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Nonlinear First Ply Failure Response of Composite Cylindrical Shells under Non-Uniform Transverse Pressure

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Abstract **-** Use of laminated composite materials in place of conventional materials in weight sensitive disciplines of engineering including structural engineering started from the second half of the twentieth century. Confident use of composite structural units necessitates comprehensive understanding of the material behaviour including failure characteristics which the researchers are focusing on in the recent papers. Failure in composite materials may initiate from the surface and from inner laminae as well which may remain undetected. Unnoticed flaw may progress leading to overall failure of the structural unit. This paper intends to explore the first ply failure characteristics of shells and takes up the industrially popular cylindrical configuration having wide industrial applications about which only a very few papers report the failure related information. Failure of clamped graphite-epoxy shells is studied using Sanders' linear strains together with von-Kármán nonlinear strain components. The shells subjected to nonuniform sinusoidal is modelled using isoparametric Serendipity element having five degrees of freedom at each node. The paper reports the gross failure behaviour of a number of shell options with varying lamination and curvature. The results are presented systematically for lucid understanding and are interpreted from practical engineering standpoints. The paper concludes with pin pointed guidelines based on which the different shell combinations may be taken up for relative performance study.

Keywords**:** Nonlinear analysis, first ply failure, cylindrical shells, nonuniform pressures, composite material

1. Introduction

The hunt in the search of advanced structural materials in the second half of the last century resulted in introduction of laminated composites in different engineering branches including civil engineering. The use of these materials having high specific strength and stiffness properties does on only help to make the building frames economical but also lead to reduced foundation costs as both dead and seismic loads on the structures are reduced drastically. Cylindrical shell are popular in the industry as these singly ruled units offer ease of fabrication. Though the laminated composites have high specific strength and stiffness, high fatigue strength, capacity of being assembled fast, less susceptibility to thermal expansion and less vulnerability to weathering action and moisture but the failure of laminated composite initiate from an inner lamina. Such a failure may remain unnoticed and may cause propagation of internal damage leading to gross failure. Hence the load at which the failure initiates (first ply failure) in the laminate has to be known to the end user.

Failure study of laminated composites is an active area of research. Progressive damage of composite plates due to bending was studied by Ferreira et al. [1] .They used a FORTRAN based code and studied the variation of different stress parameters considering sinusoidal surface pressure. Ghosh and Chakravorty [2] reported failure initiation of laminated hypar shell roofs of antisymmetric cross and angle ply laminations considering different boundary conditions. They used nonlinear strains and the recently proposed Puck's criterion of failure together with serviceability failure in terms of permissible deflection limits. Nonlinear first ply failure behaviour of shallow thin composite conoidal shells subjected to central point load was reported by Bakshi and Chakravorty [3]. Kumar et al. [4] worked on the failure loads of laminated composite and sandwich cylindrical shells using finite element model based on higher order zig-zag theory. They considered different loading and boundary conditions. Well established failure theories were used by Kumar and Srivastava [5] to study first ply failure load of cross ply stiffened plates under to uniformly distributed and sinusoidal load. Prusty [6] continued with evaluation of first ply failure loads for laminated stiffened and unstiffened panels under different loading conditions considering maximum stress, maximum strain, Hoffman, Tsai-Wu and Yeh-Stratton failure criteria. First ply failure of conoidal shells subjected to uniformly loading was reported by Bakshi and Chakravorty [7].

A thorough survey of the literature reveals that research on first ply failure of industrially popular composite cylindrical shells under non uniform load using the nonlinear strains has not received due attention.

Only Ferreira et al. [1] reported failure of composites under non-uniform transverse loading but that too for plates. This paper reports nonlinear first ply failure characteristics of cross ply clamped composite cylindrical shells subjected to sinusoidal loading to partially fulfill the above mentioned lacuna.

2. Finite element mathematical formulation

The present finite element formulation uses the modified Sanders' first approximation theory for thin shells and von-Kármán type geometric nonlinear shell kinematics to study first ply failure analysis of laminated composite cylindrical shells. Fig.1 represents a cylindrical shell panel of uniform thickness h and radius of curvature R_{yy} where thickness h may consist of any number of thin laminae oriented at an angle 'θ' with respect to the global '*x*' axis. The plan dimensions of the cylindrical shell are represented as 'a' and 'b' respectively in this figure.

