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Integrating Random FEM and CNN for Efficient Slope Stability Analysis with Spatially Variable Soil Properties

Abhijith Ajith[1], Bhawesh Kumar[2], Rakesh J Pillai[3]
[1]Research Scholar, Department of Civil Engineering, Indian Institute of Technology Palakkad
Kerala, India
abjitajit@gmail.com
[2]UG Student, Department of Civil Engineering, Indian Institute of Technology Palakkad
Kerala, India
102001006@smail.iitpkd.ac.in
[3]Associate Professor, Department of Civil Engineering, Indian Institute of Technology Palakkad
Kerala, India
rakeshpilla@iitpkd.ac.in

Abstract - Slope stability analysis that accounts for spatial variability of soil properties is computationally intensive. This study presents a hybrid approach integrating the Random Finite Element Method (RFEM) with Convolutional Neural Networks (CNN) and data augmentation to address this challenge. Using random field theory, random field samples for soil cohesion and friction angle are generated, for which RFEM is used to calculate the factor of safety. A data augmentation technique was applied to expand the RFEM dataset, generating up to 10,000 training samples from 200 initial simulations, significantly enhancing model performance. The CNN model, trained on this augmented dataset with a magnification factor of 50, achieved an R-squared value of 0.82 on cross-validation, demonstrating high accuracy. This approach drastically reduces computational time, with 200 RFEM simulations requiring about 10 hours while enabling the CNN to perform 10,000 stochastic analyses in mere minutes. The hybrid RFEM-CNN model was applied to a typical c-ϕ slope and predicted a probability of failure of 0.12%, closely aligning with reliability estimates from established finite difference methods and avoiding overestimations common in limit equilibrium methods. The findings highlight the model's potential as an efficient and robust tool for slope reliability studies, reducing computational costs while maintaining high prediction accuracy.

***Keywords*:** Convolutional Neural Network, Probabilistic slope stability, Spatial variability, Stochastic analysis, Random Finite Element Method

1. Introduction

The variability of natural soil properties is a major cause of uncertainty in soil engineering [1]. Traditional slope stability analyses often ignore this variability, leading to inaccurate results. Probabilistic slope stability analyses using random field theory and the random finite-element method (RFEM) have become popular [1], [2], [3]. RFEM involves generating random fields for soil properties and mapping them to a numerical mesh, requiring many simulations for reliable results. Monte Carlo Simulation (MCS) is often used but is inefficient for low probability failure scenarios [2].

Alternative methods for reliability calculations include the first-order reliability method, response surface methods [4], subset simulation, etc. aiming to reduce the computational effort. Machine-learning algorithms, particularly Convolutional Neural Networks (CNNs), have shown promise in handling high-dimensional data and complex problems in geotechnical engineering [2], [5]. While CNNs benefit from more training data, generating these data using traditional sampling methods like MCS can be computationally expensive. Additionally, complex CNN architectures may not always improve accuracy and can be difficult to implement.

This paper proposes enhancing CNN performance in probabilistic slope stability analysis using a data-centric approach. A small initial dataset of spatially varying random field sample data of cohesion (c) and angle of internal friction (ϕ) is generated and analyzed to obtain factors of safety using RFEM. A supplementary sampling data augmentation technique based on theoretical relationships in the shear strength reduction method is then used to expand the initial dataset without incurring external computational cost. This augmented dataset is used to train the CNN model. The RFEM-CNN model is validated with a c-ϕ soil slope and the trained CNN model is utilized to carry out MCS on random field to calculate probability of failure of the slope. Results show improved accuracy and efficiency, especially for low probability failure scenarios, compared to conventional methods.

