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Shear Strains in Cracked Reinforced Concrete Beams

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Abstract – The contribution of shear strains to the vertical deflections in reinforced concrete (RC) beams is typically neglected for two main reasons. First, this contribution is relatively smaller than the contribution from flexure. Second, the calculation of the shear strains in cracked RC beams is complex and requires iterations. This paper presents a simple model for the calculation of the shear stress-strain response of beams after cracking of the concrete and before yielding of the steel. It represents this part of the response using a straight line whose slope and intercept stress are related to the ratios of the reinforcement and to the concrete compressive strength respectively. The results of the proposed model are shown to be simple and to compare well with experimental results.

***Keywords*: ** Beams, concrete, cracking, deformations, shear, strains.

1. Introduction

The contribution of shear deformations to the vertical deflections in reinforced concrete (RC) beams is typically neglected. The procedures for the calculation of maximum deflections in codes such as the ACI 318 building code [1] and the CSA A23.3 standard [2] are based on the flexural deformations only. It is to be noted that deflections are not calculated with the same accuracy that is achieved when the flexural strength of beams is calculated. In addition, the contribution of the shear deformations is typically considerably smaller than that of the flexure. With the complexity of calculating the shear deformations, it was logical to neglect their effects in code procedures.

On the other hand, research has shown that the response of RC membrane elements subjected to shear is characterized by three distinct parts [3–5]. Fig. 1a shows the shear stress (ν) versus shear strain (γ) response of membrane element A3 tested by Pang and Hsu [4]. Before cracking, the response is relatively stiff, and the deformations can be accurately calculated based on linear elastic behavior of a homogeneous and isotropic material. The stiffness of the element decreases considerably after cracking. In under-reinforced elements, the stiffness is further decreased upon yielding of the reinforcement. Fig. 1b shows the $(\nu-\gamma)$ response in an RC beam near the point of contraflexure of a symmetrically reinforced beam [6]. The behavior is characterized by three parts similar to those observed in membrane elements.

The drop in stiffness after diagonal cracking indicates that the shear deformations can be significant, especially when compared to those before cracking. In fact, numerous experimental studies have shown that the contribution of shear deformations can be significant not only in shear–critical elements but also in flexure–critical elements [e.g. 7 – 9]. Advanced models [e.g. 3,10,11] can calculate the shear deformations. Simpler models are also available. Zhu et al. [12] listed thirteen models that are capable of calculating the deformations after cracking, but all these models required iterations to reach the solution.

The stiffness of the part of the response between the cracking stress and the yielding stress can be assumed to be linear. Rahal [5] presented a simple equation for the slope of this line. He further refined this model [13] by characterizing the slope (G_{cr}) and intercept stress (ν_0) for the straight line using the following equations:

$$G_{cr} = 32500\,(\rho_\ell\,\rho_t)^{0.42} \qquad \text{in MPa} \tag{1}$$

$$\nu_0 = \frac{2}{3}\nu_{cr} \cong 0.3(f_c')^{0.4} \qquad \text{in MPa} \tag{2}$$

where ρ_ℓ and ρ_t are the reinforcement ratios in the longitudinal and transverse directions respectively and ν_{cr} is the cracking shear strength. For RC membrane elements subjected to shear, Rahal [13] proposed using $[\nu_{cr} \cong 0.45(f_c')^{0.4}]$, recommended by Bentz [14], which leads to the equation shown in Eq. (2). Fig. 1a includes the calculations of the post-cracking stiffness based on Eqs. (1) and (2). It is shown that the post-cracking pre-yielding part of the response is accurately modelled.

