Proceedings of the 10th World Congress on Civil, Structural, and Environmental Engineering (CSEE 2025)
Barcelona, Spain - April, 2025
Paper No. ICGRE 174
DOI: 10.11159/icgre25.174

Constitutive Model for Cemented-Soil Based on a Dynamical Systems Approach under Monotonic Loading

Milind Amin[1], Rakesh J Pillai[2]
[1]Research Scholar, Department of Civil Engineering, Indian Institute of Technology, Palakkad
India - 678623
milindsamin@gmail.com
[2]Associate Professor, Department of Civil Engineering, Indian Institute of Technology, Palakkad
India - 678623
rakeshpilla@iitpkd.ac.in

Abstract **-** Soil stabilization using cement is a widely adopted geotechnical technique for improving mechanical properties, such as settlement and shear strength. However, cemented soils exhibit unique behaviours, including post-peak strain-softening, that are not always captured by conventional models. Several constitutive model frameworks, such as the critical state models and Discrete Element Method (DEM), have been widely explored to simulate soil behaviour, but the Dynamical Systems Soil Mechanics (DSSM) framework, despite its potential, has been less extensively studied, especially in the context of cemented soils. In its current form, the DSSM framework does not adequately represent the behaviour of cement-treated soils. This study proposes modifications to the DSSM framework to incorporate strain-dependent cohesion, enabling it to simulate the degradation of cementation bonds under monotonic loading. The modified model introduces an exponentially decaying cohesion term to represent the progressive breakdown of interparticle cementation, ensuring compatibility with the original DSSM framework's dynamic equations. Validation was conducted using triaxial compression test data from literature, spanning different curing times and confining pressures (CP). The modified model closely replicates the stress-strain response, achieving high correlation coefficients (0.98–0.99) for experimental and predicted results. These findings establish the model as a computationally efficient and accurate tool for simulating cemented-soil behaviour.

Keywords**:** Cemented-Soil; Constitutive models; DSSM; Shear stress; Steady state

1. Introduction

Soil stabilization through the addition of a binding material, such as cement, is a commonly used ground improvement method in geotechnical engineering. This technique is effective in mitigating excessive displacements or settlements in both shallow and deep foundations, as well as enhancing The stability of earth dam slopes and pavement applications [1], [2]. The primary aim of this method is to increase the stiffness and compressive strength of the treated soil [3], [4]. However, some studies have indicated that cementing soil may lead to significant nonlinear behaviour, which must be considered in geotechnical design and analysis [5], [6]. As a result, there is a need to develop an appropriate constitutive model to accurately capture the mechanical response of cemented-soil under various loading conditions.

According to experimental research, cementation can improve the engineering properties of soil, it also introduces unique mechanical characteristics compared to untreated soil. For instance, cementing soft clay can notably increase its stiffness and shear strength. However, this treatment may also lead to post-peak brittleness during monotonic shear, which is a critical issue that requires careful consideration in geotechnical engineering [5], [7]. Several research, including those by Kamruzzaman et al. [7], and Yu et al. [8], have demonstrated through mesoscopic testing that the unique mechanical behaviour of cemented-soils is primarily due to the breakdown of cement bonds between soil particles. This bond degradation occurs under high mean effective stresses or shear stresses, a phenomenon known as cementation degradation.

The significance of degradation of cementation has driven the development of constitutive frameworks and models to better simulate the behaviour of cement-treated soils. Several scholars have made significant contributions to the study and development of these constitutive models [5], [9], incorporating the effects due to cementation—such as increased compressive and tensile strength—into the yield function. Rahimi et al. [10] developed a bounding surface model for cement-treated sand. The model effectively captures the mechanical behaviour of cemented-sand in monotonic loading conditions.

The bounding surface model can effectively capture the macroscopic behaviour of cemented-soils, is inherently complex and computationally demanding [10], [11], [12]. This model operates on the macro-scale, focusing on overall material behaviour and large-scale stress-strain relationships. Additionally, due to its reliance on multiple yield surfaces and the need for detailed input parameters, the bounding surface model can become cumbersome. In contrast, discrete element method (DEM) a meso-scale modeling approach, offers a more granular perspective by simulating the interactions between individual soil particles.

DEM allows for the detailed study of particle-level behaviour, providing insights into how cemented-soil particles interact under stress and how cementation influences the internal structure of the material. While DEM provides valuable microstructural information, it is not without its challenges. The method demands substantial computational power and time, especially when simulating large numbers of particles over extended periods of loading. As a result, DEM may not always be practical for routine engineering applications or large-scale geotechnical studies.

