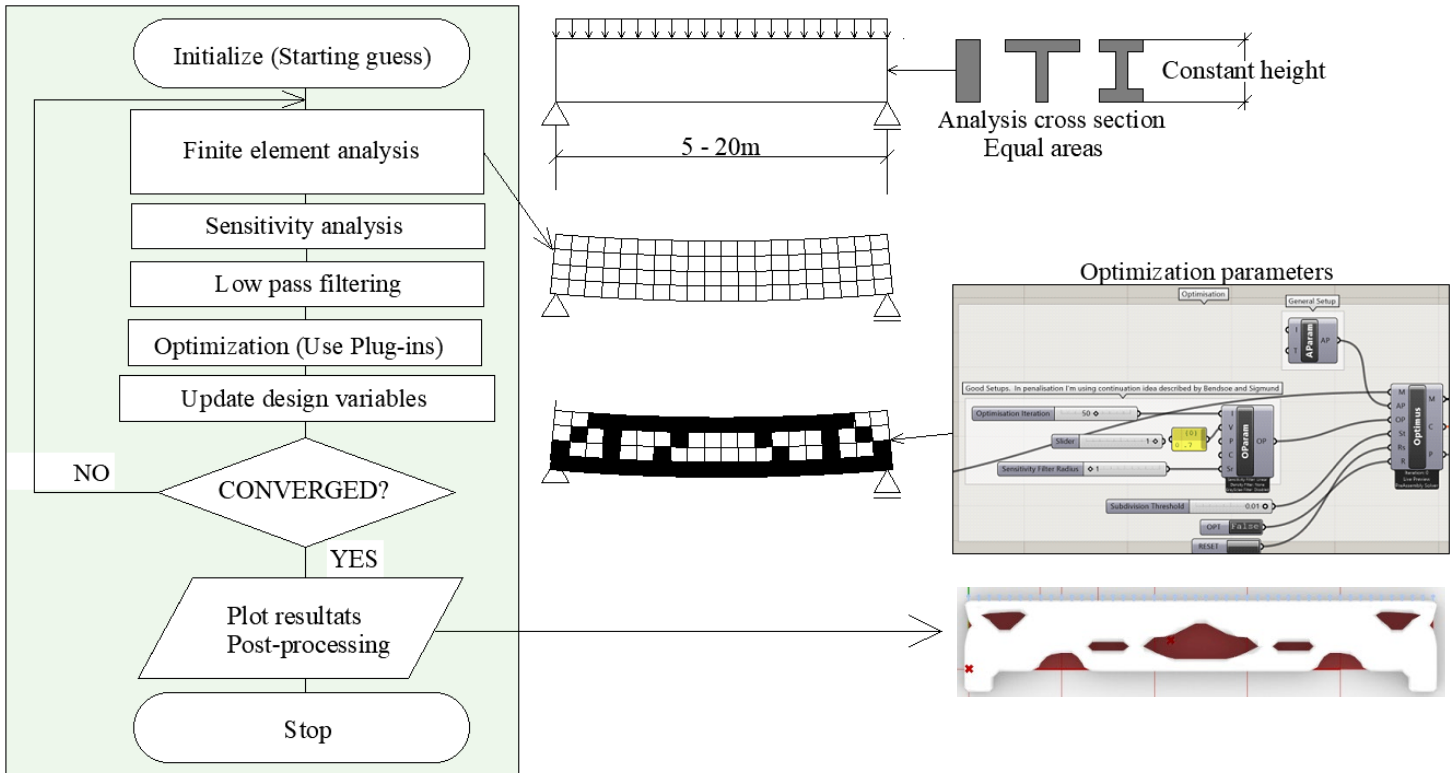


Fig. 3: Flowchart for topology optimization adapted from [2] (on the left); graphical representations (on the middle); input, optimization and rendered results (on the right).

The Karamba3D plugin (BESO-based approach) implements a topology optimization method where inefficient material is iteratively removed from the design domain while efficient material is simultaneously added to it [8]. The criterion used to indicate the efficiency of the material is usually the corresponding stress level. Regions with stresses that are too low are classified as inefficient and can be removed [9].

The Karamba3D plugin has a simple-to-use interface, with an intuitive programming. In contrast to Topos, it is based on a 2D finite element analysis. The 2D shell element should be specified including the thickness, the material mechanical properties, the boundary conditions and the definition of the optimization parameters, as shown in Figure 4.

In both analyses (Topos and Karamba3D), Galapagos plugin was used to provide an automated and integrated graphical solution, which is aimed to find the minimum volume of concrete for the stated problem described in Section 2.1.

