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Scalable Fuzzy Systems

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Abstract **–** In this paper a new class of fuzzy systems called *scalable fuzzy* (SF) systems are proposed. The SF design is built upon the idea of extending the conventional fuzzy logic approach to linguistic variables to all numbers. This leads to a new set of infinite continuous rule-base and membership functions which are located on all rational numbers and are defined based on scale, position, and input variables. The consequent of rules in the Takagi-Sugeno form are then modified, and a mathematical solution based on the convolution theorem is employed for SF modeling purposes. The discrete form of SF systems is developed, and its application is exemplified using several case studies.

Keywords**:** Fuzzy logic; system modelling; scalable fuzzy sets.

1. Introduction

Fuzzy systems have a wide range of applications in control [1] and system modeling [2]. Fuzzy logic was first introduced by Zadeh [3] and was further developed in the Mamdani form [4] and the Takagi-Sugeno (T-S) form [5]. In a Mamdani fuzzy system, similar to premises, consequent of each rule is based on sets for output of each rule (membership function),

Mamdani. *rule index:*id, If x_1 is A and x_2 is B then y is C. (R1)

However, this form is difficult to work with because the output of the system is based on membership functions which limited the application of the Mamdani form. In order to have more room for different types of equations in the system the T-S model was developed with an equation as the consequent and became rapidly popular in control and modeling [6],

Takagi-Sugeno. *rule index:*id, If x_1 is A and x_2 is B then $y_{id} = a_1x_1 + a_2x_2$. (R2)

While the T-S model improved the fuzzy systems by making the consequents easier to work with, it also changed the nature of the consequent from a fuzzy logic-based form to a pseudo-crisp logic form; i.e., an equation. Furthermore, if a system required more accuracy, either the number of membership functions, or the number of premise input variables, or the number of parameters in the consequent had to increase, all of which significantly increased the computational cost of obtaining the defuzzified output or in case of a modeling application, the computational cost of training a fuzzy based adaptive model. Moreover, while one of the advantages of the fuzzy systems is that they recognize the fuzziness of linguistic variables, the same advantage causes an inherent drawback of the fuzzy systems in both forms which is that the accuracy of the fuzzy system is directly affected by the shape and the number of membership function and how they are scattered in the input space [6].

Finally, in the conventional fuzzy system the membership functions would generally be concentrated around the operating range of the device which means that the input membership functions needed to be designed by an expert or with an optimization algorithm. Even then, if a system had a large enough operating range the fuzzy model could come short of the design criteria or require a significantly high computational effort.

In this paper we introduce a new form of fuzzy inference systems based on an infinite number of rules and membership functions with the consequent of each rule, a function of rule indices. The proposed form is shown to be unlimited in range, free of membership function design requirements, employing only fuzzy variables, and of low complexity for calculating the defuzzified output(s). One obvious application of the new class of fuzzy systems is in modelling of different processes. The modelling technique is developed using the convolution theorem and is also presented in this paper, and its performance is demonstrated for three case studies.

The remainder of this paper is organized as follows. Section 2 describes the concept and design of SF systems. Section 3 is dedicated to developing a technique for system modeling using SF systems and convolution theorem as a proof of concept. The SF modeling technique is then tested in Sec. 4 and the results are discussed. Finally, the conclusion of this paper is presented in Sec. 5.

2. Scalable Fuzzy Systems

Going back to the inspirations for developing fuzzy logic, as stated by Zadeh [3], often the classes of objects such as "warm", "tall" and "numbers greater than 1" have imprecise definitions yet are commonly used and have important implications. The extension of the same idea, that the numbers themselves are imprecise, is the inspiration of this paper. Following the same logic of stating that if a temperature of 40 degrees is called "hot" thus 39 degrees is almost as hot, it could be stated that if we define a set called "numbers close to 40", 39 is not exactly equal to 40 but it is fairly close. In other words, 39 is a member of the "close to 40" set, for instance, to the degree of 0.9. At the same time, it should be noted that based on the nature of the system the degree of closeness is dependent on scale. For instance, while 39 degrees is fairly close to 40 degrees, 39 days of bacteria population growth and 40 days of population growth are relatively distant due to the exponential behaviour of their growth.