2. Random Finite Element Method

2.1. Modeling Spatial Variability

To model the spatial variability of soil properties, random field theory and the Gaussian autocorrelation function is utilized. The Gaussian autocorrelation function provides a smooth and continuous representation, which is mathematically expressed in Eq. (1).

$$\rho[(x_1, y_1), (x_2, y_2)] = exp\left[-\pi\left(\frac{|x_1-x_2|^2}{\delta_h^2} + \frac{|y_1-y_2|^2}{\delta_v^2}\right)\right] \tag{1}$$

where (x_1, y_1) and (x_2, y_2) denotes the coordinates of any 2 points in the spatial domain; δ_2 and δ_h are the vertical and horizontal scales of fluctuation, respectively.

This function defines the spatial relationship between points based on their scaled distances, capturing the horizontal and vertical fluctuation scales [5]. For each iteration, a distance matrix is generated, and the Gaussian autocorrelation function is used to form the autocorrelation matrices for the varying parameters: cohesion (c) and angle of internal friction (ϕ). The variability of c-ϕ are assumed to be lognormally distributed, specified based on their mean and coefficient of variation (CoV). The cross-correlation between these properties is incorporated to form a comprehensive covariance matrix, which is then regularized slightly to ensure numerical stability.

For spatial variability modeling, initially the spatial domain of the slope is discretized into clusters in proportion with the scale of fluctuation. Based on the coordinates of these soil clusters, random fields are generated. The variability is quantified based on the corresponding mean and CoV values for c and ϕ parameters, assuming a lognormal distribution. Cholesky decomposition is then performed on the covariance matrix to produce a lower triangular matrix. This matrix is used to transform uncorrelated standard normal random variables into correlated variables that reflect the spatial variability and cross-correlation of the soil properties. These correlated random variables are subsequently converted to lognormal values using the previously calculated mean and variance parameters. The generated random field realizations for cohesion and friction angle are saved iteratively, providing a comprehensive dataset for geotechnical reliability analysis. This methodology ensures accurate and efficient modeling of spatially variable soil properties, crucial for probabilistic slope stability analyses.

2.2. Slope Stability Analysis

The Strength Reduction Method (SRM), also known as c-ϕ reduction, is a widely adopted technique in slope stability analysis, frequently employed using Finite Element Method (FEM) software like PLAXIS 2D. This method systematically reduces the soil strength parameters (cohesion, c and internal friction angle, ϕ) until the slope reaches a failure state, thus determining the factor of safety, FoS. In the c-ϕ reduction approach, the total multiplier $\sum M_{sf}$ is used to define the value of the soil strength parameters at any given stage in the analysis. The relationship between the reduced and initial strength parameters is given in Eq. (2).

$$\sum M_{sf} = \frac{tan\,\phi_{input}}{tan\,\phi_{reduced}} = \frac{c_{input}}{c_{reduced}} \tag{2}$$

Here, ϕ_{input} and c_{input} are the initial input values of friction angle and cohesion, respectively, and $\phi_{reduced}$ and c_{reduced} are their reduced values. The incremental multiplier M_{sf} is specified to control the stepwise reduction in strength, typically starting with a value of 0.1. This reduction continues until a fully developed failure mechanism is observed, ensuring convergence in the calculations [6]. The factor of safety, FoS is determined by the Eq. (3) once failure occurs.

$$FoS = \frac{available\ strength}{strength\ at\ failure} = value\ of \sum M_{sf}\ at\ failure \tag{3}$$

The c-ϕ reduction method in PLAXIS 2D employs a step-by-step procedure, where the soil strength parameters are reduced automatically until the model reaches failure. If failure is not fully developed, the process is repeated with a larger number of steps.

2. Machine Learning Integration

2.1. Data Augmentation

In order to use machine learning for predicting factor of safety, numerous model evaluations are required to achieve reliable training database. To overcome this challenge, a data augmentation technique was proposed, with its mathematical basis derived from the Strength Reduction Method (SRM) equation (Eq. 2). This theoretical relationship between FoS and shear strength parameters is expressed in Eq. (4), as proposed by [5], [7].