In Galapagos, it is necessary to define the optimization criteria (see Figure 5). In this case, the two criteria were the achievement of the minimum volume after the topology optimization and the attendance of the maximum vertical allowable displacement for bridges, as recommended in ABNT NBR 6118:2023 [10]. Therefore, the maximum allowable displacement was used to reduce or increase the volume of the concrete beam. For simplicity, only the instantaneous elastic deflection was determined, by means of a linear-elastic analysis.

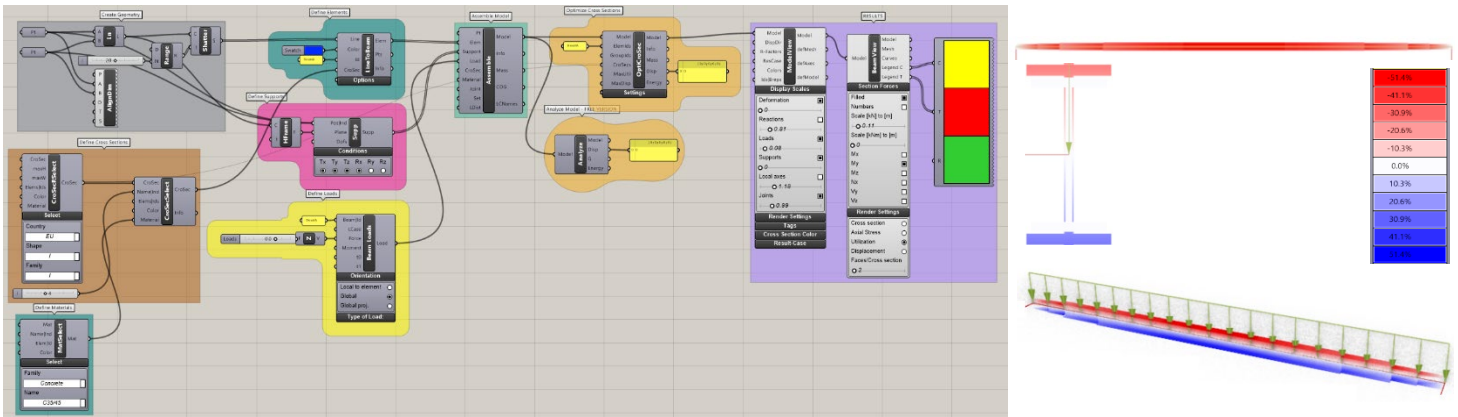


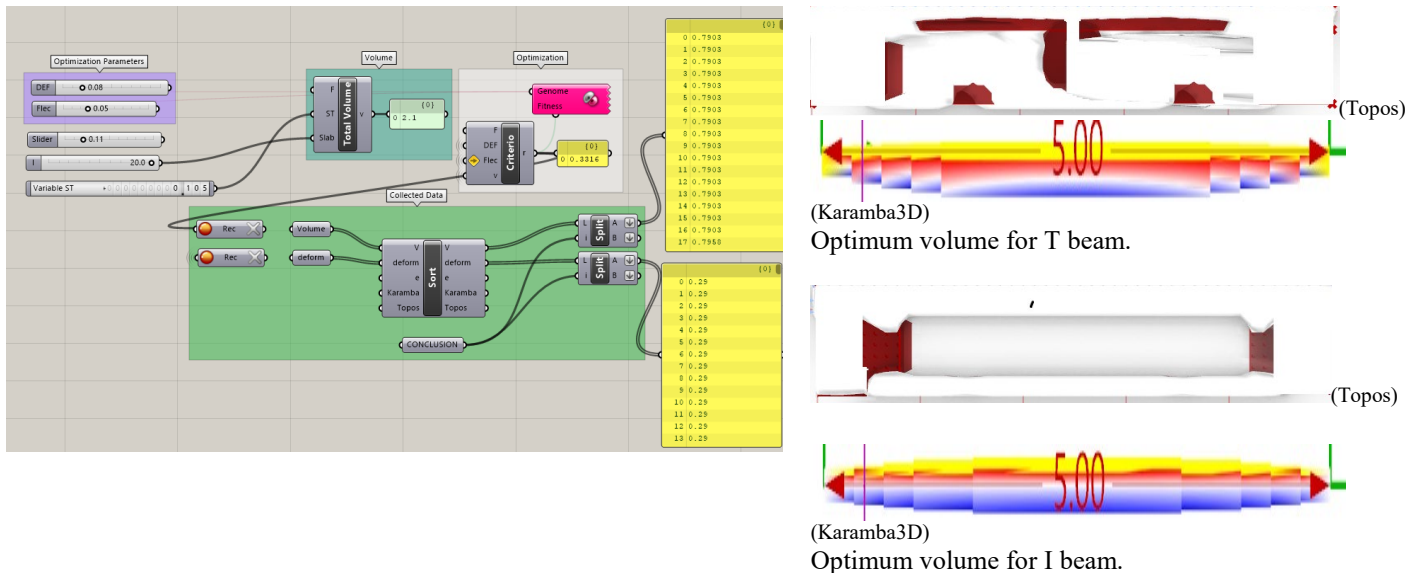
Fig. 4: Grasshopper panel with the input data and algorithm for the creation of the 2D beam model and definition of the material mechanical properties and optimization parameters (on the left). Top, cross-section and the isometric views in Rhinoceros (on the right).

