

# **Development of a Physical Apparatus and Computational Program Employing a Genetic Algorithm and Least-Squares Method for Measuring the Frictional Coefficient of the Human Ocular Surface**

**Shingo Okamoto<sup>1</sup>, Sarwo Pranoto<sup>2</sup>, Yuto Ohwaki<sup>3</sup>, Jae Hoon Lee<sup>4</sup>, Atsushi Shiraishi<sup>5</sup>, Yuri Sakane<sup>6</sup>, Kiyohiko Ohta<sup>7</sup>, Yuichi Ohashi<sup>8</sup>**

<sup>1, 2, 3, 4</sup>Mechanical Engineering Course, Graduate School of Science and Engineering, Ehime University  
3 Bunkyo-cho, Matsuyama 790-8577, Japan  
okamoto.shingo.mh@ ehime-u.ac.jp; c861004y@mails.cc.ehime-u.ac.jp; b840008c@mails.cc.ehime-u.ac.jp;  
lee.jaehoon.mc@ehime-u.ac.jp

<sup>5, 6, 7, 8</sup>Department of Ophthalmology, Ehime University School of Medicine  
Shitsukawa, Toon, Ehime 791-0295, Japan  
shiraia@m.ehime-u.ac.jp; y-sakane@ m.ehime-u.ac.jp; k-oota@menicon-net.co.jp; ohashi@ m.ehime-u.ac.jp

**Abstract** - The purpose of this research is to develop a frictional coefficient measurement unit to measure the frictional coefficient of the human ocular surface and to develop a computational program using a genetic algorithm to create empirical formula. The measurement unit, which measures the normal force, the frictional force, and the velocity of the probe simultaneously, was initially developed. Then measurements of the friction at the ocular surface of a normal adult male were carried out. A computational program combining a genetic algorithm and the least-squares method was subsequently developed in order to process the experimental data. Finally, an empirical formula for the frictional coefficient of the normal adult male was created using the developed program. The authors have succeeded in measuring the frictional coefficient of the human ocular surface for the first time ever reported.

**Keywords:** Dry eye syndrome, Empirical formula, Frictional coefficient of human ocular surface, Genetic algorithm

## **1. Introduction**

In recent years, the use of computers and digital electronic devices has significantly increased. Staring at computer monitors and screens of smartphones and other mobile devices for hours at a time has become a part of the modern workday. Thus, people who stare at computer monitors for a long time may experience a change in their tear production, which is symptom of dry eye syndrome.

M. C. Acosta et al. [1] studied the changes in blink frequency and the levels of ocular discomfort during work at a video display terminal. S. Patel et al. [2] found that using a visual display unit decreases the blink rate and possibly reduces the stability of tear film. The results of these studies support the conclusion that working with visual displays causes excessive tear evaporation, thereby decreasing the tear amount covering the ocular surfaces and reducing the stability of tear film. This causes some people to develop dry eye syndrome.

In dry eye syndrome, the tears on the ocular surfaces dry, resulting in increased frictional forces of the ocular surfaces. I. Cher [3] explored ocular surface disorders that potentially arise from mechanical friction or dysfunctional lubricity within the eyes. Sakai et al. [4] assessed a newly developed eyelid pressure measurement system that uses a tactile pressure sensor to evaluate the pressure of the eyelids on the ocular surface in normal and diseased eyes. Furthermore, E. Yoshioka et al. [5] found that the eyelid pressure becomes significantly higher in dry-eyed patients than in patients with normal eyes, especially when the patients are older than 50 years. Frequently, people suffering from dry eyes may develop epithelial disorders of the ocular surface and various symptoms such as visual impairment as well as foreign body sensation and discomfort. These symptoms may cause deterioration in the quality of life. Effects of dry eye syndrome on the quality of life have been reported in other studies [6] [7]. Therefore, it is important to identify a solution to alleviate the dry eye symptom.

In general, the problem of friction is extremely complex. It is necessary to investigate the plight of ocular surfaces where friction is generated and the associated frictional characteristics in order to solve the dry eye problem. A genetic

algorithm to solve optimization problems has been studied by various researchers. It has been used by Liu [8] and L. Wu [9] to identify the parameters of friction on mechanical servo systems based on the LuGre model and the Tustin model, respectively. In addition, a combination of the genetic algorithm and the least-squares method has been used by F. Alonge et al. [10] to identify an induction motor parameter. Despite copious amounts of research existing related to dry eye syndrome, no studies have been published on the frictional coefficient of human ocular surfaces. Neither the genetic algorithm nor the least-squares method has been implemented to solve problems related to friction on human ocular surfaces. Therefore, these frictional characteristics have yet to be clarified.