Given these limitations of both macro-scale and meso-scale models, there is a growing need for a modeling approach that balances between computational efficiency and the ability to capture particle-level interactions. Joseph (2009) [13] introduced a phenomenological model. A phenomenological model for soil under shear behaviour is based on two key hypotheses. First, the rates of change of shear stress, and effective normal stress, are proportional to their applied values, with the initial constants decaying exponentially with shear strain, ultimately reaching zero at steady-state conditions. Second, the shear stress contributes to the destruction of soil structure, while confining pressure (CP) works to preserve it.

DSSM model does not account for elastic strains and is not applicable to cement-treated soils, or soils that experience significant particle breakage during shearing. Currently, it is limited to monotonic condition-based shear tests conducted on non-cemented clays, silts, sands, and their mixtures, under tension and compression conditions. In this study, based on Joseph's (2009) [13] DSSM model, a constitutive model introduced to understand the mechanical behaviour of cement-treated soils under monotonic loading. A variable representing the degree of cementation in terms of cohesion is incorporated into the model, allowing for the representation of cementation degradation. The performance of the model rigorously validated through comparison with experimental results from monotonic loading tests.

2. Dynamical Systems Soil Mechanics (DSSM)

According to Poulos (1981) [14], when a soil is subjected to shear, it progresses from an initial state of deformation toward a steady-state condition. The steady-state of deformation is characterised by continuous changes in the soil mass at constant volume, normal effective stress, shear stress, and deformation velocity. This condition is attained only when the particle orientations reach a statistically steady-state, and any particle breakage, if present, has been completed. At this point, the shear stress required to maintain deformation and the velocity of deformation remain constant.

The steady-state concept is commonly used in earthquake engineering, large-strain analysis, and soil dynamics research [15], [16], [17], [18], [19]. Joseph (2009) [13] applied this concept in a phenomenological soil shear model, which allows for modeling the full shear stress-strain behaviour, including the whole shear strain range, not just up to the development of shear failure planes.

Joseph (2009) [13] model for shear tests in monotonic condition, the model that the rates of change in shear stress (q), and effective normal stress ($\bar{p}$) are proportional to the applied normal and shear stresses. Initially, these proportionality constants decay with shear strain (γ), ultimately reaching zero at the steady-state condition. This decay follows an exponential pattern, consistent with other natural systems, as the soil structure evolves from its initial state to its final steady-state, flow-like structure. The rate of structural change is driven by shear stress, though effective normal stress counter this change at a different rate.

Mathematically, this behaviour is expressed as:

$$\frac{dq}{d\gamma} = \bar{p}Ae^{-B\gamma} - qJe^{-D\gamma} \tag{1}$$

$$\frac{d\bar{p}}{d\gamma} = \bar{p}Le^{-B\gamma} - qKe^{-D\gamma} \tag{2}$$

where A, L, J and K are proportionality constants representing the initial resistance offered by the soil structure towards $\bar{p}$ and q, respectively; B and D are the exponential rates at which these constants decay with strain. The model does not account for initial elastic deformation, and the parameters A, L, J, K, B and D are all independent of strain rate. These equations satisfy the steady-state requirement of zero change at ultimate conditions and do not include error correction terms. Also, this model does not describe elastic strains and is not applicable to cemented soils, meta-stable soils, or soils that experience significant particle breakage during shearing. Currently, it is designed for monotonic shear tests on soils that do not undergo such structural changes.

3. Proposed Model

DSSM model is effective in predicting the behaviour of non-cemented soils under monotonic shear by assuming a steady-state structural change due to shear. However, it does not account for the unique response of cemented soils, which exhibit distinct mechanical behaviour due to the presence of cohesion. This cohesion is mobilised during shear and gradually diminishes with increasing strain, leading to a strain-softening behaviour. This characteristic, often represented as a "hump" in stress-strain curves, is not captured by the original DSSM model. To address this gap, the model is modified to include a strain-dependent cohesion component.

Cemented soils derive their strength from the bonds formed between soil particles through cementation. These interparticle bonds provide additional resistance to shear stress (q) and effective normal stress ($\bar{p}$), but this resistance decreases as the cementation is progressively broken down with increasing strain. The destruction of cementation follows a gradual, exponential decay, which aligns with the structural decay mechanism already incorporated in the original DSSM model.