3. Results and Discussions

The analyses in both Topos and Karamba3D plugins, considering different beam spans and heights, were automated using Galapagos, as shown in Figure 5.

The results using Topos plugin were, on average, 2.85% lower than the results generated in Karamba3D. An example of the solution for the three cross-sections investigated is shown in Figure 6. In Topos plugin, the remaining material is represented in white and the removed material is indicated in red. Karamba3D displays the results by means of a scale of colors, in which yellow represents the region where the load is transmitted to the supports and, therefore, there is no reduction in volume.

Galapagos returns the optimum volume according to the input data described in section 2.1. Figures 7 to 10 shows the results of the beams with different heights according to different spans. The analyses demonstrate that it is possible to reduce the concrete volume of the beam (span of 5 m and, in some specific conditions, 10m). In contrast, for spans of 15 m and 20 m, the initial volumes adopted will not satisfy the design criteria adopted in this work (maximum allowable displacement). Hence, in these cases, it is necessary to increase the initial volume.



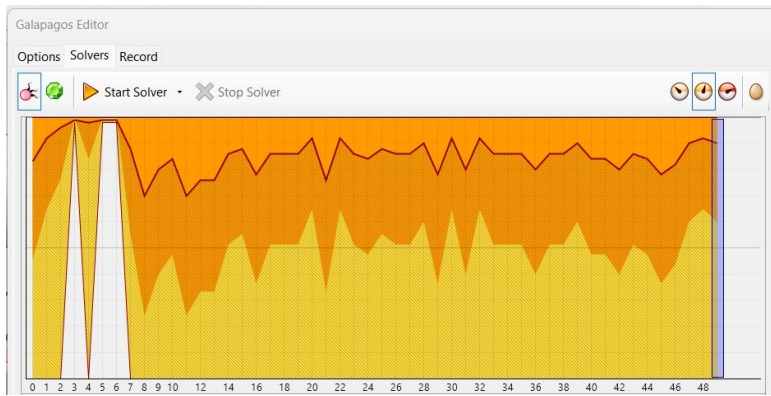
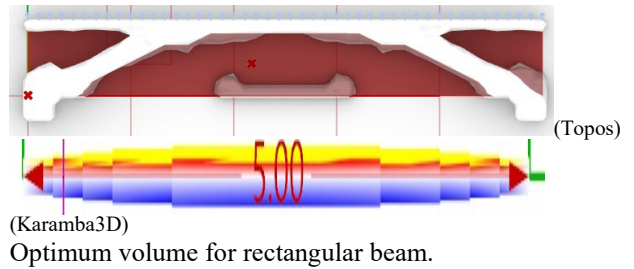


Fig. 5: Example of the analysis in Galapagos.



Optimum volume for rectangular beam.

Fig. 6: Comparison between the results of Topos and Karamba3D for a beam with height of 70 cm and span of 5m.

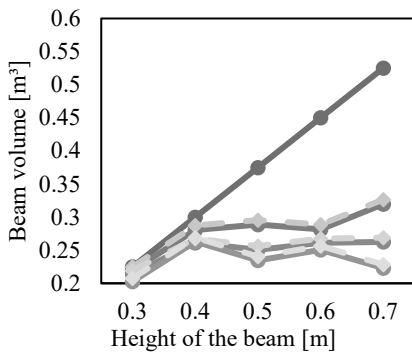


Fig. 7: Topos and Karamba3D optimum volume for beams with 5 m span.

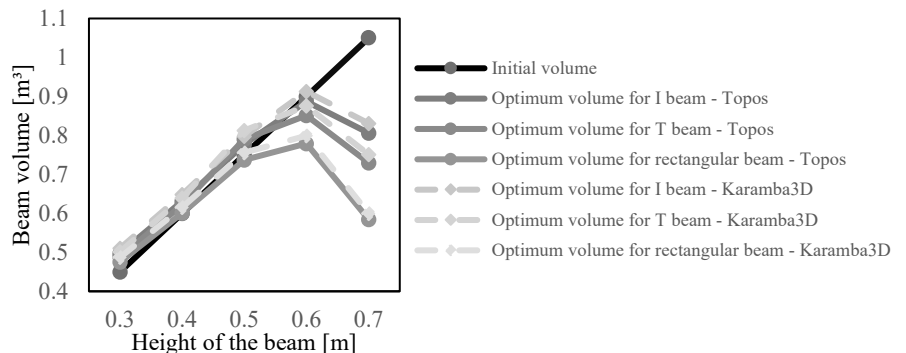


Fig. 8: Topos and Karamba3D optimum volume for beams with 10 m span.