The principal contribution of this research is the novel development of a frictional coefficient measuring unit for human ocular surfaces. The frictional coefficient of human ocular surfaces was considered related to the viscosity of tear fluid, the velocity of eye blink, and the palpebral pressure. The measuring unit was used to measure the normal force, the frictional force, and the velocity of the probe simultaneously. The measurement of ocular surface friction of a normal adult male was conducted. In addition, a computational program using a combined genetic algorithm and least-squares method was developed in order to configure the experimental data. Finally, an empirical formula for the frictional coefficient of the normal adult male was created using the developed program.

## 2. Development of the Frictional Coefficient Measuring Unit on Human Ocular Surface

### 2.1. Device to Measure the Moving Velocity of the Probe

It is well known that frictional coefficients of journal bearings can be identified using the Hersey Number in the field of mechanical engineering. The Hersey Number [11] is expressed by

$$H_s = \frac{\eta\omega}{p} \tag{1}$$

where  $\eta$ ,  $\omega$  and,  $p$  denote the viscosity of lubricating oil, the rotational speed of a shaft, and the pressure of lubricating oil behind the location of the minimum separation between the shaft and the bearing, respectively.

It is considered that frictional coefficient of the human ocular surface is related to the viscosity,  $\eta$ , of the tear fluid, the velocity,  $V$ , of nictation, and the palpebral pressure,  $p$ . A device capable of measuring the moving velocity of the probe used to measure frictional force,  $F$ , and normal force,  $N$ , was developed.

Figure 1 shows the schematic representation of the device used to measure the moving velocity of the probe. This device was composed of a frame, an encoder (Omron, E6H-C) to measure the rotational angle and angular velocity, two pulleys, a belt, probe housing, a microcontroller (STMicroelectronics, STM32F4 Discovery), and a laptop computer. The frame was the component used to position the human head in the proper place to fit the probe to the eye position.

The encoder was connected to the microcontroller to convert the angular velocity,  $\omega$ , of the pulleys to the corresponding linear velocity,  $V$ , of the probe connected to the belt to rotate the pulleys.

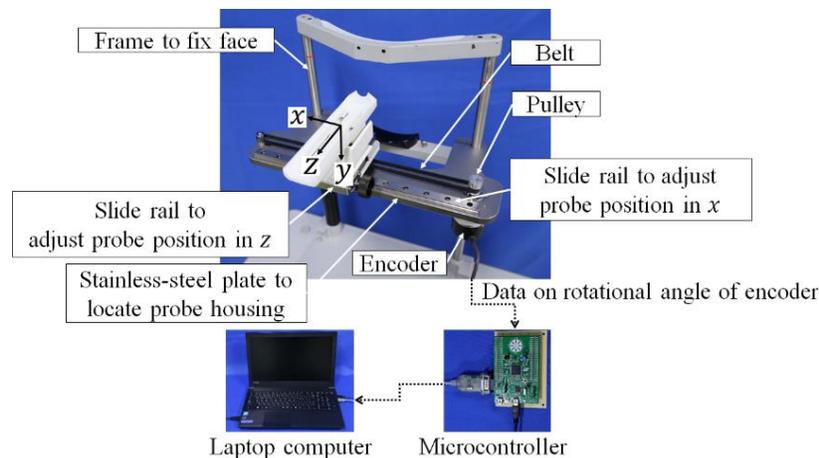


Fig. 1: Device to measure the moving velocity of the probe.

## 2.2. Assembly of the Frictional Coefficient Measuring Apparatus and Device to Measure the Moving Velocity of the Probe

Figure 2 shows the frictional coefficient measuring unit assembly including the frictional coefficient measuring apparatus and the developed device shown in Fig. 1. The frictional coefficient measuring apparatus (Trinity Lab, TL701 Handy Rub Tester) consisted of a cylindrical stainless steel probe that contacted the human ocular surface at a specified normal force to measure the encountered normal and frictional forces. The core device was responsible for collecting the normal force,  $N$ , and the frictional force,  $F$ , acquired by the probe. Thereafter, both  $N$  and  $F$  were transferred to the laptop computer directly via the serial cable.