To incorporate this behaviour, a cohesion term, $c(\gamma)$ is introduced to the modified model. This term accounts for the additional strength provided by the cementation bonds and their strain-dependent decay. The cohesion $c(\gamma)$ decreases exponentially with strain (γ) similar to the decay of structural resistance in the original DSSM framework. This modification ensures that the conceptual consistency of the original model is maintained while allowing it to capture the mechanical behaviour of cemented-soils.

The modified equations now include the additional cohesion term:

$$\frac{dq}{d\gamma} = \bar{p}Ae^{-B\gamma} - qJe^{-D\gamma} + c(\gamma) \tag{3}$$

$$\frac{d\bar{p}}{d\gamma} = \bar{p}Le^{-B\gamma} - qKe^{-D\gamma} + c(\gamma) \tag{4}$$

where the cohesion $c(\gamma)$ is defined as:

$$c(\gamma) = c_0Re^{-s\gamma} \tag{5}$$

Where c_0 represents the initial cohesion, or the strength contribution of cementation at the start of shearing. The parameter R is a non-dimensional coefficient that quantifies the initial resistance to shear provided by cementation, while s is the rate at which cohesion decays with strain, reflecting the progressive destruction of cementation. The parameters A, L, J, K, B and D are retained from the original model, representing the initial resistance to changes in $\bar{p}$ and q, as well as the exponential decay rates of these resistances with strain. This modified model now captures the full mechanical behaviour of cemented-soils under monotonic shear, including the strain-dependent changes in cohesion and the progressive degradation of cementation as the soil is subjected to strain.

4. Model Validation

This article validates the proposed model by applying it to undrained shear tests on cement-treated soil at varying curing period and various CP. The tests include conventional compression triaxial setups. The model is fitted to the entire stress-strain curve, including the post-failure regions.

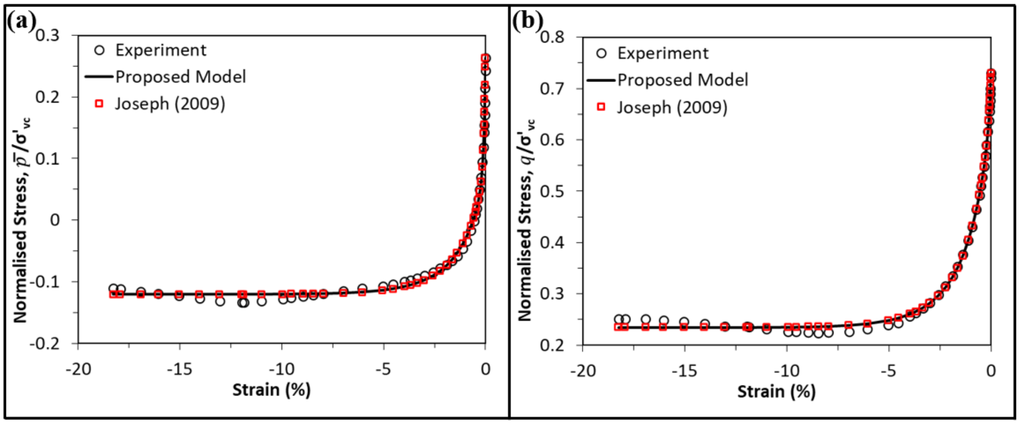

Fig. 1: Comparison of the Joseph (2009) [13] model, the proposed model, and experimentally observed data from Sheahan (1991) [20] for residual non-cemented soil in a triaxial extension test. (a) Normalised Stress, $\bar{p}/\sigma'_{vc}$; (b) Normalised Stress, q/σ'_{vc}.

To fit the model to experimental data, MATLAB tool was used, with standard 4th-order Runge–Kutta numerical integration for the iterative process. The Solver was configured to maximise the coefficient of determination (r^2) for the combined stress-strain response, considering both the shear stress (q) and effective normal stress ($\bar{p}$). The parameter values of (A, B, D, J, K, L, s, R) were initially set to (1, 0.1, 0.1, 0.5, 0.5, 1, 0.1, 1) and then iterated towards the optimal