$$\tan\phi_i = \frac{FoS_i \cdot \tan\phi}{FoS}, \quad c_i = \frac{FoS_i \cdot c}{FoS} \tag{4}$$

where c_i and ϕ_i represent the reduced friction angle and cohesion for a specified intermediate FoS, FoS_i. This technique allows for the generation of additional random field realizations of soil properties without performing further numerical calculations, thereby significantly reducing computational effort. Using this SRM relationship, it is possible to compute the random field of material property corresponding to any factor of safety F_{Mg}, based on the RFEM factor of safety F_0 calculated for a specific realization of the random fields of shear strength properties. In particular, for a random field realization $x_0 = r(x_0^1, x_0^2, \dots, x_0^D)^T$, where D denotes number of varying input parameters, and calculated RFEM factor of safety F_0, the new random field sample, x_{Mg} corresponding to a given factor of safety F_{Mg}, where $x_{Mg} = r(x_{Mg}^1, x_{Mg}^2, \dots, x_{Mg}^D)^T$ is calculated from Eq. (5).

$$x_{Mg}^1 = \frac{F_{Mg} x_0^1}{F_0}, \quad x_{Mg}^2 = \frac{F_{Mg} x_0^2}{F_0}, \dots, \quad x_{Mg}^D = \frac{F_{Mg} x_0^D}{F_0} \tag{5}$$

where, x_0 and x_{Mg} can be replaced with $\tan\phi_0$ or c_0 and $\tan\phi_{Mg}$ or c_{Mg}, corresponding to the shear parameter for which the additional random fields are generated. This technique allows generating multiple random fields from a single initial sample. For instance, if Mg = 10, to obtain 10 additional random fields corresponding to F_{Mg} = 0.5, 0.75, …, 2.75. By applying these relationships iteratively, a diverse set of random field realizations can be generated for different factors of safety. This supplementary sampling technique, based on equations 5 and 6, significantly expands the dataset available for training machine learning models, such as Convolutional Neural Networks (CNNs), used in geotechnical reliability analysis. The resulting larger dataset improves the robustness and accuracy of predictive models without the need for extensive additional numerical simulations [3], [5].

2.3. Convolutional Neural Network

Convolutional Neural Networks (CNNs) are deep-learning network architecture to process and interpret high-dimensional data using operations like convolution and pooling. A typical CNN architecture includes layers such as input, convolutional, pooling, activation, fully connected, and output layers (Jiang et al. 2023). CNNs handle high-dimensional data using kernels, parameter-sharing, and pooling, which reduce the number of learnable parameters and prevent overfitting.

This study utilized the CNN toolbox available in Python's TensorFlow library. The CNN input layer configuration for handling random fields differs from conventional configurations used for digital images. Instead of images and channel, a 3D NumPy array was used. In finite-element analyses, material parameter values are assigned to discretized spatial domain with x y coordinates based on the specified scale factor. For CNNs processing random fields, element in the spatial domain is represented as an element of a 2D array, and the spatially variable soil parameters included within the 2D array. A soil layer with varying friction angle (ϕ) and cohesion (c) involves two such 2D arrays representing each random field. These fields are combined into a 3D NumPy array, with each layer in the array corresponding to a 2D representation of the random fields of ϕ and c.

The specific CNN architecture used in this study starts with a convolutional layer of 32 filters (3x3) with ReLU activation, capturing spatial patterns in the input data. This is followed by a 2x2 MaxPooling layer to reduce spatial dimensions. Three additional convolutional layers with 64, 128, and 64 filters, all using ReLU, further refine features. The output is flattened into a single vector for the fully connected layers. The first fully connected layer has 64 neurons with ReLU activation, and the final output layer has one neuron for slope stability predictions. The Adam optimizer is used for training, and the model's performance is evaluated using the R^2 score. This CNN effectively handles high-dimensional data for geotechnical reliability calculations, providing a robust approach to slope stability analysis.