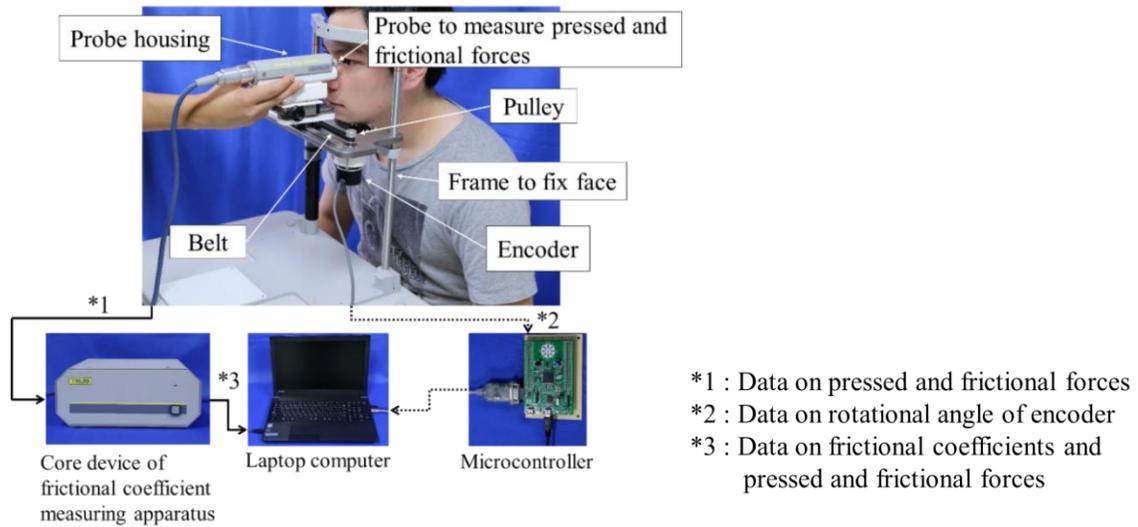


Fig. 2: Frictional coefficient measuring unit assembly including the frictional coefficient measuring apparatus and the device shown in Fig. 1.

## 3. Method of Establishing Empirical Formula for Frictional Coefficient of Human Ocular Surface

### 3.1. Mathematical Model for Frictional Coefficient of Human Ocular Surface

In the field of mechanical engineering, it is known that the frictional coefficients of bearings can be identified using the Hersey Number (eq. (1)) relating the viscosity,  $\eta$ , of the lubricating oil, the rotational speed,  $\omega$ , of a shaft, and the pressure,  $p$ , of the lubricating oil behind the location of the minimum separation between the shaft and the bearing.

In this case, the frictional coefficient of the human ocular surface is considered related to the tear viscosity, the blink velocity, and palpebral pressure.

In a normal eye, a tear layer exists between the ocular surface and the eyelid; however, in a dry eye, some areas of the two surfaces directly contact each other. For the above conditions, the frictional coefficient is considered to be within the range of fluid lubrication when the surfaces are fully separated by the tear layer and is considered to be within the range of mixture lubrication when the ocular surface is dry.

The authors propose a new number,  $X$ , that is capable of calculating the frictional coefficient of the human ocular surface as follows:

$$X = \frac{\eta^{p_1} V^{p_2}}{N^{p_3}} \quad (2)$$

where parameters,  $p_1$ ,  $p_2$ , and  $p_3$  denote arbitrary real numbers.

Then the authors propose the mathematical model describing the frictional coefficient,  $\mu$ , of the human ocular surface by incorporating the proposed new number,  $X$ , as follows:

$$\mu = p_4 X^{n-4} + p_5 X^{n-5} + \dots + p_{n-1} X + p_n \quad (3)$$

where the parameters,  $p_4, p_5, \dots$ , and  $p_n$  also denote arbitrary real numbers. In this present paper, it is assumed that  $\eta$  is constant and equal to 1, in other words  $p_1 = 0$ .

### 3.2. Identifying Parameters of Frictional Coefficient of Human Ocular Surface Using Genetic Algorithm

The algorithm combining the genetic algorithm (GA) and the least-squares method was proposed in order to identify proper values for parameters  $p_1, p_2, p_3, p_4, \dots$ , and  $p_n$  in eq. (2) and eq. (3). The computational program was subsequently developed using the proposed method.

The first step in the proposed method was to generate parent chromosomes, namely initial ones on individuals at random. Figure 3 shows the gene expressed in binary values and the chromosome composed of the parameters. The chromosome consisted of an array of parameters  $p_{i1}, p_{i2}, \dots$ , and  $p_{in}$ , and was represented by the binary encoding shown in Fig. 3. Parameter  $p_{ij}$  had a value provided by a string of bits having a length of 16. The most significant bit was used to indicate whether the sign was positive or negative. The remaining 15 bits were used to express the decimal value from 0 to 32,767, which was then multiplied by  $10^{-4}$ . Therefore, parameter  $p_{ij}$  became a value falling between -3.2767 and 3.2767.