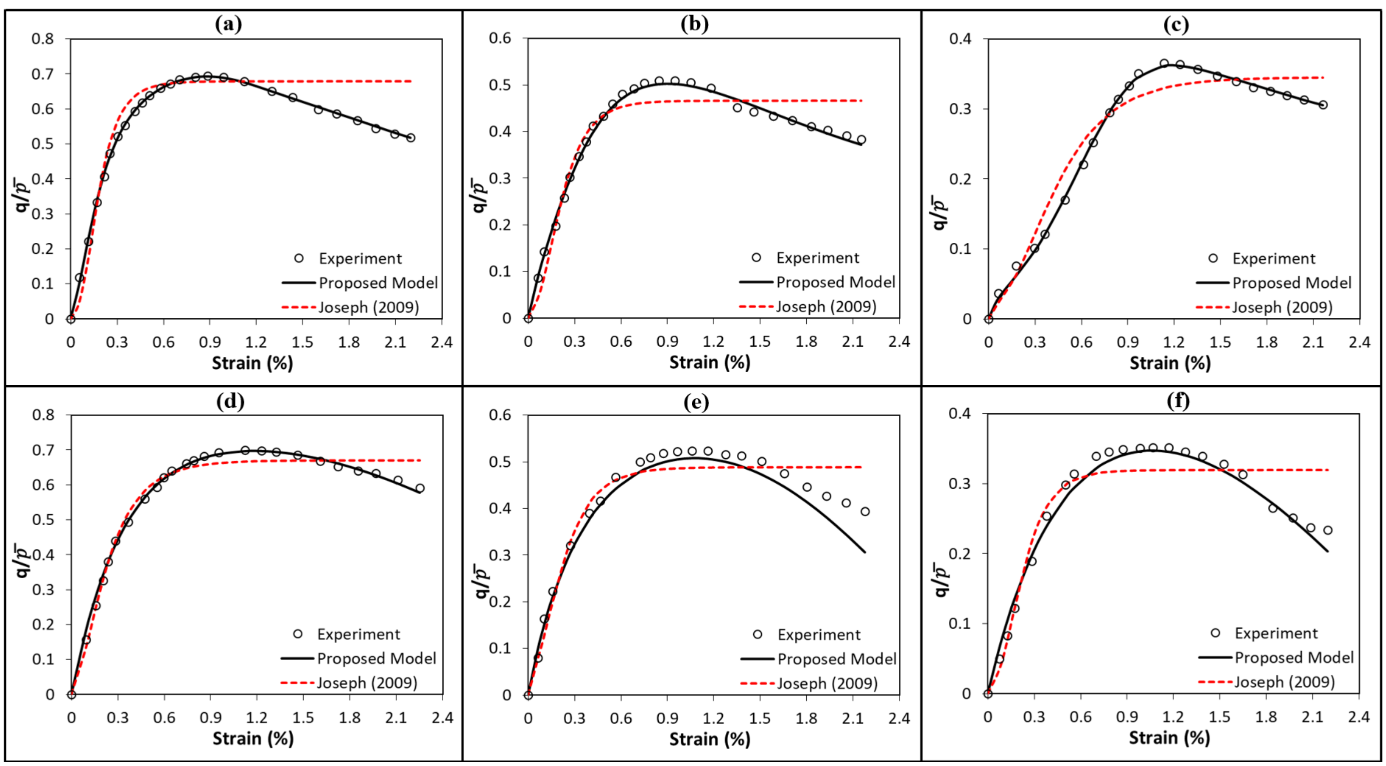

Fig. 2: Comparison between the Joseph (2009) [13] model, the proposed model, and the experimentally observed data from Royal et al. (2013) [21] for cement-bentonite samples with 28% cement content: (a) 50 kPa CP after 90 days of curing; (b) 100 kPa CP after 90 days of curing; (c) 200 kPa CP after 90 days of curing; (d) 50 kPa CP after 60 days of curing; (e) 100 kPa CP after 60 days of curing; (f) 200 kPa CP after 60 days of curing.

values, with the parameters B and D converging to similar values. And c_0 can be calculated from UCS.

Figure 1 represent a prediction using model to the stress-strain behaviour obtained from Sheahan's triaxial test on on untreated soil. The results demonstrate that both the Joseph (2009) [13] model and the proposed model accurately fit the fit the observed data for non-cemented soil. Royal et al. (2013) [21] tested three cement-bentonite slurry mixtures containing containing 28% Pulverised Fuel Ash (PFA) by mass of cementing material, using UCS and triaxial compression test apparatus to study its stress-strain and shear strength characteristics. The specimens were cured for 60, and 90 days. Each mixture was prepared with 4% bentonite and 20% cementitious materials (by mass of water), and allowed to cure underwater after being extruded from sealed moulds. Displacement rates of 1.0 mm/min and CP of 50, 100, and 200 kPa were used during testing. The shear strength and strain at peak deviator stress for the specimens did not vary significantly with CP. For the 28% PFA mixture, most physical properties of the cement-bentonite specimens showed significant changes during the first 60 days of curing. After this point, the properties stabilised and became similar to those of samples cured for 90 days. Figure 2 illustrates the typical fit of the stress-strain curves from Royal et al.'s [21] compression tests.