Based on this analogy a category of membership function called *closing* functions are defined as follows

$$C_{(t)}^{x,s} = C\left(\frac{t-x}{s}\right) \tag{1}$$

where $C(\bullet)$ is the *mother membership function*, t is the input variable, s is scaling factor, and x is a constant called the *membership index* which is an indication of where the closing function is positioned with respect to the variable-axis. Here, the membership degree represents the closeness of variable t, to membership index x, at scale s. Also, the mother membership function could be any even function with the properties of a normalized membership function at zero.

Now since the relation of closeness exist for all numbers in the continuous rational space $t \in \mathbb{R}$ for any variable t, the proposition of this paper is that there should be an infinite number of membership function at each number in the $x \in \mathbb{R}$. Figure 1 represents symbolically the arrangement of the infinite membership functions where x represent the mean value (position) of membership function $C_{(t)}^{x,s}$. Note that the x-axis in the membership functions represents the variable.

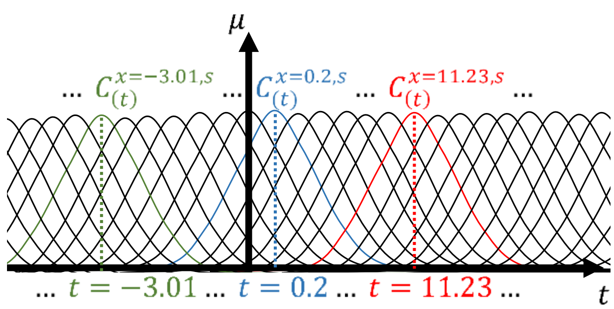

Fig. 1. Infinite closing membership function for variable t.

Thus, since there is an infinite number of membership functions, there would be an infinite number of rules each corresponding to one of the membership functions. Consider a fuzzy system with n variables, $t_1, t_2, \dots, t_n$, the range for each of them is $(-\infty, +\infty)$. The infinite fuzzy rule base is defined as follows

SF. *rule index:* $X = (x_1, x_2, \dots, x_n)$, If t_1 is $C_{(t_1)}^{x_1,s_1}$ and t_2 is $C_{(t_2)}^{x_2,s_2}$ and … and t_n is $C_{(t_n)}^{x_n,s_n}$ then $y^X = f_{(x_1,x_2,\dots,x_n)}$. (R3) where $X = (x_1, x_2, \dots, x_\mathrm{n})$ is the rule index, y^X is the output of rule X, and $f_{(x_1, x_2, \dots, x_\mathrm{n})}$ is a function of rule index. These rule indices are indicatives of which membership function is present in a rule and consequently where each membership function is located on the variable space, $T = \{t_1, t_2, \dots, t_\mathrm{n}\}$.

There are two key features in the SF-form fuzzy system. First, the whole fuzzy system including membership functions and consequent output is a function of rule indices which, compared to the conventional T-S system, relieves the fuzzy rule base of their dependency to the input variables, $t_1, t_2, \dots, t_\mathrm{n}$, except for their place as the inputs to the fuzzy system as a whole.

Second, the firing degree of the scalable fuzzy system is the same function as the multiplication of membership functions. Considering a Gaussian mother membership function (2) and using a multiplication AND operator, this is shown in Fig. 2 for a single input SF system.

$$C_{(t)}^{x,s} = e^{-\frac{1}{2}\frac{(t-x)^2}{s^2}} \tag{2}$$

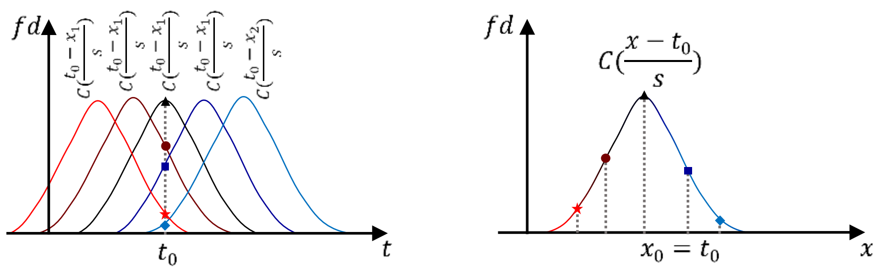

Fig. 2. Firing degrees with respect to x and t.