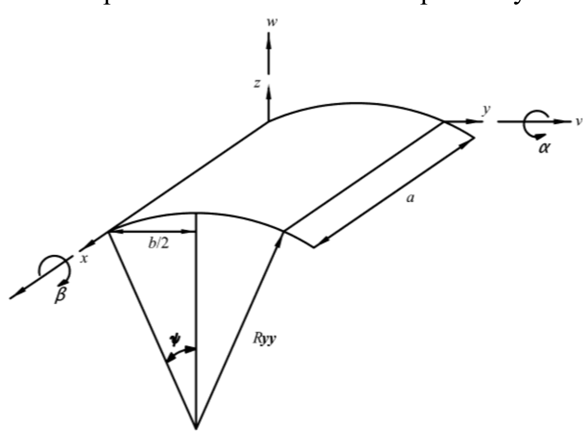

Fig. 1: Typical cylindrical shell surface

The present isoparametric finite element formulation uses eight noded doubly curved elements with C0 continuity. Five degrees of freedom u, v, w, α_x and α_y (Fig. 1) are set at each node of the element. The strain displacement relation of cylindrical shell may be expressed as a combination of mid-surface strains and curvatures which is given below.

$$\left\{\varepsilon'_x \quad \varepsilon'_y \quad \gamma'_{xy} \quad \gamma'_{xz} \quad \gamma'_{yz}\right\}^T = \left\{\varepsilon_x \quad \varepsilon_y \quad \gamma_{xy} \quad \gamma_{xz} \quad \gamma_{yz}\right\}^T + z\left\{\kappa_x \quad \kappa_y \quad \kappa_{xy} \quad \kappa_{xz} \quad \kappa_{yz}\right\}^T \tag{1}$$

The above equation can be expressed in the three-dimensional field of strain problem as,

$$\begin{aligned}\{\varepsilon\} = \{\varepsilon_x \quad \varepsilon_y \quad \gamma_{xy} \quad \kappa_x \quad \kappa_y \quad \kappa_{xy} \quad \gamma_{xz} \quad \gamma_{yz}\}^T &= \begin{Bmatrix}\varepsilon_{inplane}\\ \varepsilon_{bending}\\ \varepsilon_{shear}\end{Bmatrix} + \begin{Bmatrix}\varepsilon'_{inplane}\\ 0\\ 0\end{Bmatrix}\\ &= \left[[B_0] + 0.5[B(\{d_e\})]\right]\{d_e\} = \{\varepsilon_{lineaar}\} + \{\varepsilon_{nonlineaar}\}\end{aligned} \tag{2}$$

where, the in-plane, bending and shear strain components are expressed as,

$$\{\varepsilon_{inplane}\} = \left\{\frac{\partial u}{\partial x} \quad \left(\frac{\partial v}{\partial y} + \frac{w}{R_{yy}}\right) \quad \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\right\}^T, \{\varepsilon_{bending}\} = \left\{\frac{\partial \alpha_x}{\partial x} \quad \frac{\partial \alpha_y}{\partial y} \quad \left(\frac{\partial \alpha_x}{\partial y} + \frac{\partial \alpha_y}{\partial x}\right)\right\}^T, \{\varepsilon_{shear}\} = \left\{\left(\frac{\partial w}{\partial x} + \alpha_x\right) \quad \left(\frac{\partial w}{\partial y} + \alpha_y\right)\right\}^T$$

Finally, the nonlinear components of in-plane strains $\{\varepsilon'_{inplane}\}$ are defined as,

$$\{\varepsilon'_{\text{inplane}}\}=\left\{\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 \quad \frac{1}{2}\left(\frac{\partial w}{\partial y}+\frac{v}{R_{yy}}\right)^2 \quad \left(\frac{\partial w}{\partial x}\right)\left(\frac{\partial w}{\partial y}+\frac{v}{R_{yy}}\right)\right\}^T \tag{3}$$

(3)

In Eq. (2), $[B_0]$ is the linear part while$[B]$ is the nonlinear part of the strain-displacement matrix and is a function of nodal displacements $\{\text{de}\}$. Thus, for geometric nonlinear analysis resultant strain-displacement matrix $[\bar{B}]$ is given as,

$$[\bar{B}]=[B_0]+[B]=[B_0]+[A][G]=[B_0]+\begin{bmatrix} \frac{\partial w}{\partial x} & 0 & \frac{\partial w}{\partial y}+\frac{v}{R_{yy}} \\ 0 & \frac{\partial w}{\partial y}+\frac{v}{R_{yy}} & \frac{\partial w}{\partial x} \end{bmatrix}^T \sum_{i=1}^{8}\begin{bmatrix} 0 & 0 & \frac{\partial N_i}{\partial x} & 0 & 0 \\ 0 & \frac{N_i}{R_{yy}} & \frac{\partial N_i}{\partial y} & 0 & 0 \end{bmatrix} \tag{4}$$

where, N_i and R_{yy} denote the shape function at i^{th} node and radius of curvature along the y axis of the shell respectively.