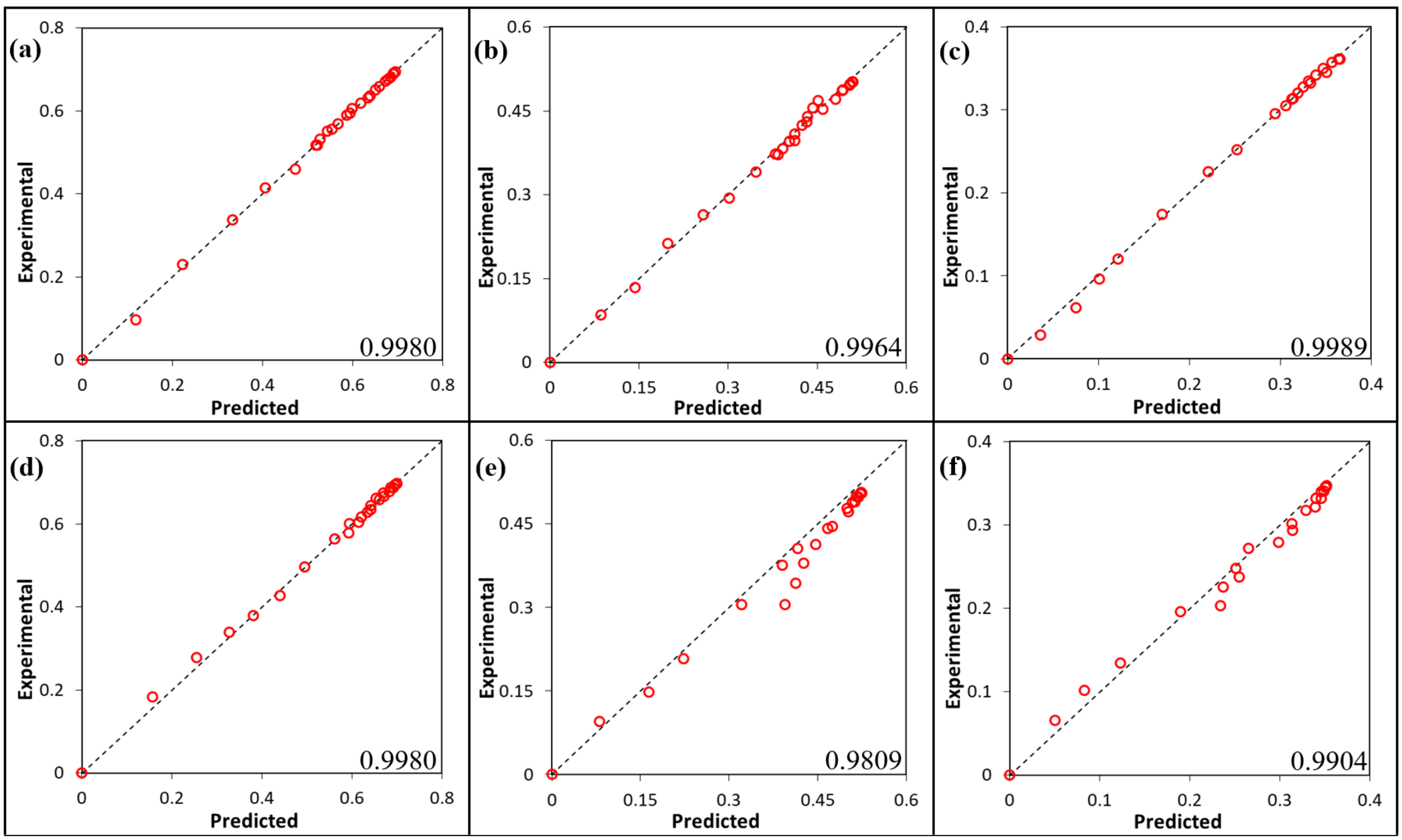

Fig. 3: Comparisons between predicted values and experimental results, along with correlation coefficients of fit, for the proposed model based on the data from Royal et al. (2013) [21]: (a) 50 kPa CP after 90 days curing; (b) 100 kPa CP after 90 days curing; (c) 200 kPa CP after 90 days curing; (d) 50 kPa CP after 60 days curing; (e) 100 kPa CP after 60 days curing; (f) 200 kPa CP after 60 days curing

