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Electric Load Estimation and Prediction Using Periodic Steady State Kalman Filter

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Abstract - The importance of electric load prediction and estimation in power system operation is undoubtable since it provides economic generation and planning. Electric load is modeled using a discrete-time model depending on previous load and weather variables. Electric load estimation is derived using time varying, time invariant, steady state and periodic steady state Kalman filters. Electric load prediction is also derived. The algorithms' efficiency is tested trough simulation results.

Keywords**:** Electric load, estimation, prediction, Kalman filter

1. Introduction

Electric load estimation has always been of indisputable importance for the reliable operation of the power grid. Forecasting techniques have been used for management and planning [1]-[5]. Renewable Energy Systems penetration created the need for accurate short-term forecasting techniques [6]-[12]. In general, the electric load models mentioned in literature are designed to solve specific load forecasting problems and are divided into three basic groups [11]: a) models that depend only on previous values of the load, called non-weather sensitive models, b) models that depend on the weather variables, called weather sensitive models and c) hybrid models. More specifically, in the cases of weather sensitive models or hybrid models, discrete time models describing the electric load have been proposed in [8], [12]. Especially, Kalman filtering techniques have been applied to forecast the electrical load [8], [12]-[13]. Also, in [14], Kalman and Lainiotis filter have been used for short-term electric load estimation.

In this paper we continue to investigate the potential of Kalman filter for short-term electric load estimation. We consider the hybrid model proposed in [8] and we focus on steady state and periodic Kalman filters development. Also a N steps ahead prediction algorithm is proposed. The model and the developed Kalman filters are presented in section 2. The contribution of the paper concerns (a) the derivation of steady state Kalman filter and Finite Impulse Response form (FIR form) of the steady state Kalman filter, (b) the derivation of periodic steady state Kalman filter. In Section 3 simulation results are presented. Section 4 summarizes the conclusions.

2. Weather Sensitive Model and Kalman Filters

In this section paper we are going to present the discrete-time model depending on previous load and weather variables, which is used for the describing the electric load. We are also going to develop Kalman filters in order to estimate and predict the electrical load.

2.1. Weather Sensitive Model

In this paper we consider the hybrid model proposed in [8], where the variation of electrical load depends on previous loads and weather data (temperature and wind). The state $\mathrm{x(k)}$ consist of has $\mathrm{n} = 10$ elements concerning weather parameters described in [8]. The measurement $\mathrm{z(k)}$ is $\mathrm{m} = 1$ element, the electric load. The discrete time varying model has as follows:

$$\mathrm{x(k+1) = F \cdot x(k) + w(k)} \tag{1}$$

$$\mathrm{z(k) = H(k) \cdot x(k) + v(k)} \tag{2}$$

for time $\mathrm{k} \geq 0$.

The model parameters are: (a) the $n \times n$ transition matrix $F = I$ (identity matrix), (b) the $m \times n$ output matrix $H(k)$ derived by the model parameter identification [8], (c) the $n \times n$ covariance $Q = I$ of the zero mean Gaussian state noise $w(k)$ and (d) the covariance $R = 1$ of the zero mean Gaussian measurement noise $v(k)$. The state $x(0)$ is a Gaussian random variable with mean x_0 and covariance P_0.

2.2. Kalman Filters

We are going to develop Kalman filters [15]-[17] in order to estimate and predict the electrical load. Kalman filter produces the state prediction $x(k+1/k)$ and the corresponding prediction error covariance $P(k+1/k)$, as well as the state estimation $x(k/k)$ and the corresponding estimation error covariance $P(k/k)$. Certainly it is possible to develop the Lainiotis filters [17]-[18], which are equivalent to Kalman filters [17], since they compute the same estimations and estimation error covariances as Kalman filters do, given the same initial conditions.

Working as in [14], we consider the noiseless electric load $y(k) = H(k) \cdot x(k)$. Then the estimation and the estimation error covariance are:

$$y(k/k) = H(k) \cdot x(k/k)$$
$$c(k/k) = H(k) \cdot P(k/k) \cdot H^T(k)$$

and the prediction and the prediction error covariance are:

$$y(k+1/k) = y(k/k)$$
$$c(k+1/k) = c(k/k) + H(k) \cdot H^T(k)$$

The time variability of $H(k)$ leads to the time varying Kalman filter derivation:

Time Varying Kalman Filter (TVKF)

$$x(k+1/k) = x(k/k)$$
$$P(k+1/k) = I + P(k/k)$$
$$K(k+1) = \frac{P(k+1/k) \cdot H^T(k+1)}{H(k+1) \cdot P(k+1/k) \cdot H^T(k+1) + 1}$$
$$x(k+1/k+1) = [I - K(k+1) \cdot H(k+1)] \cdot x(k+1/k) + K(k+1) \cdot z(k+1)$$
$$P(k+1/k+1) = [I - K(k+1) \cdot H(k+1)] \cdot P(k+1/k)$$
$$y(k/k) = H(k) \cdot x(k/k)$$
$$c(k/k) = H(k) \cdot P(k/k) \cdot H^T(k)$$
$$y(k+1/k) = y(k/k)$$
$$c(k+1/k) = c(k/k) + b(k)$$

where

$$b(k) = H(k) \cdot H^T(k)$$

Assuming that $H(k)$ is periodic with period $p = 24$ as derived by the model parameter identification [8] (the period can be assumed in a week / month / year basis). In this case TVKF is developed with $H(k) = H(k \bmod 24)$.

The assumption that $H(k) = H$ computed as the mean of $H(k)$, leads to the time invariant Kalman filter derivation:

Time Invariant Kalman filter (TIKF)

$$x(k+1/k) = x(k/k)$$
$$P(k+1/k) = I + P(k/k)$$
$$K(k+1) = \frac{P(k+1/k) \cdot H^T}{H \cdot P(k+1/k) \cdot H^T + 1}$$
$$x(k+1/k+1) = [I - K(k+1) \cdot H] \cdot x(k+1/k) + K(k+1) \cdot z(k+1)$$
$$P(k+1/k+1) = [I - K(k+1) \cdot H] \cdot P(k+1/k)$$

$y(k/k) = H \cdot x(k/k)$
$c(k/k) = H \cdot P(k/k) \cdot H^T$
$y(k+1/k) = y(k/k)$
$c(k+1/k) = c(k/k) + b$
where
$b = H \cdot H^T$
$H = [27.22 \quad 0.54 \quad 0.83 \quad -0.43 \quad 0.02 \quad -0.64 \quad 0.19 \quad 0.38 \quad 0.08 \quad 0.03]$

The observation that there exists steady state [14] leads to the steady state Kalman filter derivation:

Steady State Kalman Filter – SSKF
$y(k+1/k+1) = A \cdot y(k/k) + B \cdot z(k+1)$
$$A = \frac{2}{2 + b + \sqrt{b^2 + 4b}} = 0.00134$$
$$B = \frac{b + \sqrt{b^2 + 4b}}{2 + b + \sqrt{b^2 + 4b}} = 0.99866$$
Note that the coefficients A and B are calculated off-line by first solving the corresponding discrete time Riccati equation emanating from Kalman filter [15], [17].

From the estimation equation of SSKF we get:
$y(k/k) = A^k \cdot y(0/0) + A^{k-1} \cdot B \cdot z(1) + \cdots + B \cdot z(k)$
We observe that $|A| < 1$. It is obvious that the integer positive powers of A tend to zero. Hence there exists a positive integer M, such that $A^{M-1} > \varepsilon$ and $A^{M+i} \le \varepsilon, i = 0,1,\ldots$, where ε is a small positive real number. It is evident that the value of M depends on the choice of the convergence criterion ε. Assuming that $z(k) = 0, k < 0$, the Finite Impulse Response form (FIR form) of the steady state Kalman filter is derived:

FIR form of Steady State Kalman (FIRSSKF)
$$y(k/k) = \sum_{j=0}^{M-1} \{A^j \cdot B \cdot z(k-j)\}$$
Remarks.
1. The FIRSSKF coefficients are calculated off-line.
2. The estimation depends only on a well-defined set of measurements.

The fact that the output matrix $H(k)$ is periodic with period $p = 24$ leads to the conclusion that we are able to derive periodic steady state filters [19]-[20] with periodic steady state coefficients, which are computed by solving the periodic Riccati equation [21]. Then the periodic steady state Kalman filter is derived:

Periodic Steady State Kalman Filter (PSSKF)
$y(k+1/k+1) = A(k) \cdot y(k/k) + B(k) \cdot z(k+1)$
where the periodic steady state coefficients are periodic with period $p = 24$:
$A(k) = A(k\bmod 24)$
$B(k) = B(k\bmod 24)$
and can be computed off-line by solving the corresponding periodic Riccati equation.

The periodic coefficients of PSSKF (period $p = 24$) are depicted in Figure 1.