As shown in Fig. 2, if the input mother membership function is Gaussian, the firing degrees of the collective rules of the rule base, as a function, would be the same Gaussian function at $x_0 = t_0$ as well. In other words, when for a specific variable value, $t = t_0$, the firing degree is calculated for membership functions starting at $x = -\infty$ and moving to $x = +\infty$ the resulting function of firing degrees will be the same closing function with respect to x, translated to position $x_0 = t_0$. This is easily proved for all mother membership functions as follows (keep in mind that mother membership function is an even function)

$$fd^x = C(\tfrac{x-t_0}{s})|x \in \mathbb{R} \equiv C(\tfrac{t-x_0}{s})|t \in \mathbb{R} \tag{3}$$

where fd^x is the firing degree for rule x. The same could be said for an n-variable SF system with the following firing degree for rule with index X at point $(t_{1,0}, t_{2,0}, \dots, t_{n,0})$

$$fd^X = C\left(\frac{x_1 - t_{1,0}}{s_1}\right) \cdot C\left(\frac{x_2 - t_{2,0}}{s_2}\right) \cdot \dots \cdot C(\tfrac{x_n - t_{n,0}}{s_n})|X \in \mathbb{R}^n \tag{4}$$

where fd^X is the firing degree. Using the obtained firing degree functions of X for the rule base the defuzzification equation for a n-variable system can be written as follows (note that $y^X = f_{(x_1, x_2, \dots, x_\mathrm{n})}$ is the function pertaining to the consequents)

$$Y_{(t_1, t_2, \dots, t_\mathrm{n})} = \frac{\int_{-\infty}^{+\infty} C_{(t_\mathrm{n})}^{x_\mathrm{n}, s_n} \dots \int_{-\infty}^{+\infty} C_{(t_2)}^{x_2, s_2} \int_{-\infty}^{+\infty} C_{(t_1)}^{x_1, s_1} \cdot y^X dx_1 dx_2 \dots dx_n}{\int_{-\infty}^{+\infty} C_{(t_\mathrm{n})}^{x_\mathrm{n}, s_n} \dots \int_{-\infty}^{+\infty} C_{(t_2)}^{x_2, s_2} \int_{-\infty}^{+\infty} C_{(t_1)}^{x_1, s_1} dx_1 dx_2 \dots dx_n} \tag{5}$$

Substituting the closing functions (5) using the mother membership functions (1) gives

$$Y_{(t_1, t_2, \dots, t_\mathrm{n})} = \frac{\int_{-\infty}^{+\infty} C(\frac{t_\mathrm{n} - x_\mathrm{n}}{s_n}) \dots \int_{-\infty}^{+\infty} C(\frac{t_2 - x_2}{s_2}) \int_{-\infty}^{+\infty} C(\frac{t_1 - x_1}{s_1}) \cdot y^X dx_1 dx_2 \dots dx_n}{\int_{-\infty}^{+\infty} C(\frac{t_\mathrm{n} - x_\mathrm{n}}{s_n}) \dots \int_{-\infty}^{+\infty} C(\frac{t_2 - x_2}{s_2}) \int_{-\infty}^{+\infty} C(\frac{t_1 - x_1}{s_1}) dx_1 dx_2 \dots dx_n} \tag{6}$$

What is appeared in (6) is a convolution of the membership function and the fuzzy consequent output function, y^X, thus it can be written as

$$Y_{(t_1,t_2,\dots,t_n)} = \frac{\left(C\left(\frac{x_n}{s_n}\right)*\dots*C\left(\frac{x_2}{s_2}\right)*C\left(\frac{x_1}{s_1}\right)*y^X\right)_{(t_1,t_2,\dots,t_n)}}{\int_{-\infty}^{+\infty} C(\frac{t_n-x_n}{s_n}) \dots \int_{-\infty}^{+\infty} C(\frac{t_2-x_2}{s_2}) \int_{-\infty}^{+\infty} C(\frac{t_1-x_1}{s_1}) dx_1 dx_2 \dots dx_n} \quad (7)$$

where $*$ is the convolution operator. Note that writing the convolution in this form could be misleading; to obtain the convolution at point $T = (t_1, t_2, \dots, t_n)$, we need to know values of each of the functions at all points $T \in \mathbb{R}^n$. Furthermore, since the denominator of the right-hand side of the equation is constant for all variable values, it can be written as

$$C_{sum} = \int_{-\infty}^{+\infty} C(\frac{t_n-x_n}{s_n}) \dots \int_{-\infty}^{+\infty} C(\frac{t_1-x_1}{s_1})\, dx_1 \dots dx_n \quad (8)$$