5. Results and discussion

The proposed model is validated using the triaxial compression tests data from Royal et al. (2013) [21]. They conducted the test on a 28% cement-bentonite mixture after the curing of 60 and 90 days, with a deformation rate of 1.0 mm/min under CP of 50, 100, and 200 kPa. Figure 3 compares the experimental results with the predicted values, showing how well the model aligns with the observed data across different stress levels and curing periods. The close agreement between the experimental and predicted curves indicates the accuracy of the prediction model in capturing the material's stress-strain behaviour under compression.

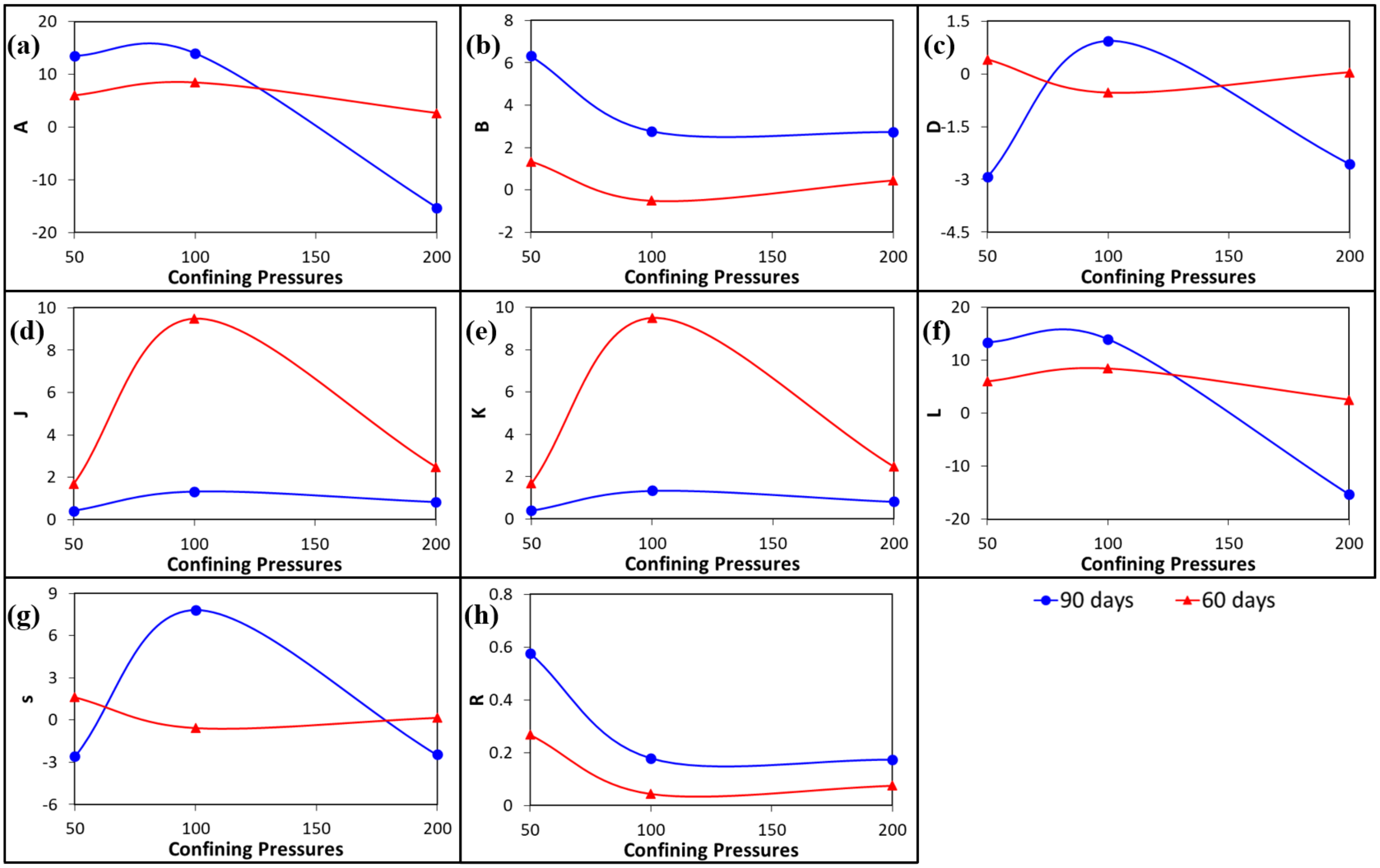

Fig. 4: Model fit parameters for triaxial compression tests conducted by Royal et al. (2013) [21].