2.3. Implementation

Figure 1 outlines the implementation procedure. The initial step involves gathering relevant geotechnical data, such soil properties and site-specific conditions, followed by a thorough statistical analysis to understand the variability and distribution of these properties. Using the specified mean, CoV and correlation functions, spatially correlated sample arrays representing the variability in the soil properties are generated. These sample arrays form the basis for the initial dataset, with a sample size denoted as i. Once the initial dataset is generated, random finite element method (RFEM) analysis is conducted based on these spatially correlated domains. The python remote scripting in PLAXIS 2D is utilized to iteratively assign the spatially varying material property from the initial dataset to assess the stability of the slope. The calculated factor of safety, FoS values from the RFEM analyses are saved with the corresponding input files to form the initial input dataset for data augmentation. Subsequently, supplementary sample points are generated by applying a magnification factor (Mg) to the initial dataset. This step aims to expand the dataset, capturing more intricate patterns and enhancing the robustness of the analysis.

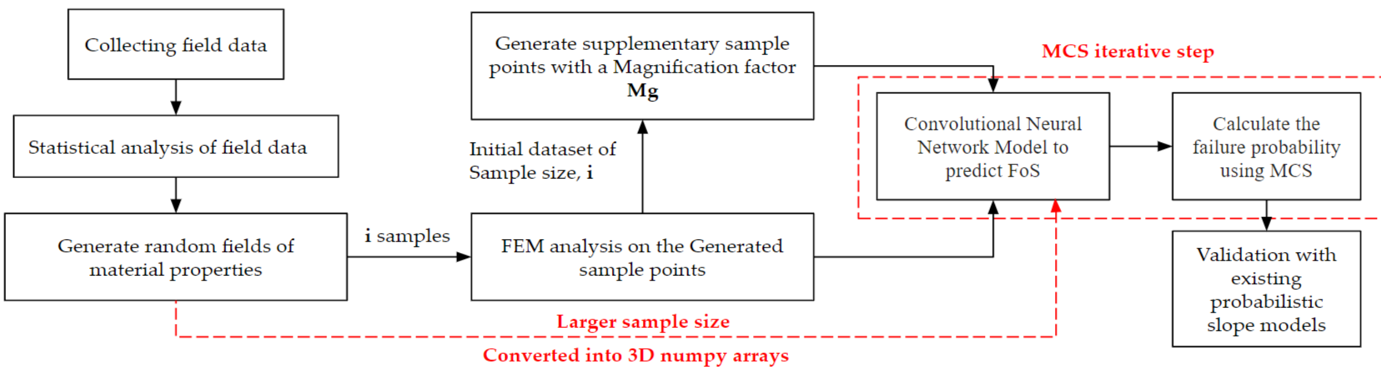

Fig. 1: Layout of proposed Methodology.

The expanded dataset is then used to train a Convolutional Neural Network (CNN) model to predict the FoS for different soil conditions. The CNN model leverages the enhanced dataset to learn and predict the stability of the slope with high accuracy. An iterative Monte Carlo Simulation (MCS) step is explicitly incorporated, where the pre-trained CNN model is used to predict the FoS for numerous scenarios, and the failure probability is calculated. The iterative MCS step ensures a comprehensive probabilistic analysis, which is validated against existing probabilistic slope models to ensure accuracy and reliability. This methodology combines advanced data augmentation techniques with machine learning and probabilistic analysis to provide a robust framework for slope stability assessment.

2. Results and Discussion

2.1. Slope Geometry

To demonstrate the effectiveness of the proposed methodology, which combines data augmentation and CNN-based reliability analysis, we use a c-ϕ soil slope example. This example has been previously studied by several researchers [4], [5], [8], [9], [10]. The slope features a height of 10 meters and an inclination of 45°. To reduce boundary effects, the right boundary extends 10 meters beyond the slope toe, and the left boundary extends 10 meters beyond the slope crest, with the bottom boundary located 5 meters below the slope toe. Figure 2 illustrates the dimensions and geometry of the modeled c-ϕ slope. The model is composed of 1,210 clusters, each containing one to two 15-noded triangular elements, with a cluster length of 0.5 meters. The deterministic FEM model is compared with a deterministic FDM analysis from [5] by calculating the factor of safety for a homogeneous soil layer with the properties listed in Table 1.