To maintain the equilibrium condition, the virtual work done by all forces applied on the system must be zero. The internal force over the domain of the system is

$$\{\sigma_{int}\} = \int_A [\bar{B}]^T\{\sigma\}\, dA = \int_A ([B_0]+[B])^T[E]([B_0]+0.5[B])\{d_e\}dA = [k]_s\{d_e\} \tag{5}$$

The secant stiffness matrix $[k]_s$ is expressed as,

$$[k]_s = \int_A [B_0]^T[E]\,[B_0]dA + 0.5\int_A [B_0]^T\,[E][B]dA + \int_A [B]^T[E][B]\,dA + 0.5\int_A [B]^T[E][B]\,dA \tag{6}$$

Externally generated force $\{P_n\}$ on the system by the applied force is expressed as,

$$\{P_n\} = \sum_{i=1}^{8}\int_A [N_i]^T\{F\}dA \tag{7}$$

where applied force $\{F\} = \{p_x \quad p_y \quad p_z \quad m_x \quad m_y\}^T$ in which p_x, p_y, p_z are the applied non-uniform pressure along x, y and z directions respectively and m_x, m_y are the applied moments per unit area along x, y axes respectively. Except p_z, all other components are assumed to be zero for the present investigation and p_z denotes the applied non-uniform sinusoidal transverse pressure expressed as,

$$p_z = p_0 sin\frac{\pi x}{a} sin\frac{\pi y}{b}, \tag{8}$$

where p_0 is the peak transverse surface pressure of laminated composite cylindrical shell surface. As per the theorem of virtual work,

$$\{\sigma_{int}\} - \{P_n\} = \{r\} \tag{9}$$

Here $\{r\}$ is the residual force in case of nonlinear equilibrium equation which is to be minimised to get improved displacement. The detailed expression for tangent stiffness matrix $[k]_t$ is given as,

$$[k]_t = \int_A [B_0]^T[E]\,[B_0]dA + \int_A [B_0]^T[E][B]\,dA + \int_A [B]^T[E][B_0]dA + \int_A [B]^T[E]\,[B]dA$$
$$+ \int_A \begin{bmatrix} \partial N_i/\partial x & \partial N_i/\partial y \end{bmatrix}_{i=1,2\ldots,8} \begin{bmatrix} N_x & N_{xy} \\ N_{xy} & N_y \end{bmatrix} \begin{bmatrix} \partial N_i/\partial x \\ \partial N_i/\partial y \end{bmatrix} dA \qquad (10)$$

Here, N_x, N_y and N_{xy} represent the components of normal stress resultant vector and the expressions of which are the same as it were reported earlier by Ghosh and Chakravorty [2]. From the same reference the laminate elasticity matrix [E] is taken including the properties of graphite-epoxy composite.

The tangent and secant stiffness matrices and external and internal force vectors are calculated by numerical integration technique using 2×2 Gauss quadrature rule. Global stiffness matrices and force vectors are obtained by assembling the element matrices with proper transformations due to the curved geometry of the shell. Then the convergence of the Newton – Raphson iterative process is checked. The converged displacements of the cylindrical shell are used to obtain the lamina stresses and strains. The first ply failure pressures are obtained by applying those stresses and strains in well-known failure theories like maximum stress, maximum strain, Tsai-Hill, Tsai-Wu, Hoffman, Hashin and Puck failure criteria.

3. Numerical Investigations with results and discussions

3.1. Results of benchmark problem

First ply failure load values of a partly fixed laminated composite square plate was obtained both experimentally and analytically by Kam et al [8] .The published results are used for validating the correctness of the present approach. Comparative results are presented in Table 1.