In Table 1, experimental and predicted peak deviatoric stress values (q) are compared for curing periods of 60 and 90 days under applied CP of 50 kPa, 100 kPa, and 200 kPa. For both curing periods, the experimental values are very close to the predicted values, indicating that the prediction model is generally accurate. Minor discrepancies are observed at higher stress levels, particularly for the 90-day curing period at 100 kPa and 200 kPa, where experimental values slightly exceed the predicted ones. These differences may reflect slight variations in material behaviour or limitations of the model, but overall, the predictions align well with experimental results.

For silt, clay, and sand, the parameter values decreased as the CP increased, while for sand, the parameter values increased with relative density (r_d). Figure 4 illustrates the relationship between CP and the parameters of model obtained from the results of compression tests. However, the model parameters for cement-treated soil do not exhibit a consistent or orderly variation, as seen in the untreated soils. There could be a number of reasons for the lack of a clear pattern in the cemented-soil. One possible reason is that the cementation process leads to a more complex behaviour compared to untreated soils, as the cement bonds alter the soil structure and its response to stress. The strength and stiffness of cement-treated soil are influenced not only by the CP and relative density but also by the degree of cementation, which varies with curing time and other factors. Additionally, the non-linear degradation of cementation under loading and the potential for microstructural changes, such as cracking or bond breakage, could contribute to the observed irregularity in the model parameters.

Table 1: Comparison of experimental and predicted peak deviatoric stress values at different confining pressures based on the Data of Royal et al. (2013) [21].

Curing Period	Peak deviatoric stress, q (kPa)					
	CP 50 kPa		CP 100 kPa		CP 200 kPa	
	Experimental	Prediction	Experimental	Prediction	Experimental	Prediction
60 days	116.43	115.67	110.04	108.67	108.65	108.62
90 days	113.93	113.73	103.90	101.85	115.20	113.57

6. Conclusion

The following conclusions can be drawn from the current study:

- The model retains the conceptual simplicity of the original DSSM framework while addressing its limitations for cemented soils. This balance ensures that the proposed model is computationally efficient and suitable for practical applications.
- The model bridges the gap between macro-scale and meso-scale modeling approaches by accurately capturing strain-softening and cohesion degradation. This ensures more reliable predictions of mechanical behaviour for cement-treated soils, benefiting geotechnical designs in foundations, slopes, and pavements.
- The modified DSSM framework successfully incorporates strain-dependent cohesion, allowing it to replicate the strain-softening behaviour observed in cemented-soils. This enhancement addresses limitations of the original DSSM model in capturing post-peak bond degradation for cement-treated soil.
- The proposed model demonstrates excellent agreement with experimental results from triaxial compression tests, achieving high correlation coefficients (0.98–0.99).

Future work could explore the application of this model to cyclic loading scenarios or its integration with elastic strain components to extend its applicability to broader geotechnical challenges.

References

[1] N. C. Consoli, D. Párraga Morales, and R. B. Saldanha, "A new approach for stabilization of lateritic soil with Portland cement and sand: strength and durability," *Acta Geotech*, vol. 16, no. 5, pp. 1473–1486, May 2021.

[2] A. Thomé, M. Donato, N. C. Consoli, and J. Graham, "Circular footings on a cemented layer above weak foundation soil," *Canadian Geotechnical Journal*, vol. 42, no. 6, pp. 1569–1584, Dec. 2005.

[3] M. Azadi, M. Ghayoomi, N. Shamskia, and H. Kalantari, "Physical and mechanical properties of reconstructed bio-cemented sand," *Soils and Foundations*, vol. 57, no. 5, pp. 698–706, Oct. 2017.

[4] S. Leroueil and P. R. Vaughan, "The general and congruent effects of structure in natural soils and weak rocks," *Géotechnique*, vol. 40, no. 3, pp. 467–488, Sep. 1990.