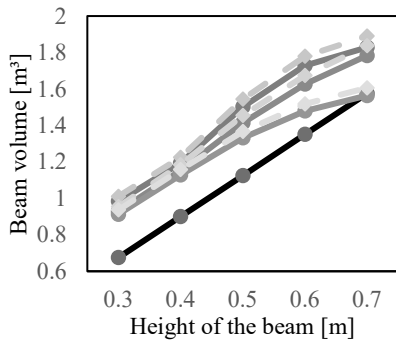


Fig. 9: Topos and Karamba3D optimum volume for beams with 15 m span.

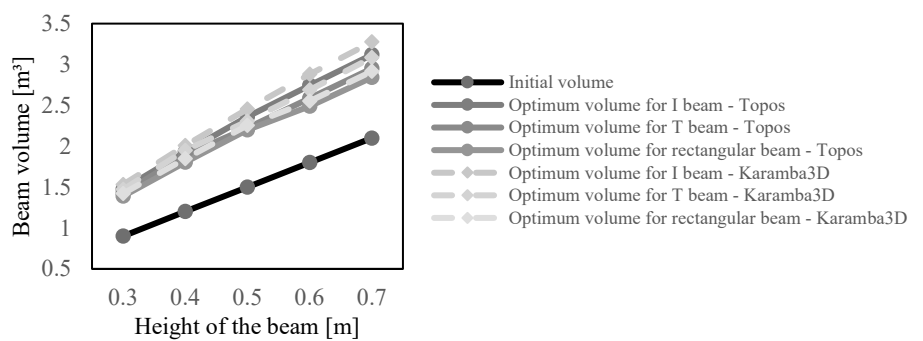


Fig. 10: Topos and Karamba3D optimum volume for beams with 20 m span.

4. Conclusions

Topology optimization applied to concrete structures is a segment not yet fully explored, having a great potential to design advanced durable elements that cope with the current demands of sustainability and performance-based analyses. The plugins Topos and Karamba3D have predicted reasonable structural layouts, with fast processing for simply supported beams with different cross-sections and spans. The results of the analyses demonstrated that, for different spans, the rectangular design domain showed to be the one with the greatest material removal, followed by the T and I geometries.

Topos plugin demanded a more time-consuming analysis compared to Karamba3D. It is worth mentioning that Karamba3D is based on a 2D BESO analysis, while Topos adopts a 3D SIMP method. Topos presented better graphical results, as well as an average 2.85% smaller concrete volume compared to Karamba3D.

Considering the optimization criteria in Galapagos (minimum volume and maximum allowable deflection), it was verified that, for a span of 5 m, it was possible to reduce the initial volume of concrete in all the beam heights adopted in the analyses. This was also true for the beam with a height of 0.70m and a span of 10m. However, for beams with spans of 15 m and 20 m, the optimization criteria were not met, therefore, it is necessary an increase in the volume to meet the deflection requirement.

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References

- [1] Lopez, R.; Beck, A. Optimization under uncertainties. In: Optimization of Structures and Components. Springer, 2013. Cap. X, pp. 117–138.
- [2] Nocedal, J.; Wright, S. J. Numerical Optimization. Second edition. Springer. 2000, pp. 5-12.
- [3] Bendsøe, M.P e Sigmund, O. Material interpolations in topology optimization, Arch. Appl. Mech. 1999, pp. 630–654.
- [4] Bendsøe, M. P. e Kikuchi, N., “Generating Optimal Topologies in Structural Design Using Homogenization Method”, Computer Methods in Applied Mechanics and Engineering, vol. 71, n° 2, pp. 201-224, 1988.
- [5] Kleiber, M.; Hien, T. The Stochastic Finite Element: Basic Perturbation Technique and Computer Implementation. John Wiley & sons. 1992.
- [6] Sudret, B.; Kiureghian, A. D. Stochastic Finite Element Methods and Reliability: A State-of-the-Art Report. 2000.
- [7] Brazilian Association of Technical Standards – ABNT (Associação Brasileira de Normas Técnicas), NBR 7188 Road and pedestrian live load on bridges, viaducts, footbridges and other structures. 2013 (in Portuguese).
- [8] Huang X and Xie YM. Topological design of microstructures of cellular materials for maximum bulk or shear modulus. Comput Mater Sci. 2011.
- [9] Xie Y.M. and Steven G. P. A simple evolutionary procedure for structural optimization, J. Comput Struct. 1993, pp. 885– 896.
- [10] Brazilian Association of Technical Standards – ABNT (Associação Brasileira de Normas Técnicas), NBR 6118 – Design of structural concrete – Procedure. 2023 (in Portuguese).