Table 1: First ply failure point loads in (N) for a ($0^0_2/90^0$)s plate

Failure criteria	Length/ plate thickness	Experimental failure load (Kam et al. [8])	First ply failure loads (Kam et al. [8])	First ply failure loads (present formulation)
Maximum stress	105.26	157.34	147.61	139.94
Maximum strain			185.31	194.58
Hoffman			143.15	137.12
Tsai-Wu			157.78	150.71
Tsai-Hill			144.42	151.22

Note: Length = 100mm, Load details = Central point load

Once the accuracy of the proposed finite element code is confirmed, first ply failure peak pressure values (p_0) for cylindrical shells of fixed edges are evaluated with different combinations of anti-symmetric and symmetric, cross and angle ply laminates. Two different curvature values are taken up. The results are presented in form of tables and figures for lucid understanding.

3.2. General first ply failure behaviour

Table 2 represents the first ply failure peak pressure values (p_0) for different combinations of anti-symmetric and symmetric cross and angle ply laminates with varying curvature. It is evident from the results that the angle ply laminates, in general, perform better than their cross ply counterparts and interestingly for angle plies Puck failure criterion always yields the governing failure load. This indicates that out of seven failure criteria, only a limited study on angle ply laminates using Puck's criterion may be recommended. It is also found that except for $R_{yy}/a = 0.5$, for each of the other curvature values the $45^o/-45^o$ laminate is the best choice offering the highest value of the failure pressure.

For $R_{yy}/a = 0.5$ even, though $0^o/90^o$ laminate gives the best result, but the failure capacity of $45^o/-45^o$ option is within 10% of the highest failure load value obtained for $0^o/90^o$ laminate. Therefore, from practical engineering point of view, it would not be wrong to infer that a design engineer may recommend the $+45^o/-45^o$ angle ply laminate to be used for best performance in terms of first ply failure.

The fact that fabrication of cross ply laminates is easier than that of angle ply ones in case cylindrical shells is well known. Hence an engineer may face a compulsion of using cross ply shells only and in that case the results of Table 2 suggest that four layered cross ply laminates are to be avoided as they offer less load resisting capacity with more fabrication effort.

Table 2: First ply failure peak pressures on clamped graphite-epoxy cylindrical panels

R_{yy}/a values	Laminations	Governing failure criteria	Governing failure peak pressure (p_0) in MPa	Failed ply number
0.5	$0^0/90^0$	Hashin	0.6257	1
	$0^0/90^0/0^0$	Hashin	0.5383	1
	$0^0/90^0/0^0/90^0$	Puck	0.4712	3
	$0^0/90^0/90^0/0^0$	Hashin	0.4890	1
	$+45^0/-45^0$	Puck	0.5778	2
	$+45^0/-45^0/+45^0$	Puck	0.5793	3
	$+45^0/-45^0/+45^0/-45^0$	Puck	0.5784	4
	$+45^0/-45^0/-45^0/+45^0$	Puck	0.5722	4
0.75	$0^0/90^0$	Puck	1.0344	2
	$0^0/90^0/0^0$	Puck	0.8798	3
	$0^0/90^0/0^0/90^0$	Puck	0.6404	3
	$0^0/90^0/90^0/0^0$	Puck	0.7376	4
	$+45^0/-45^0$	Puck	1.1839	2
	$+45^0/-45^0/+45^0$	Puck	1.0626	3
	$+45^0/-45^0/+45^0/-45^0$	Puck	1.0288	4
	$+45^0/-45^0/-45^0/+45^0$	Puck	1.0110	4
1.0	$0^0/90^0$	Puck	0.7253	2
	$0^0/90^0/0^0$	Puck	0.9103	3
	$0^0/90^0/0^0/90^0$	Puck	0.5338	3
	$0^0/90^0/90^0/0^0$	Puck	0.7903	4
	$+45^0/-45^0$	Puck	1.0304	2
	$+45^0/-45^0/+45^0$	Puck	0.9522	3
	$+45^0/-45^0/+45^0/-45^0$	Puck	0.7989	3
	$+45^0/-45^0/-45^0/+45^0$	Puck	0.7988	3
1.25	$0^0/90^0$	Puck	0.7565	2
	$0^0/90^0/0^0$	Puck	0.7653	3
	$0^0/90^0/0^0/90^0$	Puck	0.5537	3
	$0^0/90^0/90^0/0^0$	Puck	0.6894	4
	$+45^0/-45^0$	Puck	0.8878	2
	$+45^0/-45^0/+45^0$	Puck	0.7425	1
	$+45^0/-45^0/+45^0/-45^0$	Puck	0.6899	4
	$+45^0/-45^0/-45^0/+45^0$	Puck	0.7160	4