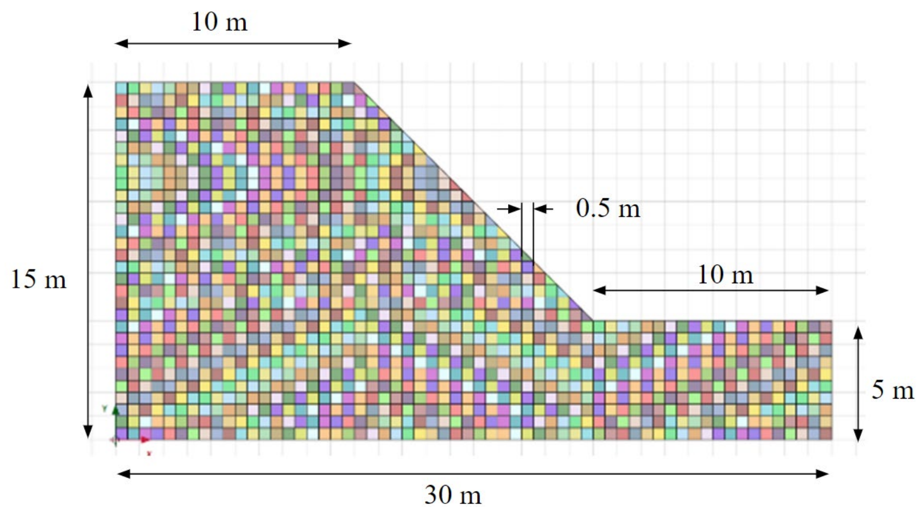

Fig. 2: Slope Geometry and dimensions

Table 1: Input Properties of RFEM model and FoS values of homogeneous case

Parameters	Value
Elastic Young's modulus	100 MPa
Poisson's ratio	0.3
Unit weight	20 kN/m³
Mean angle of internal friction	30°
Mean cohesion	10 kPa
CoV of angle of internal friction	0.2
CoV of cohesion	0.3
Cross-correlation coefficient	-0.7
Factor of safety for homogeneous case (Current study: using FEM)	1.216
Factor of safety for homogeneous case using FDM [5]	1.219

2.4. Dataset and Training

To develop a robust CNN model, a systematic approach was employed, focusing on two key parameters: the sample size of initial datasets (i) and the magnification factor (Mg) applied to each of these datasets. The initial dataset includes the spatially varying material properties saved in the form of 3D NumPy arrays. Figure 3 shows the visualization of a typical realization of the spatially varied material properties in each layer of this 3D array. This methodology could achieve an R-squared value exceeding 0.975, a benchmark based on previous modeling efforts.

Firstly, the sample size of initial datasets (i) was varied from 60 to 150. This dataset is subjected to the data augmentation based on supplementary sampling. Different magnification factors (Mg) are applied to each of the initial datasets to understand the minimum sample size required for effective training of CNN. The magnification factor controls the amplification of input and output data extracted from the initial datasets, enabling the model to capture intricate patterns and nuances within the data.