The second step was to execute a crossover function. Figure 4 shows the uniform crossover in the genetic algorithm. The uniform crossover with random masks was used to exchange the genes in the current chromosomes as shown in Fig. 4. Masks were randomly implemented within each chromosome, and crossover points were established with respect to these masked genes. Crossover was then performed between the two chromosomes.

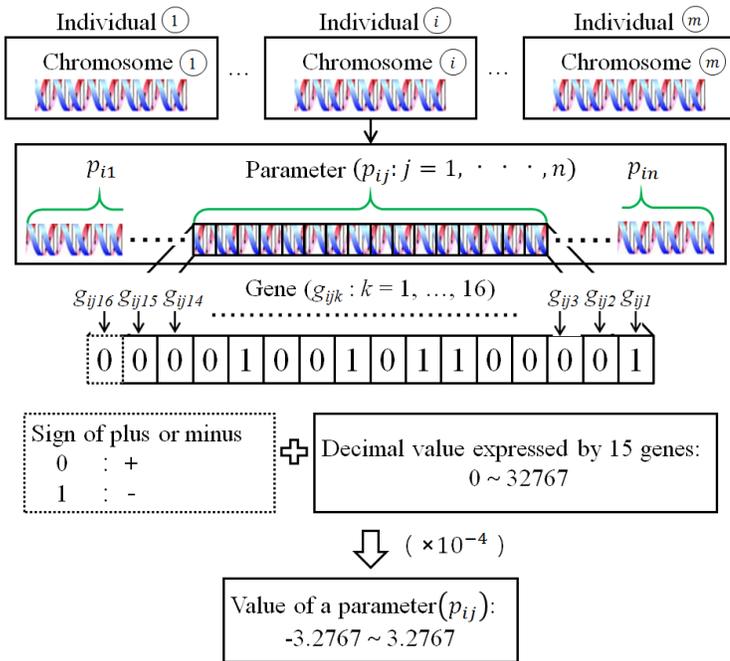


Fig. 3: Gene expressed by binary values and chromosome composed of parameters  $p_{ij}$  in the genetic algorithm.

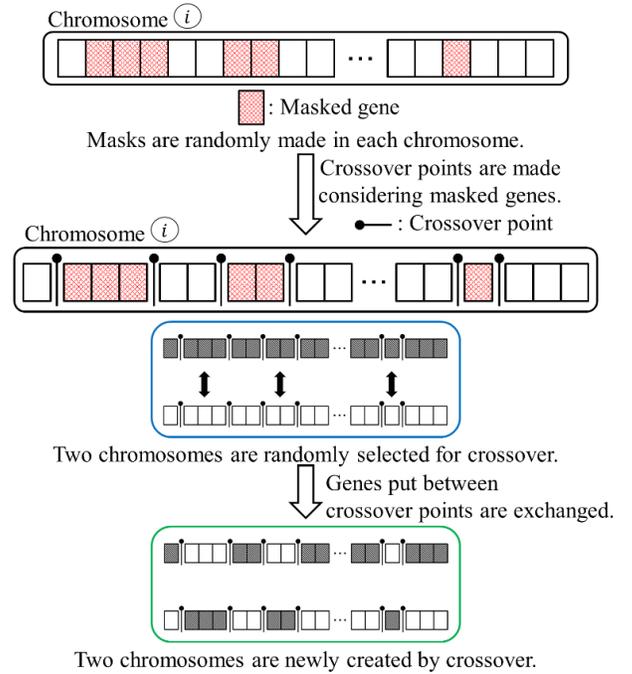


Fig. 4: Uniform crossover achieved through application of the genetic algorithm.

The third step consisted of the mutation operation. Figure 5 shows the mutation of the chromosomes in the genetic algorithm. The mutation was achieved by changing a gene from 1 to 0 or vice versa. The position of each mutated gene was determined by generating random numbers. The mutation rate,  $\alpha$ , which determined the probability that the genes of chromosomes would change, was defined by an operator of the program. An  $\alpha$  value between 0.5% and 5% was generally used.

The fourth step was to execute the selection of chromosomes. Figure 6 shows the selection of chromosomes in the genetic algorithm. In this step, the fitness (evaluation value),  $J_i$ , was calculated by the following equation:

$$J_i = 100 - \sum_{l=1}^n \sqrt{(\mu_i - \mu_l)^2} \quad (i = 1 \sim m) \quad (4)$$

where  $\mu_l$  is the actual experimental value of the human ocular surface frictional coefficient and  $n'$  is the number of experimental values.