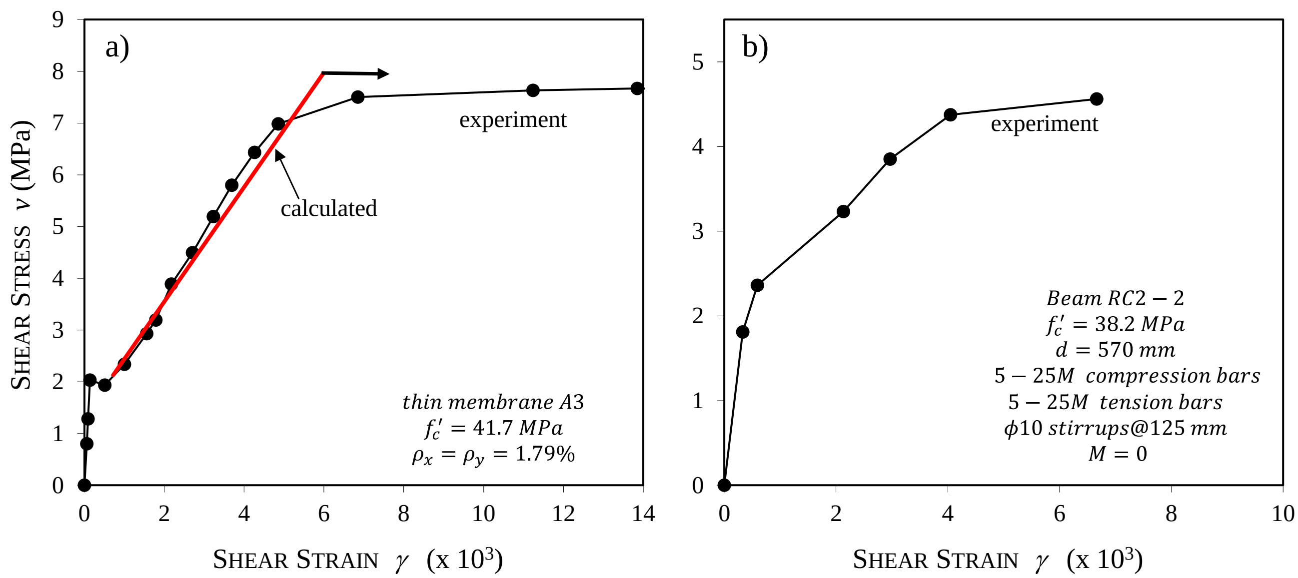

Fig. 1: Shear response of a) thin RC membrane [4] and b) RC beam near point of contra-flexure [6].

It is of interest to calculate the contribution of shear deformations to the vertical deflections in RC beams. The shear behavior of beams differs from that of membrane elements, mainly due to the presence of the bending moment and unsymmetrical reinforcement.

The objective of this paper is to evaluate the adequacy of Eqs. (1) and (2) for application to beams reinforced in the longitudinal and transverse directions. It is to be noted that the shear contribution to the overall deformations is added to the flexural contribution. Since the latter contribution in the code procedures [e.g. 1,2] has been calibrated to match the experimental results of the total deflections, it is preferable if the shear deformations are not over-estimated.

2. Model for Shear Stress-Strain Relationship in Cracked RC Beams

Fig. 2 shows a reinforced concrete beam element subjected to a shear force (V) and a bending moment (M). The width of the web is (b_w), the effective depth is (d) and the effective shear depth is (d_v). The area and the spacing of the stirrups are A_v and s respectively. The areas of the tensile and compressive flexural reinforcement are (A_s) and (A_s') respectively. Typically, (A_s) is significantly larger than (A_s'). The shear force causes tension in the tensile and the compressive reinforcement while the bending moment adds tension in the tensile reinforcement and compression in the compression reinforcement.

It has been shown that the shear resistance and stiffness increase with an increase of the transverse as well as the longitudinal reinforcement [10,15]. Reinforcement in both directions restricts the opening of the diagonal cracks, which enhances the aggregate interlock and the dowel action. The author has proposed that in an unsymmetrically reinforced cross section subject to pure shear, the smaller amount of reinforcement (the compression reinforcement in Fig. 2) is critical instead of the bottom reinforcement [16]. He also proposed that the amount of longitudinal reinforcement effective in resisting the shear force can be takes as ($A_s' + A_s'$) instead of ($A_s + A_s'$). A relatively small bending moment counteracts the effect of the shear force on the compression side of the section and hence increases the strength and stiffness. A relatively larger bending moment weakens the ability of the bottom reinforcement to resist the axial tension caused by shear, rendering this reinforcement critical in determining the strength and stiffness. Hence, the presence of a bending moment can in many cases counteracts the effect of using lower amounts of reinforcement in the compression zone.

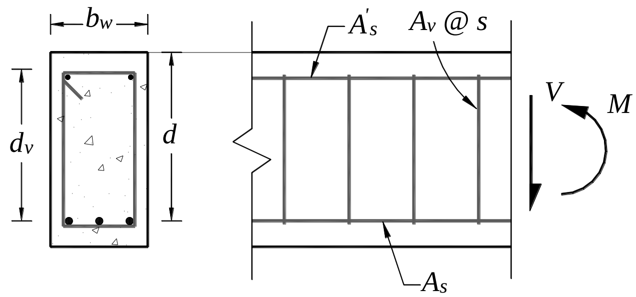

Fig. 2: A reinforced concrete cross section subjected to shear and bending.

