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Improving the Accuracy of Dynamical System Based Optical Flow Estimation by Central Difference Discretization of Time Derivative

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Abstract **-** Optical flow is a velocity vector which represents the motion of objects in images. For the dynamical system based optical flow estimation, this paper proposes a new dynamical system model to improve the estimation accuracy. Specifically, the central difference discretization is applied instead of the forward difference discretization to time derivative in the optical flow equation. As the central difference discretization reduces the approximation error, the estimation accuracy might be improved. We validate the effectiveness of the proposed method by numerical experiments.

Keywords**:** Optical Flow Estimation, Dynamical System Model, Discretization.

1. Introduction

Optical flow is a velocity vector which represents the motion of objects in images. The estimation of optical flow enables us to detect and track the motion of the object in the image. As an optical flow estimation method, there is the gradient method which assumes that the brightness does not change after the image moves, and estimates optical flow based on the partial derivative equation, called the optical flow equation [1]. Since there are not enough constraints on the optical flow, the optical flow is not uniquely determined. In order to solve this problem, Horn and Schunck [1] and Lucas and Kanade [2] proposed two different estimation methods. The method in [1] assumes that the optical flow changes smoothly and minimizes both the error of optical flow equation and the difference of optical flow. On the other hand, the method in [2] assumes that the optical flows of the neighborhood are all same and gather all the optical flow equations in the neighborhood as a simultaneous equation, which determine the optical flow uniquely. Based on these essential works, enormous attempts have been made to improve the estimation accuracy and to overcome the difficulties caused by occlusion and/or large replacement, e.g., [3,4,5,6], etc.

The dynamical system based optical flow estimation method, which regards the optical flow equation as a dynamical system and applies the Kalman filter to the dynamical system, has been proposed [7]. The dynamical system based estimation in [7] is for the point estimation, i.e., the optical flow at a fixed pixel. In [8], the dynamical system based estimation is applied to the dense estimation of optical flow. The dynamical system based estimation method has some advantages. This method improves the accuracy of optical flow estimation in the rotating and shrinking images. Also, by applying the Kalman filter, the covariance matrix of the estimation error is obtained simultaneously with the optical flow, and the estimation accuracy can be evaluated even if the true value of optical flow is unknown. Moreover, by using the measurement residual, the flow boundary can be estimated simultaneously with the optical flow. Also, a method to introduce artificial diffusion term parameters [9] has been proposed to perform smooth estimation like the estimation method based on the variational method [10]. All the models used in those estimation method are based on the discretized both in the time and space. The time derivative has been discretized by the forward difference discretization and the spatial derivative by the central difference discretization.

In this paper, in order to improve the accuracy of optical flow estimation, we propose to apply the central difference discretization instead of the forward difference discretization to time derivative in the optical flow equation. As the central difference discretization reduces the approximation error, the estimation accuracy might be improved. We validate the effectiveness of the proposed method by numerical experiments.

2. Dynamical-System-Based Estimation Model

In this section, the quasi-dense estimation model proposed in [8] is reviewed.

2.1. The Optical Flow Equation and Dynamical System based Estimation

The gradient method is an estimation method using the conservation law of brightness. The conservation law is based on the assumption that the brightness does not change after moving the image in minute time, and is expressed by

$$\frac{dI}{dt} = \frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v + \frac{\partial I}{\partial t} = 0, \tag{1}$$

where $I(x, y, t)$ is the brightness; (x, y) represents the coordinates on the image, t is time, and $[u \;\; v]^{\mathrm{T}}$ represents optical flow. The dynamical system based estimation regards the equation (1) as a dynamical system, and applies the Kalman filter to the dynamical system [7]. The dynamical system model in [7] can be expressed as follows:

$$\frac{\partial}{\partial t}\begin{bmatrix} I \\ I_x \\ I_y \\ I_{xx} \\ I_{xy} \\ I_{yy} \\ u \\ v \\ u_x \\ v_x \\ u_y \\ v_y \end{bmatrix} = -\begin{bmatrix} I_x u + I_y v \\ I_{xx}u + I_{xy}v + I_x u_x + I_y v_x \\ I_{xy}u + I_{yy}v + I_x u_y + I_y v_y \\ 2I_{xx}u_x + 2I_{xy}v_x \\ I_{xx}u_y + I_{xy}\left(v_y + u_x\right) + I_{yy}v_x \\ 2I_{xy}u_y + 2I_{yy}v_y \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \tag{2}$$