Note: a = b = 1000 mm, h = 10 mm

3.3. Effect of curvature

The basic shell behavior due to curvature, calls into play a major contribution of the inplane stiffness being added flexural stiffness making these curved shapes structurally more efficient than the flat plates. Hence, growth of first ply peak pressure with increase in curvature is an interesting point of study and it is found that introduction of curvature the load resistance capacity no doubt, but when R_{yy}/a becomes less than 0.75, the load capacity suffers a decline (Refer 2). Among the class of shells taken up here a value of R_{yy}/a = 0.75 may be recommended.

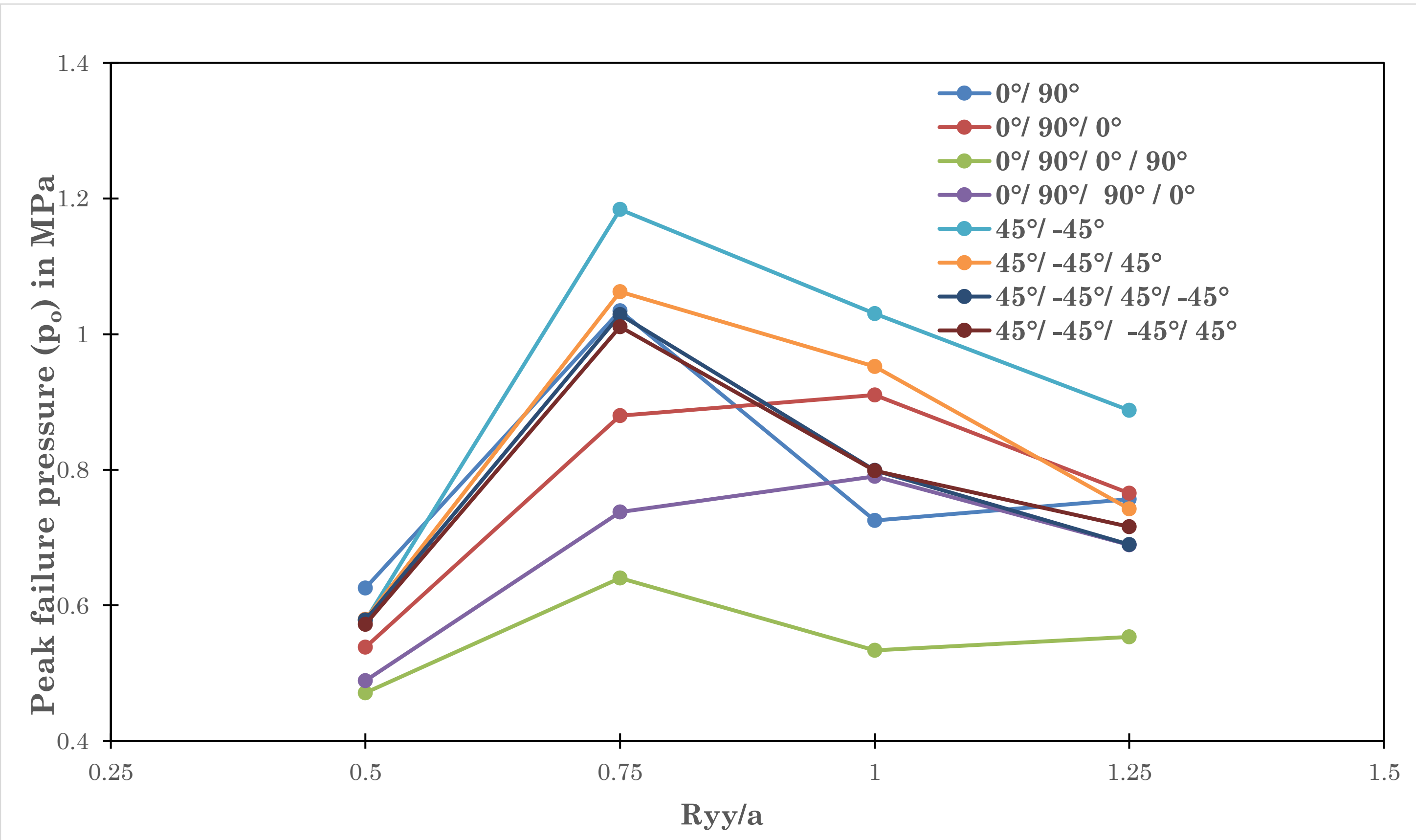

Fig. 2: Variation of peak failure pressure with curvature

3.4. Selection guidelines considering different practical factors

In practical engineering application different factors like ease of fabrication and maintenance are to be taken into account together with the load carrying capacity. Table 3 is a relative performance matrix presenting the ranks of angle ply laminates (which show superior performance compared to cross ply ones) taking into consideration the ease or difficulty in introducing a curvature in fabrication, the effort in fabricating with more number of thin laminae and also the shell options are ranked based on the fact whether damage initiation is superficial or latent.