To maintain the simplicity of the model, it is suggested to neglect the effects of both the bending moment and the unsymmetrical reinforcement on the calculation of G_{cr}. The following equations are proposed:

$$\rho_\ell = \frac{A_s + A_s'}{b_w\, d_v} \tag{3}$$

$$\rho_t = \frac{A_v}{b_w\, s} \tag{4}$$

In Eq. (3), it is assumed that the shearing stresses are uniform over an effective width equal to the width of the web b_w and an effective depth equal to the flexural lever arm d_v, taken as $(0.9d)$.

The intercept stress (v_0) depends on the cracking stress, which is significantly affected by bending. In rectangular sections, the cracks first develop on the tension side of the cross section due to the bending moment. At higher loads, they propagate towards the neutral axis at an inclination due to the effect of shear. Hence, in the parts of the beam where the bending moment is significant, (v_0) is reduced and can reach zero. If the effect of flexural cracking is neglected, Eq. (2) overestimates the intercept stress, and the shear strains are underestimated. Since it was proposed that it is preferred to underestimate the shear strains at a specific shear stress rather than overestimating them, it is suggested to maintain Eq. (2) for the calculation of the intercept stress in beams.

3. Comparison with Experimental Results

The calculations of Eqs. (1) to (4) are compared to the experimentally observed post-cracking response of three beams. For the first beam, the calculations of the are given in details to show the simplicity of the application of the equations.

Rahal [6] tested full scale beams subjected to shear and torsion and double flexure. Beam RC2-2 was not subjected to torsion. At the center of the test region, the section was subjected to pure shear and was reinforced with five top and five bottom bars. Fig. 3a shows the cross-section details. The concrete surface strains were measured in six 200 mm square grids, three on each side of the beam. At the two ends of the test region, strains were also measured where the ratio (M/V) at the center of the grids was 1.05 m. The compressive strength of the concrete was 38.2 MPa.

The reinforcement ratios are calculated as follows: $\rho_\ell = (A_s + A_s')/(b_w\, d_v) = (2500 + 2500)/(340 \times 0.9 \times 570) =$ 2.87% and $\rho_t = A_v/(b_w s) = 200/(340 \times 125) = 0.4706\%$. Hence, $G_{cr} = 32500\,(2.87\% \times 0.4706\%)^{0.42} =$ 770 MPa. The intercept stress is calculated as: $v_0 = 0.3(f_c')^{0.4} = 0.3(38.2)^{0.4} = 1.29$ MPa.

Fig. 3a compares the calculated results to the observed ones. The linear post-cracking response ends at the level of stress equal to the ultimate strength. In this case, the strength is calculated using the procedure proposed by Rahal [16]. The figure also plots the response assuming a linear elastic response of homogeneous and isotropic material. The uncracked modulus of rigidity is calculated based on a Poisson's ratio equal to 0.2 and a modulus of elasticity given by the ACI 318 [1] equation: $E_c = 4700\sqrt{f_c'}$. It is shown that the proposed equations provide significant improvements to the conventional elastic model, especially at service load levels. The latter is taken as equal to 70% of the calculated ultimate strength.

Fig. 3b provides a similar comparison for the part of the beam subjected to shear and bending. The larger amount reinforcement slightly increased the slope of the post-cracking calculated response while the presence of the bending moment increased the strains. Despite that, the calculated results compare well with the observed ones. This shows that in spite of neglecting the presence of the bending moment and the lack of symmetry in the reinforcement, the proposed equations provide improvement over the conventional procedure.