Substituting (8) into (7) gives

$$Y_{(t_1,t_2,\dots,t_n)} = \frac{\left(C\left(\frac{x_n}{s_n}\right)*\dots*C\left(\frac{x_2}{s_2}\right)*C\left(\frac{x_1}{s_1}\right)*y^X\right)_{(t_1,t_2,\dots,t_n)}}{C_{sum}} \quad (9)$$

Thus (9) and (6) both can be used for obtaining a defuzzified output from an SF system in continuous space. However, in order to make this tool useful in practice a discrete form should be introduced. The discrete form of the defuzzification expression (6) can be written as

$$Y_{[\tau_1,\tau_2,\dots,\tau_n]} = \frac{\sum_{x_n=1}^{N_n} \dots \sum_{x_2=1}^{N_2} \sum_{x_1=1}^{N_1} C_{[\tau_n]}^{[x_n],s_n} \dots C_{[\tau_2]}^{[x_2],s_2} C_{[\tau_1]}^{[x_1],s_1} y^X}{\sum_{x_n=1}^{N_n} \dots \sum_{x_2=1}^{N_2} \sum_{x_1=1}^{N_1} C_{[\tau_n]}^{[x_n],s_n} \dots C_{[\tau_2]}^{[x_2],s_2} C_{[\tau_1]}^{[x_1],s_1}} \quad (10)$$

where $N_n, \dots, N_2, N_1$ are the number of closing functions for variables, $[\tau_1, \tau_2, \dots, \tau_n]$, respectively. Since the denominator of the right-hand side of the equation is constant for all variable values, it can be rewritten as

$$Y_{[\tau_1,\tau_2,\dots,\tau_n]} = \frac{\sum_{x_n=1}^{N_n} \dots \sum_{x_2=1}^{N_2} \sum_{x_1=1}^{N_1} C_{[\tau_n]}^{[x_n],s_n} \dots C_{[\tau_2]}^{[x_2],s_2} C_{[\tau_1]}^{[x_1],s_1} y^X}{C_{sum}} \quad (11)$$

where

$$C_{sum} = \sum_{x_n=1}^{N_n} \dots \sum_{x_1=1}^{N_1} C_{[\tau_n]}^{[x_n],s_n} \dots C_{[\tau_2]}^{[x_2],s_2} C_{[\tau_1]}^{[x_1],s_1} \quad (12)$$

In summary, the proposed new fuzzy system has an infinite number of membership functions, each located on a rational number. Each rule of the SF system is denoted by a rule index and the consequent outputs of the SF system are functions of these rule indices. As shown by (4), the firing degree for each variable is the same function as the mother membership function at $X_0 = T_0$. This means that the fuzzification of inputs, inferencing and generating the defuzzification of the SF system from the input variables is a fast process, summed up in the convolution (9), due to the consistency in the firing degree functions for the whole support of variables and the constant value in the denominator of the defuzzification equation.

3. Proposed Fuzzy Modeling and Convolution Theorem

One of the more common applications of the fuzzy systems is in system modeling. Different iterations of the fuzzy systems have been introduced for this purpose such as recurrent fuzzy systems [7] and Adaptive Neuro-Fuzzy Systems [8]. However, as already mentioned, these systems are limited to range, the input membership functions needed to be designed by an expert or with an optimization algorithm and if more accuracy was required the computational cost would increase significantly.