Then, the probability of a selected chromosome was calculated by

$$x_i = \frac{J_i}{\sum_{i=1}^m J_i} \quad (i = 1 \sim m) \quad (5)$$

where  $J_i$  is the fitness of chromosome  $i$ , and  $m$  is the number of chromosomes in the current population.

After calculating the fitness of each chromosome and the probability of the selected chromosome, the selection process using the roulette wheel selection was performed.

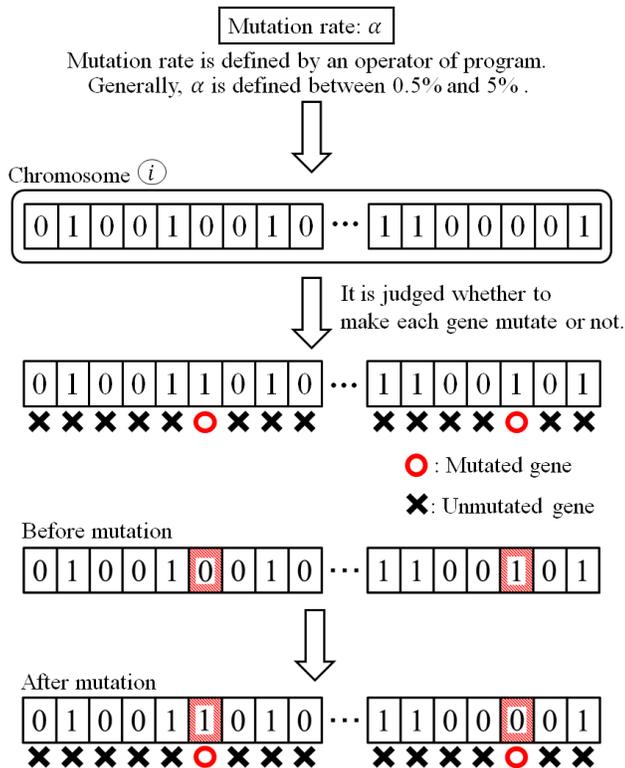


Fig. 5: Mutation of chromosomes in the genetic algorithm.

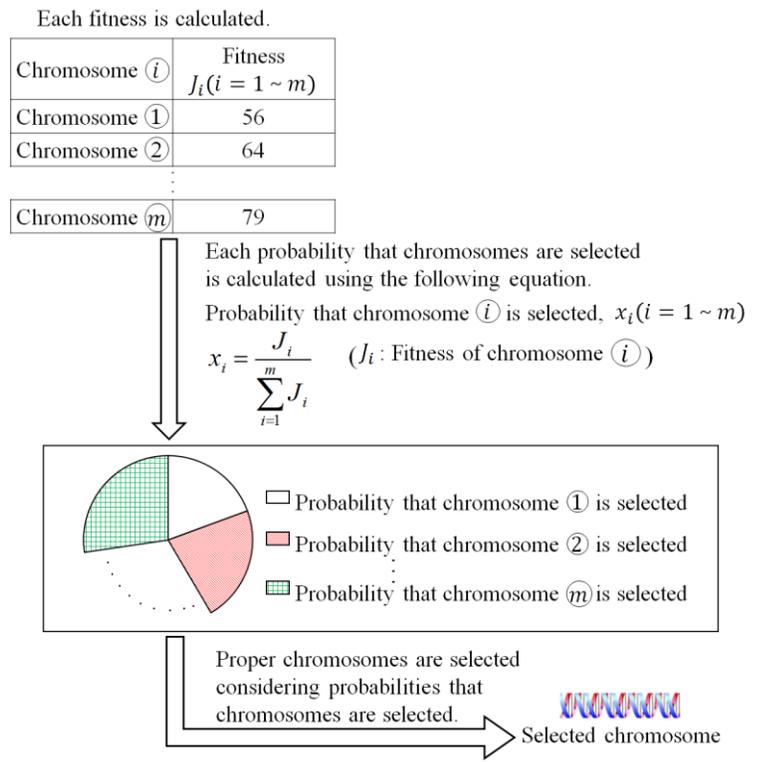


Fig. 6: Selection of chromosomes in the genetic algorithm.

### 3.3. Procedure to Identify Parameters of Human Ocular Surface Frictional Coefficient Using Genetic Algorithm

The flowchart of the proposed method illustrating all steps in the previous sections is shown in Fig. 7. In the present study, 100 ( $= m$ ) was set as the number of initial parent chromosomes. The adoption of parameters defined in eq. (2) and eq. (3) was first performed, followed by the initialization of all parameters. The four basic operations of the proposed method, crossover, mutation, calculation of fitness (evaluation value), and selection, were processed sequentially. Then, the termination criterion was checked. If the number of generations was less than the termination criterion, then a new generation was created using the four operations.