The method in [7] enables us to estimate not only the optical flow $[u \;\; v]^{\mathrm{T}}$, but also its spatial derivatives simultaneously. Furthermore, as proposed in [7], if we apply Kalman filter to estimate the optical flow, we can also obtain the covariance matrix of the estimation error. This implies that we can evaluate the confidence of the estimation without the knowledge of true optical flow values.

In order to estimate the optical flow using a computer, it is necessary to discretize the partial derivative of the equation (1) both in space and time. Therefore, the time derivative is discretized by the forward difference discretization and the spatial derivative is discretized by the central difference discretization. Accordingly, the discretized optical flow equation becomes as follows:

$$I(x,y,k+1) - I(x,y,k) = -\left\{I_x(x,y,k)u(x,y,k) + I_y(x,y,k)v(x,y,k)\right\}, \tag{3}$$

where I_x, I_y are the spatial derivatives of the brightness on x and y. The discrete time system model can be obtained by applying similar discretization to the left-hand side of (2). With the discrete time system model, we can implement the proposed method in [7].

2.2. Dense Estimation

The dynamical system based estimation in [7] is for the point estimation, i.e., the optical flow (and its spatial derivatives) at a fixed pixel is estimated. In [8], the dynamical system based estimation is applied to the dense estimation of optical flow. The key idea is replacing the spatial derivative of the brightness at the pixel with the difference of the brightness of the neighborhood of the pixel. This replacement can be realized as follows:

$$
\begin{bmatrix} \vdots \\ I(x,y,k+1) \\ \vdots \\ u(x,y,k+1) \\ v(x,y,k+1) \\ \vdots \end{bmatrix} = \begin{bmatrix} \vdots \\ I(x,y,k) - \left\{ \dfrac{I(x+1,y,k) - I(x-1,y,k)}{2} u(x,y,k) + \dfrac{I(x,y+1,k) - I(x,y-1,k)}{2} v(x,y,k) \right\} \\ \vdots \\ u(x,y,k) \\ v(x,y,k) \\ \vdots \end{bmatrix}. \tag{4}
$$

Note that all the elements in the right-hand side of the equation consist of the state variables, i.e., the variables in the left-hand side of the equation, at time *k*. Accordingly, the equation (4) can be regarded as a discrete time dynamical system. In order to guarantee the observability of the model (4), the quasi-dense estimation in [8] proposes to decimate the optical flow of some pixels and estimate the optical flow of the remaining pixels, not estimate with all pixels of the estimation area. The optical flows of decimated pixels are interpolated by applying the interpolation operator after estimation.

3. Proposed Method

In this section, we describe a new estimation model that improves the accuracy of the optical flow estimation.

3.1. Improvement of Discretization Accuracy of Time Derivative

In order to improve the accuracy of optical flow estimation, we propose to apply the central difference discretization instead of the forward difference discretization to time derivative in the equation (1). As the central difference discretization reduces the approximation error, the estimation accuracy might be improved. With the central difference discretization, equation (1) is discretized as follows:

$$
\frac{I(x,y,k+1) - I(x,y,k-1)}{2} = -\left\{ I_x(x,y,k) u(x,y,k) + I_y(x,y,k) v(x,y,k) \right\}. \tag{5}
$$