The proposed fuzzy modeling technique can deal with all three of those disadvantages; It is not limited to any range, or any level of resolution since the membership functions exist for all variables. Also, for adaptive modeling purposes the system can be trained instantly from a gathered set of data. This is done using the convolution theorem. Multiplying both sides of (9) by C_{sum} gives

$$Y^p_{(t_1,t_2,\dots,t_n)} = \left(C\left(\frac{x_n}{s_n}\right) * \dots * C\left(\frac{x_2}{s_2}\right) * C\left(\frac{x_1}{s_1}\right) * y^{x_1,x_2,\dots,x_n}\right)_{(t_1,t_2,\dots,t_n)} \quad (13)$$

where $Y^p_{(t_1,t_2,\dots,t_n)} = C_{sum} Y_{(t_1,t_2,\dots,t_n)}$. Using the convolution theorem for (13)

$$\hat{Y}^{p}_{(k_1,k_2,\dots,k_n)} = \hat{C}(k_n)\cdot \dots \cdot \hat{C}(k_2)\cdot \hat{C}(k_1)\cdot \hat{y}^{k_1,k_2,\dots,k_n} \quad (14)$$

where $\hat{Y}^{p}_{(k_1,k_2,\dots,k_n)}$, $\hat{C}(k_n)$, … , $\hat{C}(k_2)$, $\hat{C}(k_1)$, $\hat{y}^{k_1,k_2,\dots,k_n}$ are Fourier transforms of $Y^{p}_{(t_1,t_2,\dots,t_n)}$, $C\left(\frac{x_n}{s_n}\right)$, … , $C\left(\frac{x_2}{s_2}\right)$, $C\left(\frac{x_1}{s_1}\right)$, $y^{X=x_1,x_2,\dots,x_n}$, respectively. Thus, the consequent of rule X could be easily obtained as

$$y^{X(=x_1,x_2,\dots,x_n)} = \mathcal{F}^{-1}\left\{\frac{\hat{Y}^{p}_{(k_1,k_2,\dots,k_n)}}{\hat{C}(k_n)\cdot\dots\cdot\hat{C}(k_2)\cdot\hat{C}(k_1)}\right\} \quad (15)$$

where $\mathcal{F}^{-1}$ represents the inverse Fourier transform. The same could be said for a discrete form of the defuzzification, (11), using the circular convolution with enough zero-padding.

$$y^{X} = D\mathcal{F}T^{-1}\left\{\frac{\hat{Y}^{p}_{[k_1,k_2,\dots,k_n]}}{\hat{C}[k_n]\cdot\dots\cdot\hat{C}[k_1]}\right\} \quad (16)$$

where $D\mathcal{F}\mathrm{T}^{-1}$ represents the inverse discrete Fourier transform, and $\hat{Y}^{p}_{[k_1,k_2,\dots,k_n]}$, $\hat{C}[k_n]$, … , $\hat{C}[k_1]$ are discrete Fourier transforms of $Y^{p}_{[\tau_1,\tau_2,\dots,\tau_n]}$, $C^{x_n,s_n}_{[\tau_n]}$, … , $C^{x_1,s_1}_{[\tau_1]}$, respectively.

This means that if a set of data corresponding to a set of variables, $Y_{[\tau_1,\tau_2,\dots,\tau_n]}$, is gathered the corresponding consequent output function of the rules (as a function of the rule indices), $y^{X(=x_1,x_2,\dots,x_n)}$, could be obtained instantly from (16) without any further adjustment of the technique.

4. Examples of SF Modeling

This section describes the effect of training data on the designed fuzzy system. Two mathematical models are used for training 2-dimentional fuzzy systems, and a hydraulic system is used for training a 3-dimensional scalable fuzzy system. Table 1 describes the two functions and the two variables which are chosen for the fuzzy model of each function.

Table 1 Test functions

	Function	t_1	t_2
f^1	$f^1 = (x_1^2 + x_2 - 11)^2 + (x_2^2 + x_1 - 7)^2$	x_1	x_2
f^2	$f^2_{i+1} = 0.2 * f^2_i + 0.4 * f^2_{i-1} - 0.3 * f^2_{i-2}$	f^2_i	f^2_{i-1}

The first function, f^1, is the Himmelblau mathematical equation with two variables $x_2, x_1 \in [-5,5]$. Since this is a static system, in order to train the SF system, the data from the equation were collected with discretization step size of $\Delta = 0.1$ and considering the entire range. Also, the mother membership function of the SF system was chosen to be triangular with the scale of $s = 0.1$. Figure 3 shows the results pertaining the modeling of the first function.