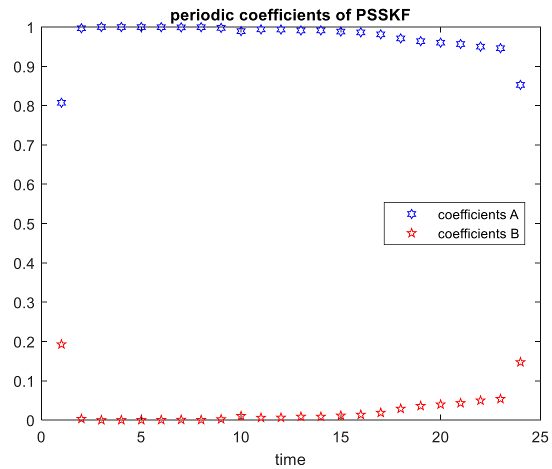

Fig. 1: The periodic coefficients of the PSSKF.

We observe that a steady state filter can be designed by defining the steady state coefficients as the mean values of the periodic steady state coefficients of the PSSKF. It is evident that the coefficients of the new filter are calculated off-line. The derived "mean" periodic steady state Kalman filter is:

Mean Periodic Steady State Kalman Filter (MPSSKF)

$y(k+1/k+1) = A \cdot y(k/k) + B \cdot z(k+1)$

$A = 0.02845$

$B = 0.97155$

Finally, we are going to develop a prediction algorithm. The prediction N steps ahead and the corresponding prediction error covariance are given in the N steps ahead prediction algorithm:

N steps ahead Prediction

$x(k+N/k) = \Phi_{k+N,k+1} \cdot x(k+1/k)$

$$P(k+N/k) = \Phi_{k+N,k+1} \cdot P(k+1/k) \cdot \Phi_{k+N,k+1}^{T} + \sum_{i=k+1}^{k+N-1} \Phi_{k+N,i} \cdot Q \cdot \Phi_{k+N,i}^{T}$$

where

$\Phi_{k,\ell} = F(k,k-1) \cdot \ldots \cdot F(\ell+1,\ell), k > \ell$

$\Phi_{k,k} = I$

In the hybrid sensitive model case, where $F = I$, we derive:

$x(k+N/k) = x(k+1/k) = x(k/k)$

$P(k+N/k) = P(k+1/k) = (N-1) \cdot Q$

Note that the one step ahead prediction is equal to the estimation, due to the fact that $F = I$.

3. Simulation Results

The behaviour of the developed Kalman filters on the same data (measurements) is investigated. For the hybrid weather sensitive model presented in section II, the TVKF, TIKF, SSKF, PSSKF and MPSSKF are implemented.

Figure 2 depicts the results for electric load estimation using TVKF, TIKF, SSKF, PSSKF and MPSSKF using measurements generated from the weather sensitive model, for 1 day (24h). All Kalman filters perform equally well and they follow the trend of the actual load curve.

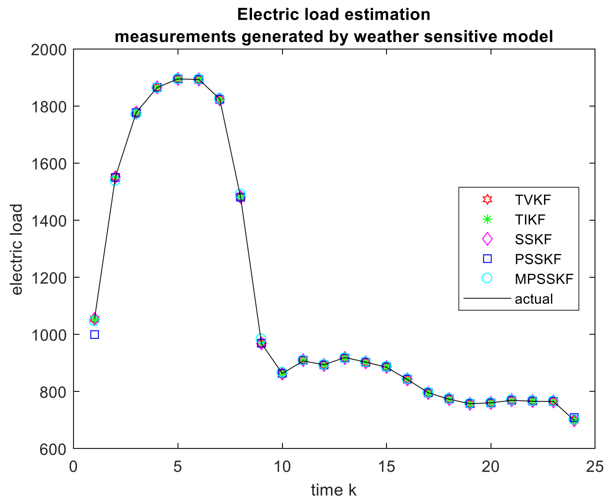

Fig. 2: Electric load estimation using measurements generated by the weather sensitive model.

For the non-weather sensitive model presented in [14], the TVKF, TIKF, SSKF, PSSKF and MPSSKF are implemented. Figure 3 depicts the results for electric load estimation using TVKF, TIKF, SSKF, PSSKF and MPSSKF using measurements generated from the non-weather sensitive model, for 1 day (24h). All Kalman filters adapt to the actual load.