Accordingly, the discrete time system model of optical flow (4) is replaced by

$$
\begin{bmatrix} \vdots \\ I(x,y,k+1) \\ I(x,y,k) \\ \vdots \\ u(x,y,k+1) \\ v(x,y,k+1) \\ \vdots \end{bmatrix} = \begin{bmatrix} \vdots \\ \begin{bmatrix} I(x,y,k-1) \\ \quad - 2\left\{ \dfrac{I(x+1,y,k) - I(x-1,y,k)}{2} u(x,y,k) + \dfrac{I(x,y+1,k) - I(x,y-1,k)}{2} v(x,y,k) \right\} \end{bmatrix} \\ I(x,y,k) \\ \vdots \\ u(x,y,k) \\ v(x,y,k) \\ \vdots \end{bmatrix}. \tag{6}
$$

Note that $I(x,y,k)$ in the left-hand side of the equation is the one time ahead of the variable $I(x,y,k-1)$ and the implementation of the central difference discretization of time derivative requires the additional brightness $I(x,y,k-1)$ as a partial state of the dynamical system model. The explicit representation of the discrete time system is given as follows. Let us assume that the estimation area is a rectangular area whose width and height are *M* and *N*, and that *L* is the number of pixels in which optical flow is actually estimated. The vector of brightness at time *k* is defined as follows:

$$
q_{I(k)} = [I_{(1,1,k)} \;\; I_{(1,2,k)} \;\; \cdots \;\; I_{(M,N,k)}]^{\mathrm{T}}. \tag{7}
$$

Also, the vectors of the optical flows are defined as follows:

$$
q_{u(k)} = [u_{(1,1,k)} \;\; u_{(1,2,k)} \;\; \cdots \;\; u_{(M,N,k)}]^{\mathrm{T}}, \tag{8}
$$

$$q_{v(k)} = [v_{(1,1,k)}\ \ v_{(1,2,k)}\ \ \cdots\ \ v_{(M,N,k)}]^{\mathrm{T}}. \tag{9}$$

Using these vectors, the state vector $q_{(k)}$ of the proposed model is defined as follows:

$$q_{(k)} = [q_{I(k)}^{\mathrm{T}}\ \ q_{I(k-1)}^{\mathrm{T}}\ \ q_{u(k)}^{\mathrm{T}}\ \ q_{v(k)}^{\mathrm{T}}]^{\mathrm{T}}. \tag{10}$$

Using these equations, the following non-linear dynamical system is considered as follows:

$$q_{(k+1)} = f(q_{(k)}), \tag{11}$$

$$z_{(k)} = Hq_{(k)}, \tag{12}$$

where $f(q_{(k)})$ is the non-linear function, and $z_{(k)}$, H are given by

$$z_{(k)} = [I_{(1,1,k)}\ \ I_{(1,2,k)}\ \ \cdots\ \ I_{(M,N,k)}]^{\mathrm{T}} = q_{I(k)}, \tag{13}$$

$$H = \left[E_{MN\times MN}\ \ O_{MN\times(MN+2L)}\right]. \tag{14}$$

The matrices $E_{m\times n}$, $O_{m\times n}$ are the $m\times n$ identity matrix and zero matrix, respectively. We apply a Kalman filter to the proposed model. The model with system noise $\eta_{(k)}$ and measurement noise $\zeta_{(k)}$ is defined as follows:

$$\begin{bmatrix} q_{I(k+1)} \\ q_{I(k)} \\ q_{u(k+1)} \\ q_{v(k+1)} \end{bmatrix} = f(q_{(k)}) + \begin{bmatrix} \eta_{I(k)} \\ O \\ \eta_{u(k)} \\ \eta_{v(k)} \end{bmatrix}, \tag{15}$$

$$z_{(k)} = Hq_{(k)} + \zeta_{(k)}. \tag{16}$$

Note that the state $q_{I(k-1)}$ is the past signal and the noises are the current signals. Accordingly, the second element of the noise vector in (15) should be zero. As the proposed model is a non-linear dynamical system model, the extended Kalman filter can be applied to estimate the state of the model. The proposed model is expected to improve accuracy of optical flow estimation. However, due to increasing the number of the state, it is expected that the proposed method requires much more computational consumption.

4. Numerical Experiments

In this section, we validate the effectiveness of the proposed model in subsection 3.1 by numerical experiments.