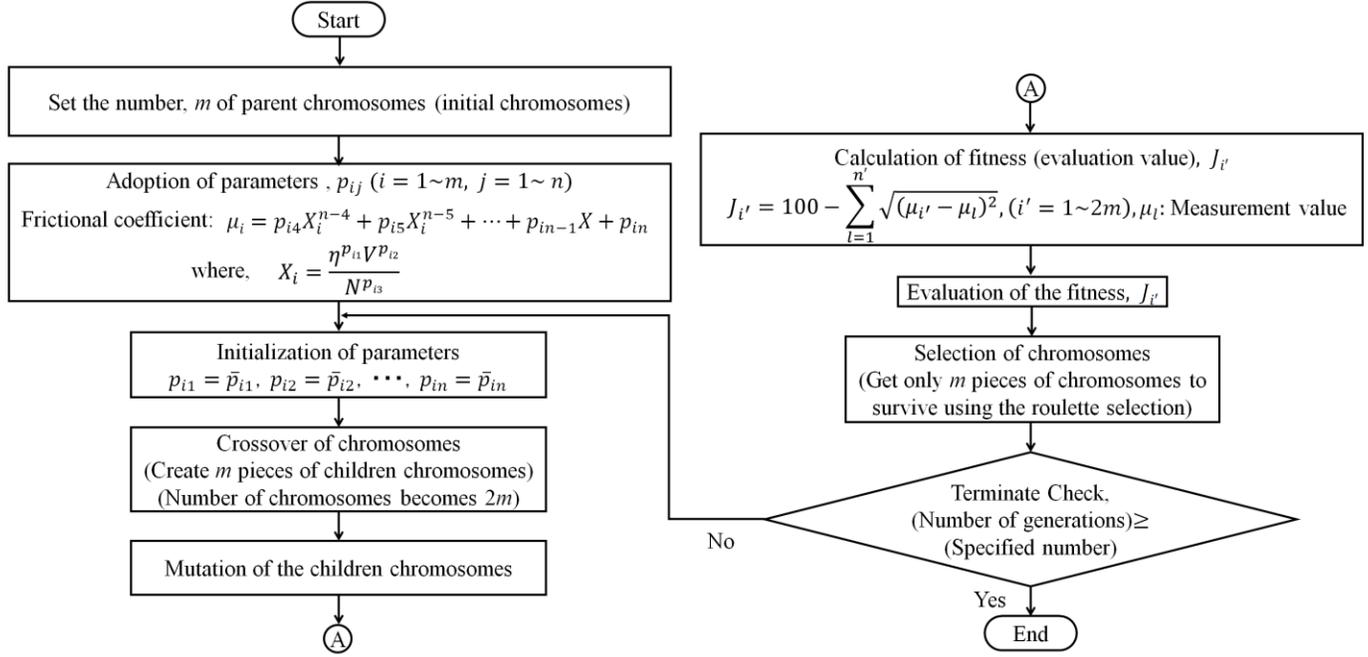


Fig. 7: Flowchart of the genetic algorithm to identify proper parameters.

## 4. Results and Discussion

### 4.1. Data Measured by the Developed Frictional Coefficient Measuring Unit

Figure 8 shows an example of the results of a cornea, measured by the frictional coefficient measurement unit. In this experiment, the normal force,  $N$ , applied to the cornea was within the range of  $1.4 \times 10^{-1}$  [N] to  $2.5 \times 10^{-1}$  [N]. The displacement,  $d$ , of the probe was controlled to be within the range of  $2.2 \times 10^{-3}$  [m] to  $5.0 \times 10^{-3}$  [m]. Results similar to those of the cornea were obtained for measurements taken on the bulbar conjunctiva.

### 4.2. Calculated Data Using the Measured Ones by the Frictional Coefficient Measuring Unit

Figure 9 shows the calculated frictional coefficient,  $\mu$ , and the velocity of the probe,  $V$ , using the measured data shown in Fig. 8. The results of the bulbar conjunctiva were similar to those of the cornea. The  $\mu$  values of the cornea ranged between 0.04 and 0.11, while the  $\mu$  value of the bulbar conjunctiva ranged between 0.04 and 0.13. The average  $\mu$  value was 0.07 for both the cornea and the bulbar conjunctiva. The velocity,  $V$ , of the probe on the cornea varied within a range of  $3.19 \times 10^{-3}$  [m/s] to  $5.95 \times 10^{-3}$  [m/s], while that of the bulbar conjunctiva varied within a range of  $3.77 \times 10^{-3}$  [m/s] to  $8.85 \times 10^{-3}$  [m/s]. The average  $V$  values for the cornea and the bulbar conjunctiva were  $4.59 \times 10^{-3}$  [m/s] and  $5.67 \times 10^{-3}$  [m/s], respectively.