[5] L. Nguyen, B. Fatahi, and H. Khabbaz, "Development of a Constitutive Model to Predict the Behavior of Cement-Treated Clay during Cementation Degradation: C3 Model," *International Journal of Geomechanics*, vol. 17, no. 7, Jul. 2017.

[6] U. Khalid, G. Ye, S. K. Yadav, A. Zhou, and L. Gu, "Consolidation Pressure Consequences on the Soil Structure of Artificial Structured Marine Clay: Macro and Micro Evaluation," *Geotechnical and Geological Engineering*, vol. 39, no. 1, pp. 247–263, Jan. 2021.

[7] A. H. Kamruzzaman, S. H. Chew, and F. H. Lee, "Structuration and Destructuration Behavior of Cement-Treated Singapore Marine Clay," *Journal of Geotechnical and Geoenvironmental Engineering*, vol. 135, no. 4, pp. 573–589, Apr. 2009.

[8] C. Yu, H. Wang, A. Zhou, X. Cai, and Z. Wu, "Experimental Study on Strength and Microstructure of Cemented Soil with Different Suctions," *Journal of Materials in Civil Engineering*, vol. 31, no. 6, Jun. 2019.

[9] S. Horpibulsuk, M. D. Liu, D. S. Liyanapathirana, and J. Suebsuk, "Behaviour of cemented clay simulated via the theoretical framework of the Structured Cam Clay model," *Comput Geotech*, vol. 37, no. 1–2, pp. 1–9, Jan. 2010.

[10] M. Rahimi, D. Chan, and A. Nouri, "Bounding Surface Constitutive Model for Cemented Sand under Monotonic Loading," *International Journal of Geomechanics*, vol. 16, no. 2, Apr. 2016.
[11] H. Xiao, F. H. Lee, and Y. Liu, "Bounding Surface Cam-Clay Model with Cohesion for Cement-Admixed Clay," *International Journal of Geomechanics*, vol. 17, no. 1, Jan. 2017.
[12] K. L. Dev, R. J. Pillai, and R. G. Robinson, "Estimation of Critical State Parameters from One-dimensional Consolidation and Triaxial Compression Tests," *Indian Geotechnical Journal*, vol. 43, no. 3, pp. 229–237, Sep. 2013.
[13] P. Joseph, "Constitutive Model of Soil Based on a Dynamical Systems Approach," *Journal of Geotechnical and Geoenvironmental Engineering*, vol. 135, no. 8, pp. 1155–1158, Aug. 2009.
[14] S. J. Poulos, "The Steady State of Deformation," *Journal of the Geotechnical Engineering Division*, vol. 107, no. 5, pp. 553–562, May 1981.
[15] P. V. Lade and J. A. Yamamuro, "Evaluation of static liquefaction potential of silty sand slopes," *Canadian Geotechnical Journal*, vol. 48, no. 2, pp. 247–264, Feb. 2011.
[16] J. A. Yamamuro and K. M. Covert, "Monotonic and Cyclic Liquefaction of Very Loose Sands with High Silt Content," *Journal of Geotechnical and Geoenvironmental Engineering*, vol. 127, no. 4, pp. 314–324, Apr. 2001.
[17] J. A. Yamamuro and P. V. Lade, "Steady-State Concepts and Static Liquefaction of Silty Sands," *Journal of Geotechnical and Geoenvironmental Engineering*, vol. 124, no. 9, pp. 868–877, Sep. 1998.
[18] R. J. Finno, W. W. Harris, M. A. Mooney, and G. Viggiani, "Strain Localization and Undrained Steady State of Sand," *Journal of Geotechnical Engineering*, vol. 122, no. 6, pp. 462–473, Jun. 1996.
[19] M. F. Riemer and R. B. Seed, "Factors Affecting Apparent Position of Steady-State Line," *Journal of Geotechnical and Geoenvironmental Engineering*, vol. 123, no. 3, pp. 281–288, Mar. 1997.
[20] T. C. Sheahan, "An experimental study of the time-dependent undrained shear behavior of resedimented clay using automated stress path triaxial equipment," PhD, Massachusetts Institute of Technology, Cambridge, MA, 1991.
[21] A. C. D. Royal, Y. Makhover, S. Moshirian, and D. Hesami, "Investigation of Cement–Bentonite Slurry Samples Containing PFA in the UCS and Triaxial Apparatus," *Geotechnical and Geological Engineering*, vol. 31, no. 2, pp. 767–781, Apr. 2013.