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Speedup of Extended Kalman Filter due to Gain Elimination

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Abstract **-** The Extended Kalman filter uses, in every iteration, the Kalman filter gain in order to produce estimation and prediction of the n-dimensional state vector using the m-dimensional measurement vector. In this paper, a variation of the Extended Kalman filter eliminating the Kalman filter gain is proposed. The proposed Extended Kalman filter gain elimination algorithm may be faster than the traditional Extended Kalman filter, depending on the model dimensions.

Keywords**:** discrete time, extended Kalman filter, Kalman filter gain, calculation burden

1. Introduction

Nonlinear estimation plays a crucial role in many fields of science and engineering. The nonlinear estimation problem is associated with discrete time systems described by the following state space equations:

$$x(k+1) = f(x(k), k) + w(k) \tag{1}$$

$$z(k) = h(x(k), k) + v(k) \tag{2}$$

where $x(k)$ is the $n \times 1$ state vector, $z(k)$ is the $m \times 1$ measurement vector, $f(x(k), k)$ is the $n \times 1$ state nonlinear vector function, $h(x(k), k)$ is the $m \times 1$ observation nonlinear vector function, $w(k)$ is the $n \times 1$ state noise and $v(k)$ is the $m \times 1$ measurement noise at time $k \geq 0$.

The statistical model expresses the nature of the state and the measurements. The basic assumption is that the state noise and the measurement noise are Gaussian white noises with known covariances $Q(k)$ of dimension $n \times n$ and $R(k)$ of dimension $m \times m$, respectively.

The following assumptions also hold: (a) the initial value of the state $x(0)$ is a Gaussian random variable with mean x_0 and covariance P_0; (b) the stochastic processes $\{w(k)\}$, $\{v(k)\}$ and the random variable $x(0)$ are independent.

The **Extended Kalman filter** [1] is the most well-known algorithm that solves the nonlinear filtering problem. The traditional Extended Kalman filter implements the linearization of the nonlinear system using the first order term of Taylor series. The Extended Kalman filter computes the state estimation $x(k|k)$ and the corresponding estimation error covariance matrix $P(k|k)$ as well as the state prediction $x(k+1|k)$ and the corresponding prediction error covariance matrix $P(k+1|k)$, using measurements until time k.

The Extended Kalman filter has been evolved as follows:

The **High Order Extended Kalman filter** [2] is based on a high order (for example on second order) Taylor expansion of a nonlinear system.

The **Square Root Extended Kalman Filter** [3] uses the square root $S(k|k)$ of the estimation covariance matrix $P(k|k)$, i.e. the covariance matrix $P(k|k)$ is written as $P(k|k) = S(k|k)S^T(k|k)$, where $S(k|k)$ is a triangular matrix. S^T denotes the transpose of matrix S.

The **Sigma-Point Kalman Filter** [4] applies a probabilistic framework to the same problem domain typically addressed by the Extended Kalan Filter.

The **Unscented Kalman Filter** [5] has as fundamental component the unscented transformation which uses a set of appropriately chosen weighted points to parameterize the means and covariances of probability distributions.

The **Extended Information Kalman Filter** [6] uses of the inverse $P^{-1}(k|k)$ of the covariance matrix and the information state vector $y(k|k) = P^{-1}(k|k)x(k|k)$.

The **Ensemble Kalman filter** [7] propagates ensembles with state realizations instead of mean values and covariance matrices.
The **Particle Kalman Filter** [8] uses weighted average of an “ensemble of Kalman filters” operating in parallel.
The **Adaptive Extended Kalman Filter** [9] is an adaptive version of the Extended Kalman Filter.
The **Adaptive Unscented Kalman Filter** is introduced in [10] with the purpose of handling time-varying or uncertain noise distribution.
The **Dual Kalman Filter** [11] estimates both the state and the model using essentially two Extended Kalman Filters that run concurrently.
The **Rhythmic Extended Kalman Filter** [12] is developed to estimate the pose, learn an individualized model of periodic movement over time, and use the learned model to improve pose estimation.
The **Fuzzy Extended Kalman Filter** [13] uses hybrid estimator based on combination of Extended Kalman Filter and fuzzy inference system.