The laminations are ranked in terms of failure loads in the following way. For p_0 value equal to or greater than 1.1 MPa, the assigned rank is 1, for p_0 value equal to or greater than 1.0 MPa but less than 1.1 MPa the assigned rank is 2 and likewise. The laminates are ranked from 1 to 7.

Fabrication difficulty increases with curvature and in Table 3 ranks are assigned between 1 to 4 corresponding to four different values of curvature. It is also well known that fabrication effort increases with increase in the number of layers. So, ranks ranging from 1 to 3 are assigned for 2, 3 and 4 layered laminates respectively.

Considering the long term behaviour of the laminates, the ones where first ply failure symptoms are superficial shall be preferred and hence those laminates are assigned with rank 1 and the ones exhibiting latent damage initiation are assigned with rank 2.

The ranks which the individual shell options are assigned from the above mentioned criteria are summed up and an overall rank is assigned to each of the laminates. It is observed that though the best choice according to this combined grading system and the one which is best in terms of first ply failure pressure are the same (+45º/-45º laminate with R/a = 0.75) but in most of the cases the overall rank and the rank in terms of load carrying capacity do not match. This means that the final selection of a particular shell combination shall not be made based on the notion that the option with highest load carrying capacity is the best practical choice.

Table 3: Relative performances of the shell options expressed in terms of ranks

R_{yy}/a values	Laminations	Ranks in terms of failure load	Ranks in terms of ease of fabricating curvature	Ranks in terms of ease of fabricating the laminate	Ranks in terms of visibility of first ply failure damage	Sum of ranks	Overall ranks
0.5	$+45^0/-45^0$	7	4	1	1	13	8
	$+45^0/-45^0/+45^0$	7	4	2	1	14	9
	$+45^0/-45^0/+45^0/-45^0$	7	4	3	1	15	10
	$+45^0/-45^0/-45^0/+45^0$	7	4	3	1	15	10
0.75	$+45^0/-45^0$	1	3	1	1	6	1
	$+45^0/-45^0/+45^0$	2	3	2	1	8	3
	$+45^0/-45^0/+45^0/-45^0$	2	3	3	1	9	5
	$+45^0/-45^0/-45^0/+45^0$	2	3	3	1	9	5
1.00	$+45^0/-45^0$	2	2	1	1	6	2
	$+45^0/-45^0/+45^0$	3	2	2	1	8	3
	$+45^0/-45^0/+45^0/-45^0$	5	2	3	2	12	7
	$+45^0/-45^0/-45^0/+45^0$	5	2	3	2	12	7
1.25	$+45^0/-45^0$	4	1	1	1	7	2
	$+45^0/-45^0/+45^0$	5	1	2	1	9	4
	$+45^0/-45^0/+45^0/-45^0$	6	1	3	1	11	6
	$+45^0/-45^0/-45^0/+45^0$	5	1	3	1	10	5

4. Conclusions

The following conclusions are evident from the present study.

The results of benchmark problem obtained through the current approach establish the correctness of the finite element code suggested by the authors in predicting the nonlinear first ply loads.

A design engineer may recommend the $+45^0/-45^0$ angle ply laminate to be used for best performance in terms of first ply failure among the laminated considered in the present study.

In case, cross ply shells are only available for use, four layered laminates are to be avoided as they offer less load resisting capacity with more fabrication effort.

The curvature of a cylindrical shell has to be judiciously chosen for improved performance as increase of curvature does not definitely imply increase or decrease of load carrying capacity corresponding to first ply failure.

The selection of a particular shell combination to be used in a given situation shall not be made based on the notion that the option with highest load carrying capacity is the best practical choice. Other practical factors like ease of fabrication, ease of identifying zones of possible first ply failure damage must be considered also for judging the overall performance of the shell option.

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