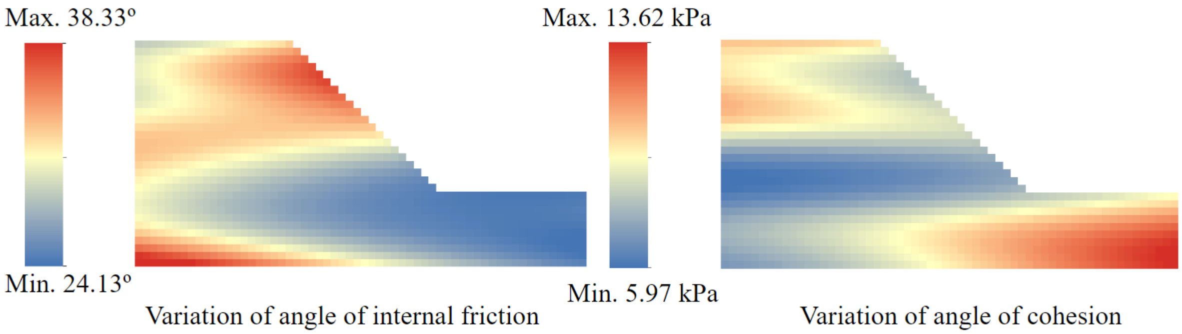

Fig. 3: Spatially varying material property for a typical realization

By systematically varying the sample size of the initial dataset across 3 different values of Mg: 10, 30 and 50, different combinations of training datasets were generated. Each combination represented a unique configuration of the augmented data, characterized by specific values of i and Mg. Subsequently, the performance of each configuration was evaluated using rigorous validation techniques, such as cross-validation or holdout validation, to ensure robustness and generalization. The primary metric for evaluation was the R-squared value, which quantifies the proportion of variance explained by the model.

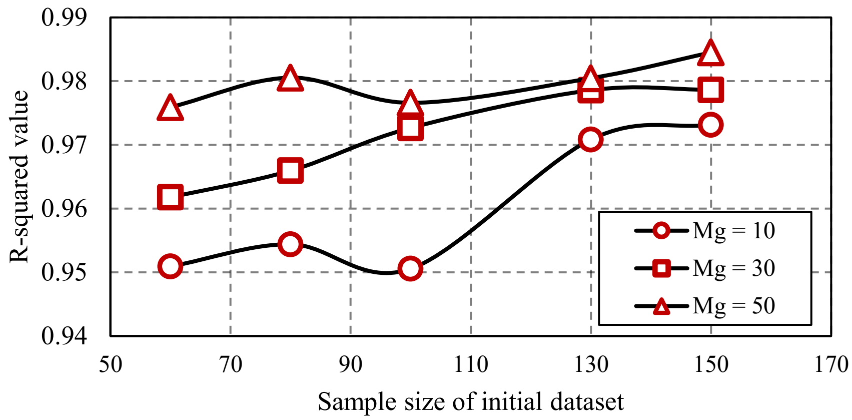

Fig. 4: Variation of R-squared value with different configuration of dataset

Through iterative experimentation and fine-tuning, the combination of parameters that yielded an R-squared value exceeding 0.975 on testing the training dataset, surpassing the performance achieved in previous studies [5], [8], [9]. Figure 4 shows the variation in R-squared value for different combination of training datasets. It is clear than a minimum of augmented sample size of 4500 was required, with at least a magnification of 30 across a minimum initial dataset sample size of 130. This evaluation ensured the development of a robust CNN model capable of accurately capturing the underlying patterns in the dataset.

2.4. Cross-Validation

The developed RFEM-CNN hybrid models with different training set were subjected to cross-validation with RFEM results. Figure 5 shows the variation in R-squared value with respective the sample size configuration used for training the CNN models. It is observed that the accuracy increases with increase in sample size, which is expected. A minimum of 7500 sample size for training with a maximum magnification factor of 50 is suggested to have an R-squared value above 0.8, shown in Fig. 5.

Based on the findings, a new RFEM-CNN model is developed for the c-ϕ slope with an initial dataset of 200 sample size magnified by a value of 50. Thus, the model was trained and validated with 10,000 datasets. Upon cross-validation, the model estimated an R-squared value of 0.82. Further, the trained RFEM-CNN model is utilized to predict the probability of failure using MCS of 10,000 simulations. Table 2 lists the probability of failure estimated from different reliability methods from previous studies. The developed hybrid model predicted a probability of failure of 0.12% which is comparable with other reliability methods using Finite Difference Methods and CNN [5], and does not overestimate the failure probability when compared with Limit Equilibrium Methods [9].