The Extended Kalman Filter uses the Kalman filter gain in order to produce estimation. In this paper, a variation of the Extended Kalman filter is developed by eliminating the Kalman filter gain from the Extended Kalman filter equations. The resulting Extended Kalman filter gain elimination algorithm may be faster than the traditional Extended Kalman filter, depending on the model dimensions.

The paper is organized as follows: Section II summarizes the conventional Extended Kalman filter. The estimation algorithm based on Extended Kalman filter gain elimination is derived in section III. In Section IV the conventional Extended Kalman filter and the proposed estimation algorithm based on Extended Kalman filter gain elimination are compared with respect to their calculation burdens. Finally, Section V summarizes the conclusions.

2. Extended Kalman Filter

In this section the Extended Kalman filter (EKF) is summarized [1]:

Extended Kalman Filter (EKF)

$$F(k) = \left.\frac{\partial f(x(k), k)}{\partial x}\right|_{x(k|k)} \tag{3}$$

$$H(k) = \left.\frac{\partial h(x(k), k)}{\partial x}\right|_{x(k|k-1)} \tag{4}$$

$$K(k) = P(k|k-1)H^T(k)[H(k)P(k|k-1)H^T(k) + R(k)]^{-1} \tag{5}$$

$$x(k|k) = x(k|k-1) + K(k)[z(k) - h(x(k|k-1), k)] \tag{6}$$

$$P(k|k) = [I - K(k)H(k)]P(k|k-1) \tag{7}$$

$$x(k+1|k) = f(x(k|k), k) \tag{8}$$

$$P(k+1|k) = Q(k) + F(k)P(k|k)F^T(k) \tag{9}$$

for $k = 0,1, \ldots$, with initial conditions $x(0|-1) = x_0, P(0|-1) = P_0$.

$K(k)$ denotes the Kalman filter gain and I is the identity matrix. ∂ denotes the derivative.

Note that the existence of the inverse of the matrices in the Kalman filter gain equation is ensured assuming that every covariance matrix $R(k)$ is positive definite; this has the significance that no measurement is exact.
Note also that the time invariant noise case does not affect the complexity of the algorithm.

3. Estimation Algorithm Based on EKF Gain Elimination

Following the idea to eliminate the Kalman filter gain computation from the linear Kalman filter [14], the quantity $\Lambda(k)$ is defined:

$$\Lambda(k) = [I - K(k)H(k)]^{-1}K(k) \tag{10}$$

Assume that $R(k)$ and $P(k|k-1)$ are positive definite matrices.

From the Kalman filter gain equation (5) we get

$\mathrm{P}(k|k-1)\mathrm{H}^{\mathrm{T}}(k) = \mathrm{K}(k)[\mathrm{H}(k)\mathrm{P}(k|k-1)\mathrm{H}^{\mathrm{T}}(k) + \mathrm{R}(k)] = \mathrm{K}(k)\mathrm{H}(k)\mathrm{P}(k|k-1)\mathrm{H}^{\mathrm{T}}(k) + \mathrm{K}(k)\mathrm{R}(k)$

$\Rightarrow \mathrm{K}(k)\mathrm{R}(k) = \mathrm{P}(k|k-1)\mathrm{H}^{\mathrm{T}}(k) - \mathrm{K}(k)\mathrm{H}(k)\mathrm{P}(k|k-1)\mathrm{H}^{\mathrm{T}}(k) = [\mathrm{I} - \mathrm{K}(k)\mathrm{H}(k)]\mathrm{P}(k|k-1)\mathrm{H}^{\mathrm{T}}(k)$

Then, using (10), we get

$\mathrm{P}(k|k-1)\mathrm{H}^{\mathrm{T}}(k) = [\mathrm{I} - \mathrm{K}(k)\mathrm{H}(k)]^{-1}\mathrm{K}(k)\mathrm{R}(k) = \Lambda(k)\mathrm{R}(k)$

and

$$\Lambda(k) = \mathrm{P}(k|k-1)\mathrm{H}^{\mathrm{T}}(k)\mathrm{R}^{-1}(k) \tag{11}$$