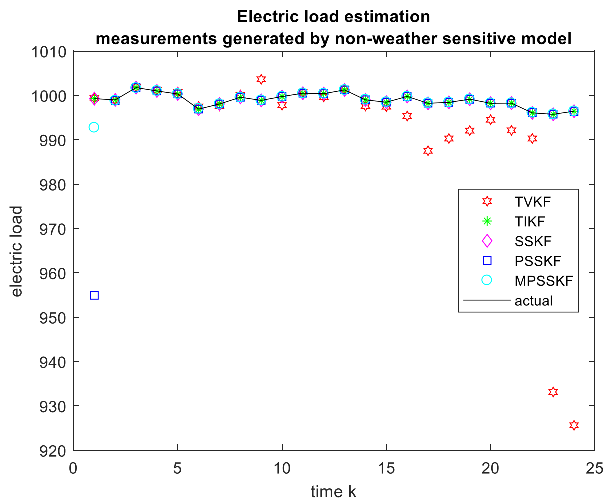

Fig. 3: Electric load estimation using measurements generated by the non-weather sensitive model.

For randomly (normal distribution) generated measurements, the TVKF, TIKF, SSKF, PSSKF and MPSSKF are implemented. Figure 4 depicts the results for electric load estimation using TVKF, TIKF, SSKF, PSSKF and MPSSKF using randomly generated measurements, for 1 day (24h). All Kalman filters adapt to the actual load.

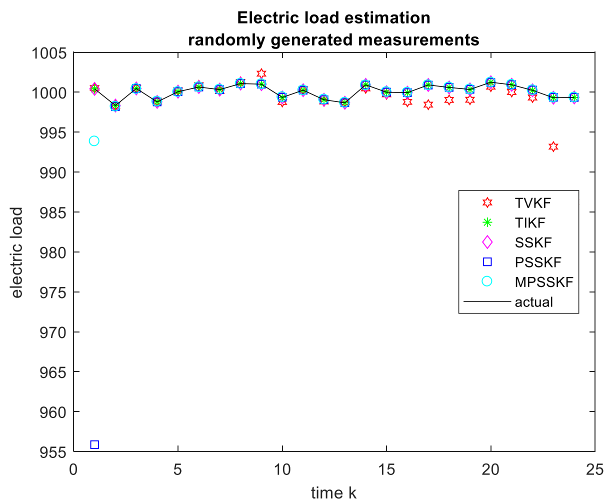

Fig. 4: Electric load estimation using randomly generated measurements.

Finally, in order to investigate the efficiency of the developed Kalman filters, we use the mean absolute percentage error (MAPE) with respect to the actual load:

$$\mathrm{MAPE} = \frac{100}{24} \cdot \sum_{j=1}^{24} \frac{|y(j/j) - y(j)|}{y(j)} \tag{3}$$

where y(j) is the actual electric load and y(j/j) is the electric load estimation.
Table 1 summarizes the results for the mean absolute percentage error with respect to the actual load for TVKF, TIKF, SSKF, PSSKF, MPSSKF on a 24h basis. All algorithms perform well. The mean absolute percentage errors between the actual and estimated loads are low.

Table 1: Mean absolute percentage error of Kalman filters.

Kalman filter	measurements generated by weather sensitive model	measurements generated by non-weather sensitive model	measurements generated randomly
TVKF	0.0776	0.8021	0.1333
TIKF	0.0782	0.0004	0.0001
SSKF	0.0784	0.0001	0.0001
PSSKF	0.3612	0.1871	0.1871
MPSSKF	0.2800	0.0390	0.0301

4. Conclusion

Load forecasting has been of paramount importance for the smooth and reliable operation of the power grid. The hybrid model proposed in [8] was considered. Kalman filters have been designed in order to estimate electric load curves.

In fact we focused on the derivation of steady state Kalman filter, FIR form of the steady state Kalman filter and periodic steady state Kalman filter. This is feasible a) by taking advantage of model parameter identification [8] and b) by converting the time varying model to time invariant model, the parameters of which can be derived by taking into account the mean of the model parameters in a period (the period can be assumed in a day/ week / month / year basis). Definitely it is possible to develop the - equivalent to Kalman filters [17] - Lainiotis filters.

More specifically time varying, time invariant, steady state and periodic steady state Kalman filters have been designed and implemented for hybrid weather insensitive model. Also a N steps ahead prediction algorithm based on Kalman filter was proposed.

All the proposed filters produce acceptable results: the filters are able to follow the load curves with a low mean absolute percentage error, as it is shown in Table 1. Time varying Kalman filter produces more accurate estimations for measurements generated by the hybrid weather sensitive model [8]. Steady state Kalman filter produces more accurate estimations for measurements generated from for the non-weather sensitive model [14] or generated randomly. Steady state and periodic steady state Kalman filters have less memory and calculation requirements than time varying and time invariant Kalman filters, as it results from the filters' equations. The results demonstrate the great potential of Kalman filters for short term load forecasting.

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