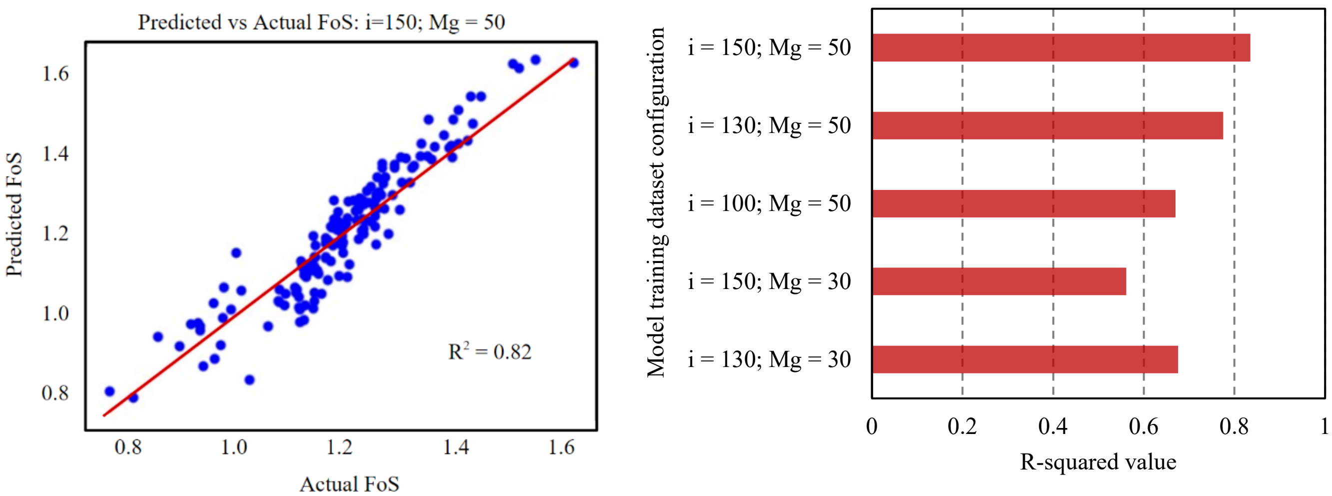

Fig. 5: Cross-Validation R-squared value with different configuration of dataset

Table 2: Comparison of probability of failure with previous studies

Simulation method	Slope stability analysis	Sample size	Failure probability
MCS [8]	Simplified Bishop method	50,000	3.9×10^{-3}
Latin-Hypercube Sampling (LHS) [9]	Simplified Bishop method	10,000	4.4×10^{-3}
MCS + Response Surfaces [9]	Simplified Bishop method	1,000	4.9×10^{-3}
Latin-Hypercube Sampling [5]	FDM	10,000	1.8×10^{-3}
LHS-CNN (6,400 training data) [5]	FDM	400	1.01×10^{-3}
MCS-CNN (200x50 initial sample) [current study]	FEM	10,000	1.21×10^{-3}

4. Conclusion

In summary, the proposed data augmentation technique significantly enhances CNN-based surrogate models for geotechnical reliability calculations, offering a practical and efficient solution for handling high-dimensional random fields and multiple uncertainties. Limiting the number of RFEM simulations eliminates the high expense of computational power required in providing a thorough statistical analysis of safety factors with comparable accuracy as traditional RFEM simulation benchmarks. The methodology is validated on a typical c-ϕ soil slope to study the performance in predicting lower value of probability of failure. The results indicate that conducting 200 RFEM simulations during the preliminary phase, which takes approximately 10 hours on a system equipped with a Ryzen 5 hexa-core processor clocked at 4 GHz and 8 GB of memory, is sufficient to adequately train the CNN model. This trained model can then predict the probability of slope failure with commendable accuracy. Additionally, an extensive Monte Carlo Simulation (MCS) can be carried out on c-ϕrandom fields explicitly using the pre-trained CNN model to calculate probability of failure of the slope. For instance, the trained model can execute 10,000 stochastic analyses within a few minutes. This rapid analysis capability is crucial for achieving a complete statistical description of the factor of safety, which demand a high volume of runs. In contrast, traditional stochastic FEM studies, necessitating such a large number of simulations, would typically span hundreds of days, underscoring the significant time-saving advantage of the present methodology.

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