Figure 4 describes the results of the simulation for the second function, f^2, which is a third order system with oscillations. In order to collect the data for modeling this function the f^2_i series was computed for only $i = 10$ iteration with the initial conditions of integers between $[-5,5]$, (i.e., $f^2_0 = -5,-4,\dots,4,5$).

The mother membership function of the SF system is a Gaussian membership function with the scale of $s = 0.001$ and the step size for discretization of variables is $\Delta = 0.01$.

The results of the simulations show that the above fuzzy model is able to estimate the behavior of the first function with a maximum estimation error of 1.51×10^{-12}. In the case of the second function the modeling error is 3.54×10^{-16} and has a maximum error of 1.01×10^{-3} for a *prediction horizon* of 4 steps. Furthermore, increasing the training data by adding increments of 0.1 instead of integers (i.e., $f^2_0 = -5,-4.9,\dots,4.9,5$) for the initial conditions of the second function, improves the performance of the SF modeling technique to have a maximum error of 4.93×10^{-3} for a prediction horizon of 7 steps.

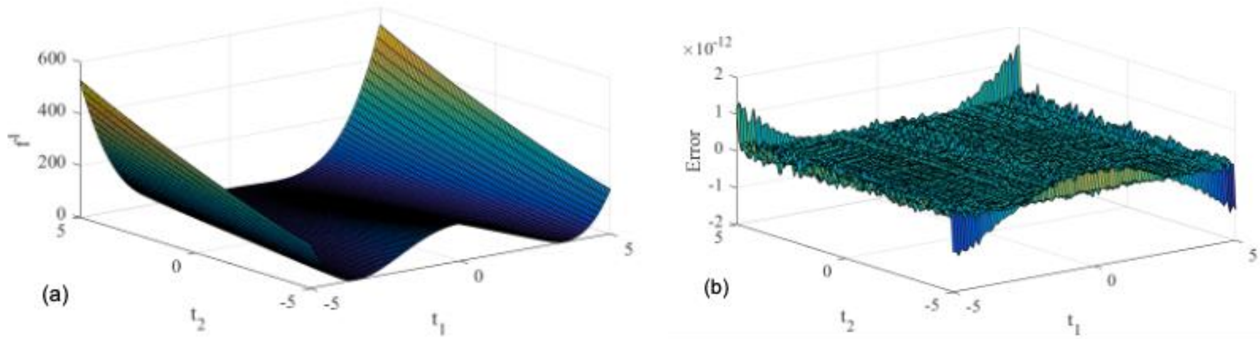

Fig. 3. Simulation results for f^1: (a) plot of function f^1; (b) error between the function output and the SF model estimation.

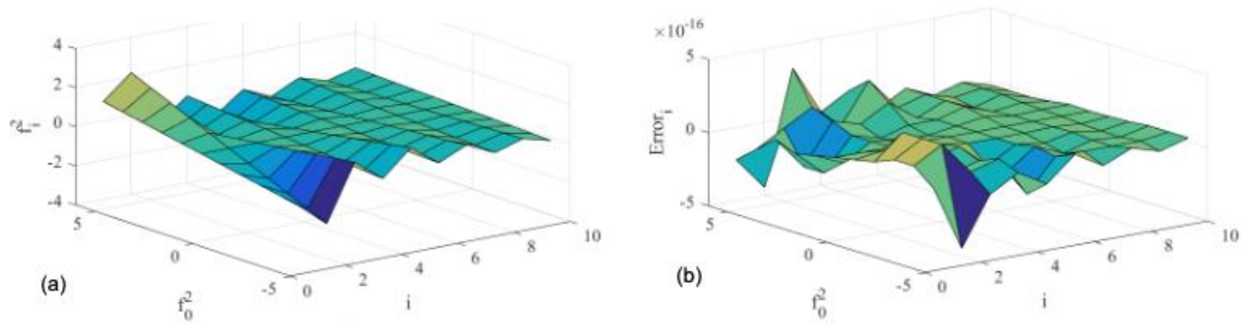

Fig. 4. Simulation results for f^2: (a) plot of function, f^2 for integer initial conditions; (b) error between function and estimation.