Also, using (7) we get

$\mathrm{K}(k)\mathrm{R}(k) = \mathrm{P}(k|k)\mathrm{H}^{\mathrm{T}}(k)$

and

$$\mathrm{K}(k) = \mathrm{P}(k|k)\mathrm{H}^{\mathrm{T}}(k)\mathrm{R}^{-1}(k) \tag{12}$$

Using again the Kalman filter gain equation (5) we get

$[\mathrm{I} - \mathrm{K}(k)\mathrm{H}(k)]\mathrm{P}(k|k-1) = \mathrm{P}(k|k-1) - \mathrm{K}(k)\mathrm{H}(k)\mathrm{P}(k|k-1)$

$= \mathrm{P}(k|k-1) - \mathrm{P}(k|k-1)\mathrm{H}^{\mathrm{T}}(k)[\mathrm{H}(k)\mathrm{P}(k|k-1)\mathrm{H}^{\mathrm{T}}(k) + \mathrm{R}(k)]^{-1}\mathrm{H}(k)\mathrm{P}(k|k-1)$

$= [\mathrm{P}^{-1}(k|k-1) + \mathrm{H}^{\mathrm{T}}(k)\mathrm{R}^{-1}(k)\mathrm{H}(k)]^{-1}$

$= [\mathrm{I} + \mathrm{P}(k|k-1)\mathrm{H}^{\mathrm{T}}(k)\mathrm{R}^{-1}(k)\mathrm{H}(k)]^{-1}\mathrm{P}(k|k-1)$

Then

$[\mathrm{I} - \mathrm{K}(k)\mathrm{H}(k)] = [\mathrm{I} + \mathrm{P}(k|k-1)\mathrm{H}^{\mathrm{T}}(k)\mathrm{R}^{-1}(k)\mathrm{H}(k)]^{-1}$

and using (11) we derive

$$[\mathrm{I} - \mathrm{K}(k)\mathrm{H}(k)] = [\mathrm{I} + \Lambda(k)\mathrm{H}(k)]^{-1} \tag{13}$$

Then, working as in [14], we are able to eliminate the Kalman filter gain from the Extended Kalman filter equations by

(a) substituting the Kalman filter gain equation (5) by (11),

(b) using (12) in the estimation equation (6),

(c) using (13) in the estimation error covariance equation (7).

For time varying noise systems, the Extended Kalman Filter gain elimination algorithm (EKFge) has been derived:

Extended Kalman filter gain elimination algorithm (EKFge)

$$\mathrm{F}(k) = \left.\frac{\partial \mathrm{f}(\mathrm{x}(k), k)}{\partial \mathrm{x}}\right|_{\mathrm{x}(k|k)} \tag{14}$$

$$\mathrm{H}(k) = \left.\frac{\partial \mathrm{h}(\mathrm{x}(k), k)}{\partial \mathrm{x}}\right|_{\mathrm{x}(k|k-1)} \tag{15}$$

$$\Lambda(k) = \mathrm{P}(k|k-1)\mathrm{H}^{\mathrm{T}}(k)\mathrm{R}^{-1}(k) \tag{16}$$

$$\mathrm{P}(k|k) = [\mathrm{I} + \Lambda(k)\mathrm{H}(k)]^{-1}\mathrm{P}(k|k-1) \tag{17}$$

$$\mathrm{x}(k|k) = \mathrm{x}(k|k-1) + \mathrm{P}(k|k)\mathrm{H}^{\mathrm{T}}(k)\mathrm{R}^{-1}(k)[\mathrm{z}(k) - \mathrm{h}(\mathrm{x}(k|k-1), k)] \tag{18}$$

$$\mathrm{x}(k+1/k) = \mathrm{f}(\mathrm{x}(k|k), k) \tag{19}$$

$$\mathrm{P}(k+1/k) = \mathrm{Q}(k) + \mathrm{F}(k)\mathrm{P}(k|k)\mathrm{F}^{\mathrm{T}}(k) \tag{20}$$

for $k = 0,1, \ldots$, with initial conditions $\mathrm{x}(0|-1) = \mathrm{x}_0, \mathrm{P}(0|-1) = \mathrm{P}_0$.