4.1. Description of Experiments

In this experiment, the sequence of generated image in which the left half part is shrinking toward the center and the right half part is rotating is used. The brightness $I_{(r,\theta,k)}$ of the left half and right half images are given respectively by

$$I_{1(r,\theta,k)} = \frac{1}{2}[\sin\{k_r \log(r - k_w)\}\sin(k_\theta\theta) + 1], \tag{17}$$

$$I_{2(r,\theta,k)} = \frac{1}{2}\{\sin(k_r \log r)\sin(k_\theta\theta - k_w) + 1\}, \tag{18}$$

where (r, θ) represents the polar coordinates. The parameter values of the equations (17) and (18) are $k_r = 8$, $k_\theta = 6$, and $k_w = -0.2k$. These parameters generate the time-invariant optical flow. We also set $k_w = -(0.2k + 0.02\sin\frac{k\pi}{5})$, which generates the time-varying optical flow. The optical flow estimation method based on the variational method [10] is known to be related to the solution of the time evolution equation including the diffusion term. Therefore, the diffusion term having the smoothing effect of the estimation is introduced [9]. At the edge of the estimation area, the diffusion term is determined by other equation. Estimation is performed on the white frame parts, whose size are 60×60 pixels, in Fig. 1. The covariance matrix of $\eta_{I(k)}$ is set to 0.001 *I*, i.e., the independent white noise with variance 0.001 is added to each pixel brightness. Similarly, the covariance matrices of $\eta_{u(k)}$ and $\eta_{v(k)}$ are set to 0.02 *I*. Also, the covariance matrix of $\zeta_{(k)}$ is set to 0.1 *I*. The Euler method with 1 time step per 1 frame is used as numerical integration. We used the endpoint error to assess the accuracy of optical flow estimation for each pixel. The endpoint error is the distance between the tips of the estimated flow vector $\widetilde{U}$ and the true flow vector *U*, and may be determined by using equation (19).

$$\sqrt{(\widetilde{U} - U)^{\mathrm{T}}(\widetilde{U} - U)}\,. \tag{19}$$

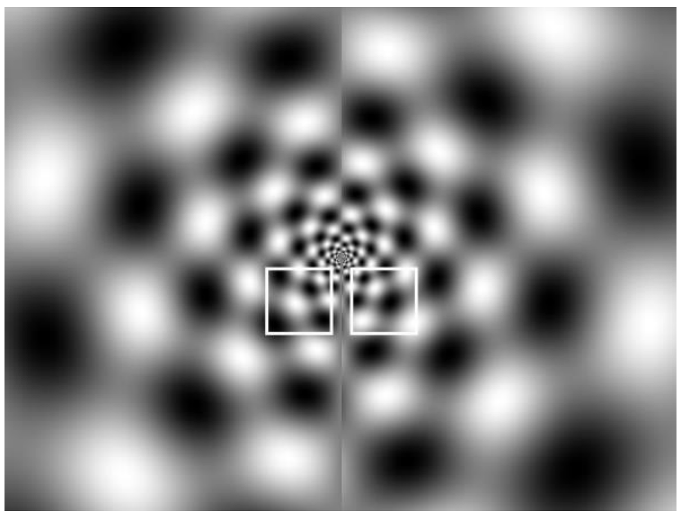
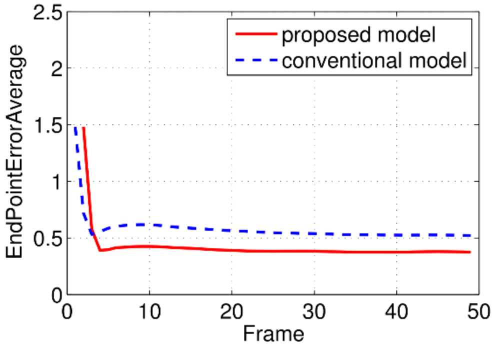


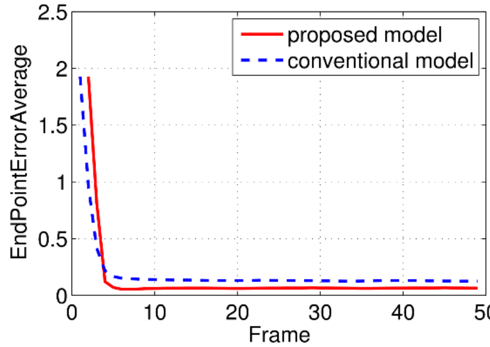
Fig. 1: The first frame of the video image.