The third system is a double-rod cylinder which is actuated by a bidirectional pump [9]. The schematics of the system under consideration is shown in Fig. 5.

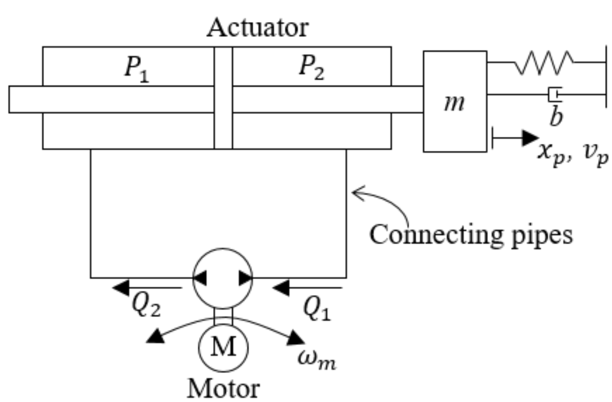

Fig. 5. Schematics of the electrohydraulic cylinder.

The nonlinear equations describing the motion of piston (moving a mass m) with respect to input signal, u, to the pump are as follows:

$$
\begin{aligned}
\dot{v}_p &= \frac{1}{m}(AP_1 - AP_2 - bv_p - kx_p) \\
\dot{P}_1 &= \frac{\beta}{V_0 + V_{pipe} + V_1 + Ax_p}\left(\frac{q_b}{2\pi}\omega_m - A\dot{x}_p\right) \\
\dot{P}_2 &= \frac{\beta}{V_0 + V_{pipe} - V_2 - Ax_p}\left(-\frac{q_b}{2\pi}\omega_m + A\dot{x}_p\right) \\
\dot{\omega}_m &= t_m(-\omega_m + k_m u)
\end{aligned}
\tag{17}
$$

where u is the input voltage to the motor, v_p is the piston velocity, x_p is the piston position, ω_m is the angular velocity of the motor, and P_1 and P_2 are left-side and right-side pressures, respectively. Table 2 describes the remaining parameters. A typical response of the hydraulic system used for training the SF system is shown in Fig. 6. The training data of the system are obtained from 3500 sets of simulations for different inputs and outputs of the system in closed-loop form each for 100 steps. The inputs to the SF model are the current input, the current velocity and the previous velocity and the output is the predicted value of the velocity for the next step. The mother membership function of the SF system is a Gaussian membership function with the scale of $s = 0.0001$ and the step size for discretization of variables is $\Delta = 0.001$.

Table 2. Parameters of the electrohydraulic cylinder

Parameter	**Symbol**	**Value**
Displacement of the pump (m^3/rev)	q_b	4.9×10^{-6}
Time constant of the servo motor (1/s)	t_m	20
Servomotor gain (rev/(s·V))	k_m	25
Piston area (m^2)	A	633×10^{-6}
Chamber volume (m^3)	V_0	192×10^{-6}
Pipe volume (m^3)	V_{pipe}	42×10^{-6}
Pump chamber side volumes (m^3)	V_1, V_2	10×10^{-10}
Effective bulk modulus (Pa)	β	689×106
Viscous damping coefficient(N·s/m)	b	250
Piston and rod mass (kg)	m	12
Spring constant (N/m)	k	125×10^{3}