Note that the existence of the inverse of the matrices in the Kalman filter gain elimination algorithm equations is ensured assuming that every covariance matrix $\mathrm{R}(k)$ is positive definite and that the initial condition $\mathrm{P}(0|-1) = \mathrm{P}_0$ is positive definite.

Note also that the time invariant noise case affects the complexity of the algorithm, due to the fact that in the time invariant noise case, the matrix R^{-1} is calculated once and *off-line*.

4. Comparison of the Algorithms

It is established that the Extended Kalman filter gain elimination algorithm equations have been derived by the Extended Kalman filter equations. Thus the Extended Kalman filter and the proposed Extended Kalman filter gain elimination algorithm are equivalent filters with respect to their behavior, since they calculate theoretically the same estimates. Both filters are iterative algorithms; then, it is reasonable to assume that all the filters compute the estimation $x(k|k)$ executing the same number of iterations. Thus, in order to compare the algorithms with respect to their computational time, we have to compare their per step (iteration) calculation burden (CB) required for the on-line calculations.

Scalar operations are involved in matrix manipulation operations, which are needed for the implementation of the filtering algorithms. Table 1 summarizes the calculation burden of needed matrix operations, for the general multidimensional case; the identity matrix is denoted by I and a symmetric matrix by S. The details are given in [15].

Table 1: Calculation burden of matrix operations.

Matrix Operation	**Matrix Dimensions**	**Calculation Burden**
$C = A + B$	$(n \times m) + (n \times m)$	nm
$S = A + B$	$(n \times n) + (n \times n)$	$\frac{1}{2}n^2 + \frac{1}{2}n$
$B = I + A$	$(n \times n) + (n \times n)$	n
$C = A \cdot B$	$(n \times m) \cdot (m \times \ell)$	$2nm\ell - n\ell$
$S = A \cdot B$	$(n \times m) \cdot (m \times n)$	$n^2m + nm - \frac{1}{2}n^2 - \frac{1}{2}n$
$B = A^{-1}$	$n \times n$	$\frac{1}{6}(16n^3 - 3n^2 - n)$

The per iteration calculation burden of the Extended Kalman filter and of the Extended Kalman filter gain elimination algorithm for the general multidimensional case, where $n \geq 2, m \geq 2$, are analytically calculated in the Appendix and summarized in Table 2.

Table 2: Per iteration calculation burden of EKF and EKF gain elimination algorithm.

Filter	**Noise**	**Calculation Burden**
EKF	*time varying or time invariant*	$CB_{EKF} = 3n^3 + m + 3n^2m + 2nm + 3nm^2 + \frac{1}{6}(16m^3 - 3m^2 - m) + CB_F + CB_H + CB_f + CB_h$
EKFge	*time varying*	$CB_{TVEKFge} = \frac{1}{2}(8n^3 - n^2 + n) + m + 6n^2m - nm + 2nm^2 + \frac{1}{6}(16n^3 - 3n^2 - n) + \frac{1}{6}(16m^3 - 3m^2 - m) + CB_F + CB_H + CB_f + CB_h$
	time invariant	$CB_{TIEKFge} = \frac{1}{2}(8n^3 - n^2 + n) + m + 6n^2m - nm + 2nm^2 + \frac{1}{6}(16n^3 - 3n^2 - n) + CB_F + CB_H + CB_f + CB_h$

From Table 2 we get:

$$CB_{EKF} - CB_{TVEKFge} = -\frac{1}{3}(11n^3 - 3n^2 + n) - 3n^2m + 3nm + nm^2$$

$$CB_{EKF} - CB_{TIEKFge} = -\frac{1}{3}(11n^3 - 3n^2 + n) - 3n^2m + 3nm + nm^2 + \frac{1}{6}(16m^3 - 3m^2 - m)$$

Then we conclude that:

(a) For time varying noise, the proposed Extended Kalman filter gain elimination algorithm may be faster than the conventional Extended Kalman filter, depending on the model dimensions. In fact, the areas depending on the model dimensions, where the proposed Extended Kalman filter gain elimination algorithm or the Extended Kalman filter is faster, are shown in Figure 1.