4.2. Comparison between Proposed Model and Conventional Model

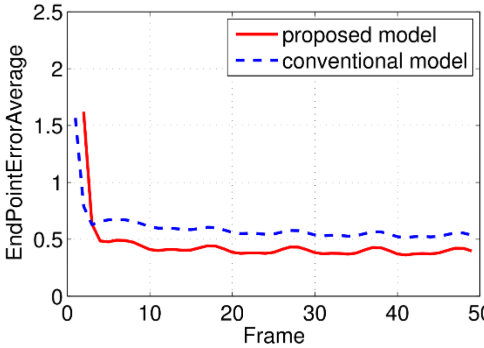
Numerical experiments are conducted by using the proposed and the conventional models. The proposed model requires the brightness at one time before as its state. Therefore, the estimation with the proposed model starts from the second frame. Figure 2 shows the time series of the average of endpoint errors; (a) time-invariant shrinking image, (b) time-invariant rotating image, (c) time-varying shrinking image, (d) time-varying rotating image. Table 1 shows the endpoint error average from 11 to 40 frames.



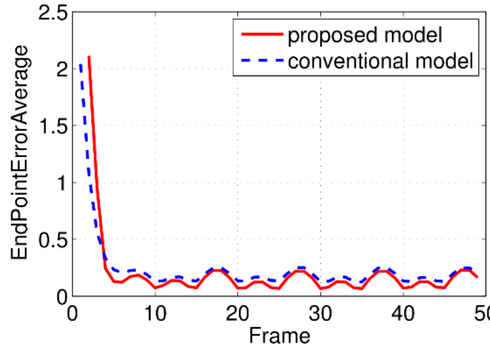
(a) time-invariant shrinking image



(b) time-invariant rotating image



(c) time-varying shrinking image



(d) time-varying rotating image

Fig. 2: Endpoint error average for each frame of the images.

Table 1: Endpoint error average and processing time of 11 to 40 frames of the images.

	Endpoint Error Average				processing time[s]			
	(a)	(b)	(c)	(d)	(a)	(b)	(c)	(d)
conventional model	0.5539	0.1310	0.5626	0.1768	23.1491	23.0325	23.0598	23.0790
proposed model	0.3884	0.0640	0.4006	0.1328	67.0975	64.9123	65.4327	66.6394

4.3. Discussion

From Fig. 2, it is confirmed that the proposed method achieves higher estimation accuracy than the conventional method in the time-invariant shrinking and rotating images. As the proposed dynamical model uses the additional past time state, it might be possible that the estimation delay becomes large and deteriorates the estimation accuracy. However, Fig. 2 (c) and (d) show that the endpoint error of the proposed method is less than that of the conventional method even for the time-varying shrinking and rotating images. From Table 1, it is also confirmed that the proposed method requires much more computational consumption. It is due to the increase of the number of the state variables. The increase of the computational consumption is the drawback of the proposed method.

5. Conclusion

For the optical flow estimation method based on the dynamical system, we proposed to improve the accuracy of optical flow estimation by the central difference discretization of time derivative. Numerical experiments confirm the effectiveness of the proposed method in most cases. However, due to increasing the number of states, the computational consumption and the processing time increase. Therefore, it is necessary to consider a method to reduce the computational consumption. Also, the proposed model can only conduct estimation after the second frame. Thus, in the first frame, if we estimate the optical flow with another method and set the estimation value as the initial value in the proposed model, improvement of estimation accuracy can be expected. Moreover, although the Euler method is used as the numerical integration method in this experiment, the Euler method is not very accurate. Therefore, it is necessary to consider integration method with more accurate numerical integration method.

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