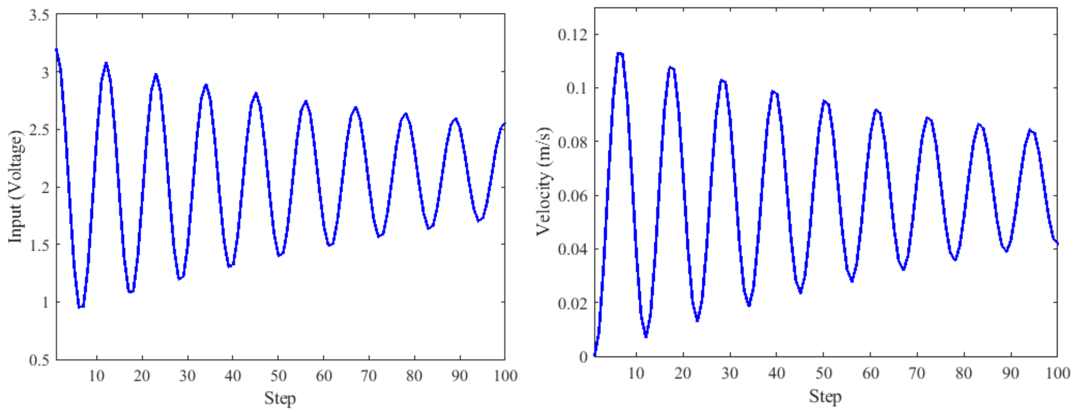

Fig. 6. Typical input and output (velocity) to the hydraulic system used for training.

Figure 7 describes the test results of the fuzzy model against the reference simulation for 20 steps. The test condition is chosen within the trained range of the system and at each step a prediction with a horizon of five steps has been performed. As shown, the prediction values are close to the actual values of the simulations. In this case the first steps of prediction have a root mean square errors of 5×10^{-4}, and the fifth steps of prediction have a root mean square errors of 1.2×10^{-3}. While the accuracy of the prediction deteriorates as the prediction horizon increases, within the first five steps the proposed model retains 98.7% accuracy. This is shown in Table 3 for predictions done at step 15.

It should be noted that errors of order 10^{-16} are due to discretization step of the system. In other words, since the input values to the fuzzy system are discretized if the inputs accurately reflect the values of the system the discretization error is the totality of the error that is observed. However, once the system diverges enough from the trained range the accuracy of the predictions decreases. Once the system moves beyond the trained range, the SF system will only generate zero (or close to zero) values.

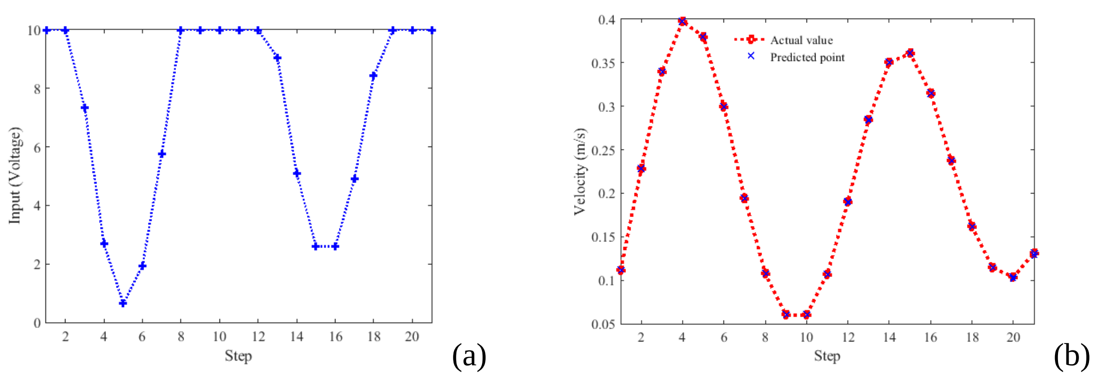

Fig. 7 Simulation results for the hydraulic system. (a) Input and (b) velocity.

Table 3. Prediction errors pertaining step 15.

Horizon	***16***	***17***	***18***	***19***	***20***	***21***	***22***
Error	1.1×10^{-16}	1.1×10^{-16}	1.4×10^{-16}	8.3×10^{-17}	2.0×10^{-3}	3.1×10^{-3}	3.6×10^{-3}

5. Conclusions

This paper proposes a novel class of fuzzy systems based on the idea of having a continuous infinite rule base and membership functions. The proposed form is shown to be unlimited in range, free of membership function design requirements, employing only fuzzy variables, and of low complexity for calculating the defuzzified output. The SF system is then used along with the convolution theorem to develop a technique for system modeling. The results of these case studies show that the presented method is capable of modeling not only static functions, but also dynamic systems.

Future research on SF systems should concentrate on four areas: (i) to use the scaling factor in the mother membership functions to develop a multiscale fuzzy system, (ii) to develop a recursive method for training the system, (iii) to develop modeling method with a second order function as the consequent of the fuzzy rules, and (iv) to investigate the effect of the size of data on the accuracy of the developed method compared to other machine learning methods.

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