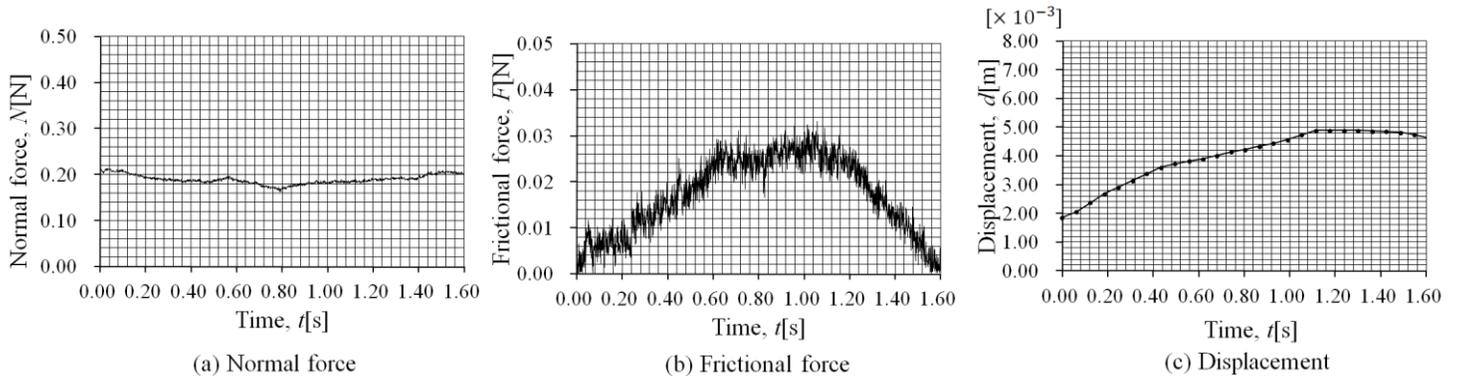


Fig. 8: Cornea results measured by the frictional coefficient measurement unit.

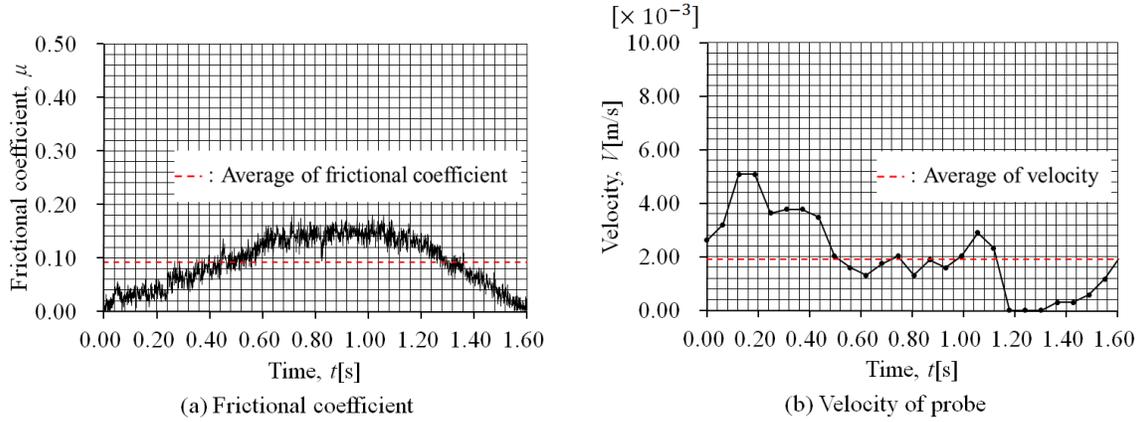


Fig. 9: Calculated cornea results using the measured values from Fig. 8.

