

Column Generation Methods of Variational Inequalities – An Extension

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Extended Abstract

Column generation (CG) is a highly effective method for solving complex mathematical programming problems with a large number of variables, including optimization models and variational inequalities (VI). These types of models are found in various fields, such as multi-cloud systems, multi-commodity economic equilibrium models, and transportation models. However, as these models become larger with the increase in data, they can become difficult to solve. For example, stochastic VI settings with large-scale scenario analysis often result in large-scale VI problems that even well-established VI solvers struggle to solve due to memory limitations.

To address this issue, column generation algorithms break down large-scale problems into smaller subproblems that can be solved on several computers. This method can also improve model development and maintenance efficiency by separating well-developed submodels when a converged solution is needed. However, the CG method is known to converge rapidly at first, but then slow down with a long tail of near-optimal solutions, which can lead to poor computational performance. Furthermore, other drawbacks such as dual oscillations, primal degeneracy, and alternative dual optimal solutions can also arise in optimization models.

Our experience with CG methods for VI problems indicates that these methods inherit the long-tail convergence property, but there are three optimization model stabilization techniques that can mitigate these drawbacks. These techniques include proximity to a stability center, smoothing techniques, and centralized prizes to stabilize iterative dual solutions from master problems. We aim to explore the use of these techniques for VI problems and develop methods to improve CG-VI computational performance by easing long-tail convergence.

Additionally, based on the literature, we know that initializing the algorithm with a larger number of subproblem columns can speed up the solution process. Thus, we aim to develop a general method that solves multiple types of subproblems in each CG iteration to address different kinds of subproblems.

Furthermore, it is worth noting that while applying CG methods to VI problems, only one stabilization technique has been employed in the literature. Therefore, we seek to fill this knowledge gap by investigating if more than one technique can be used at the same time to solve VI problems using the CG method.

We provide an illustrative example to demonstrate this method.