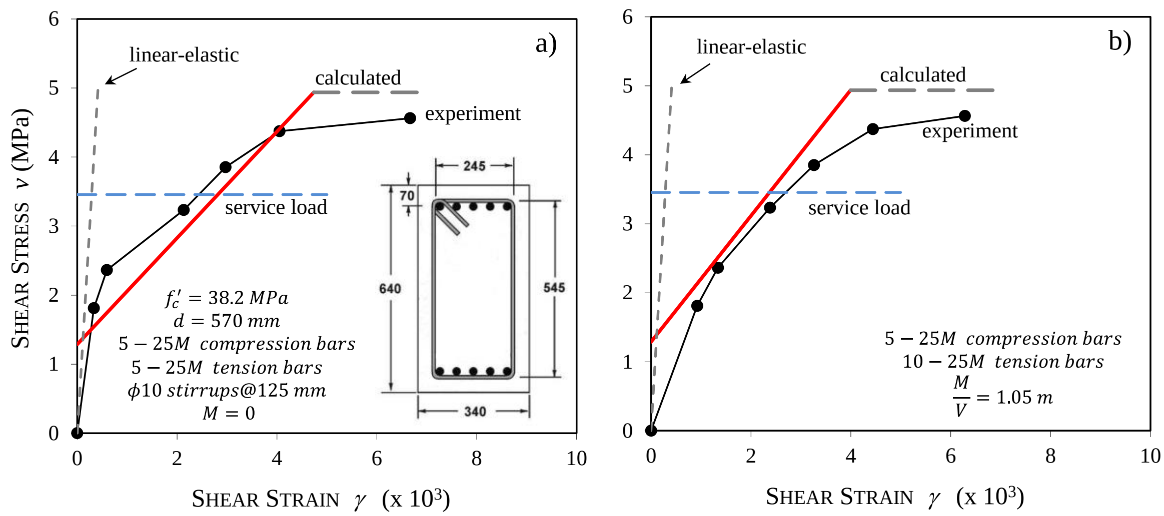

Fig. 3: Experimental and calculated shear response in beam RC2-2 [6] a) at point of contraflexure and b) at location of shear and bending.

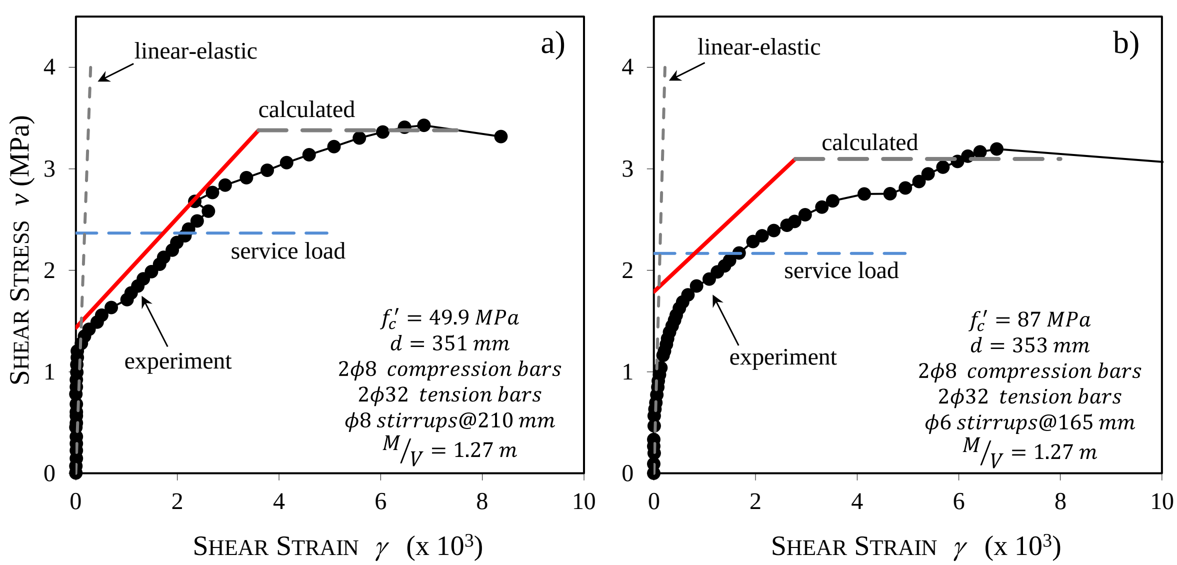

Fig. 4: Experimental and calculated shear response in beams tested by Cladera [17] a) beam H50/3 and b) H100/2.

Fig. 4 compares the results of the proposed equations with the experimental results from a beam tested by Cladera [17]. The beam was 200 mm in width and 400 mm in height and was tested in a three-point loading setup where the shear span to depth ratio was about 3. The compressive strength of the concrete was ($f_c' = 49.9$ MPa). Fig. 4 compares the observed and the calculated response. The presence of the bending moment counteracted the effect of the unsymmetrical reinforcement. Overall, the proposed model provided significant improvement over the conventional method based on linear elastic behavior.

4. Conclusions

The shear stress – shear strain response of RC beams and membrane elements after cracking and before yielding can be represented by a linear relationship. For design purposes, simple equations that can characterize this part of the response are useful to calculate the contribution of the shear strains to the overall vertical deflections in beams. This paper proposed that a set of two equations developed for the post-cracking shear response of thin membrane elements can be applied to RC beams reinforced in the longitudinal and transverse directions. The results of the proposed equations were compared with the experimental results from three RC beams and good results were obtained. A numerical example showed the simplicity of the application of the equations.

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