#### 4.3. Frictional Coefficient Curve using Proposed New Number

Figure 10 shows the distribution of values of  $p_2$  and  $p_3$  obtained after 10,000 generations in the GA calculation using the experimental results. In the GA calculation, the parameters of GA calculation were set as follows: size of population  $p_{size} = 100$ ; mutation rate  $\alpha = 0.05$ ; fitness  $J > 92.50$ . The value of  $\eta$  was assumed constant.  $p_1$  was determined to be equal to 0. The values of  $p_2$  and  $p_3$  converged to a range of 0.15 to 1.33 and 0.00 to 0.76, respectively. The parameters  $p_2 = p_3 = 1.00$ , representing the Hersey Number, were eliminated. The values of  $p_2$  and  $p_3$  are shown inside the boundary of the closed dashed curve in Fig. 10. The most densely distributed region of  $p_2$  and  $p_3$  is represented by solid circle. Being the most densely distributed region, the center of the solid circle,  $p_2 = 0.85$  and  $p_3 = 0.25$  were considered to be the optimal values for the proposed new number,  $X$ .

Figure 11 shows the frictional characteristic curve of the cornea and the bulbar conjunctiva, calculated by the developed program using the proposed new number,  $X$ . The calculated frictional coefficient,  $\mu$  is plotted as a function of  $X$ , based on the values of  $V$  and  $N$ . During measurement acquisition, the cornea was measured first and the bulbar conjunctiva was measured second. The data from the cornea revealed the friction coefficient fall within both the mixture lubrication and the fluid lubrication. However, the data from the bulbar conjunctiva revealed the friction coefficient to fall within only the mixed lubrication. The measurement data from the cornea showed it to be in wet and dry conditions, while that of the bulbar conjunctiva showed it only to be in dry conditions.

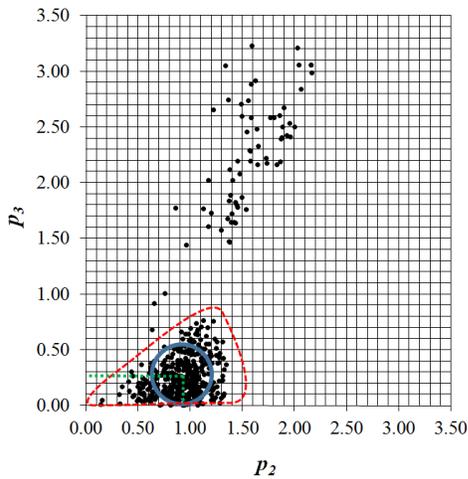


Fig. 10: The distribution of values of  $p_2$  and  $p_3$  obtained after 10,000 generations in the GA calculation using the measured data (Fitness,  $J > 92.50$ ).

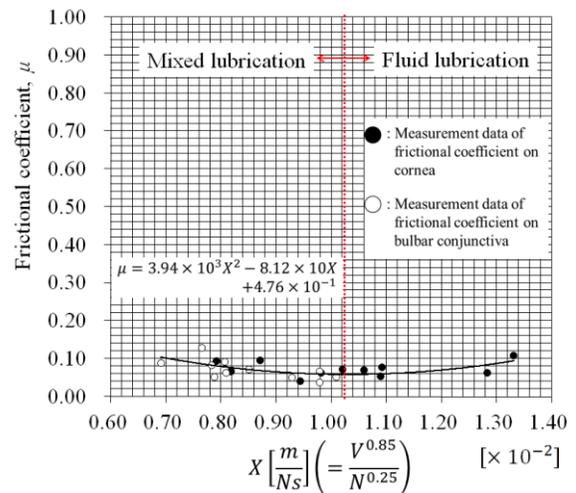


Fig. 11: Frictional characteristic curve of the cornea and bulbar conjunctiva established using the proposed new number.

The characteristics of the frictional coefficient of human ocular surface obtained in this research can be applied for clinical care of dry eye syndrome patients. They may be used by ophthalmologists in treating dry eye syndrome using lubricant eye treatments or artificial tears as appropriately required.

## 5. Conclusion

The summary of the results is shown below.

- (1) A measurement unit was developed in order to measure the frictional coefficient of the human ocular surface.
- (2) A new number relating to the tear viscosity, blink velocity, and palpebral pressure was proposed in order to identify the frictional coefficient of the human ocular surface.
- (3) A computational code combining the genetic algorithm (GA) and the least-squares method was developed to create an empirical formula.
- (4) The empirical formula describing the relationship between the frictional coefficients of the cornea and bulbar conjunctiva and the proposed new number were established using the developed computational code and the measurement data. This means that the developed measurement unit might be able to measure the frictional coefficient of the human ocular surface.
- (5) The authors succeeded in measuring the frictional coefficient of human ocular surface for the first time reported.
- (6) In future works, frictional characteristics of the human ocular surface will be further elucidated by improving the developed measurement unit and computational program.

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