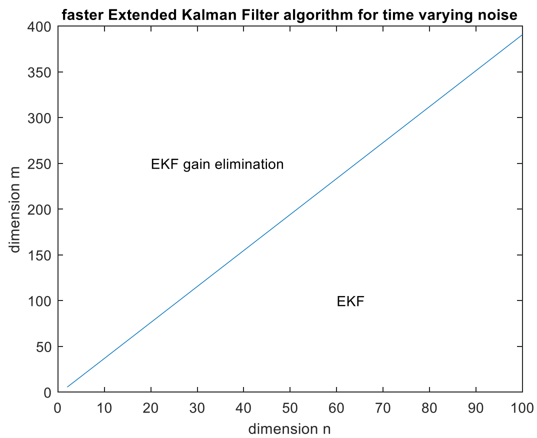

Fig. 1: Time varying noise: The faster filter depends on the model dimensions.

The following Rule of Thumb for time invariant noise systems is derived: The Extended Kalman filter gain elimination algorithm is faster than the traditional Extended Kalman filter, when

$$\frac{m}{n} > 4 \quad (21)$$

(b) For time invariant noise, the proposed Extended Kalman filter gain elimination algorithm may be faster than the conventional Extended Kalman filter, depending on the model dimensions. In fact, the areas depending on the model dimensions, where the proposed Extended Kalman filter gain elimination algorithm or the Extended Kalman filter is faster, are shown in Figure 2.

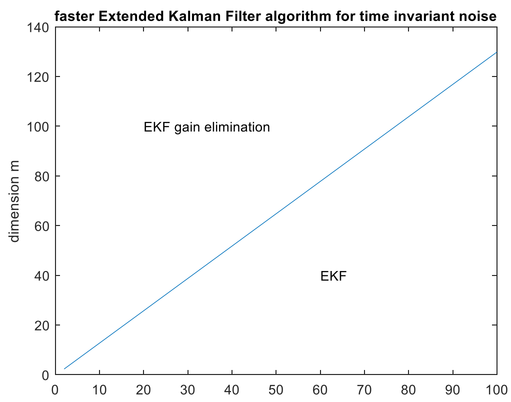

Fig. 2: Time invariant noise: The faster filter depends on the model dimensions.

The following Rule of Thumb for time invariant noise systems is derived: The Extended Kalman filter gain elimination algorithm is faster than the traditional Extended Kalman filter, when

$$\frac{m}{n} > 1.3 \quad (22)$$

4. Conclusion

The Extended Kalman filter uses, in every iteration, the Kalman filter gain in order to produce estimation and prediction of the n-dimensional state vector using the m-dimensional measurement vector.

The basic idea was to eliminate the Kalman filter gain computation. A variation of the Extended Kalman filter eliminating the Kalman filter gain has been derived. It was shown that the derived Extended Kalman filter gain elimination algorithm is equivalent to the traditional Extended Kalman filter. The proposed Extended Kalman filter gain elimination algorithm may be faster than the conventional Extended Kalman filter, depending on the model dimensions; this is very important because the knowledge of the system dimensions leads to the ability to a priori (before the filter implementation) determine which filter is faster.

Appendix

The per iteration calculation burden of Extended Kalman filter and the estimation algorithm based on Extended Kalman filter gain elimination for the general multidimensional case, where $n \geq 2, m \geq 2$, are analytically calculated in Tables 3 and 4, respectively.

Table 3: Calculation burden of Extended Kalman Filter.

Matrix Operation	**Calculation Burden**
$F(k) = \left.\frac{\partial f(x,k)}{\partial x}\right\vert_{x(k\vert k)}$	CB_F
$H(k) = \left.\frac{\partial h(x,k)}{\partial x}\right\vert_{x(k\vert k-1)}$	CB_H
$H(k)P(k\vert k-1)$	$2n^2m - nm$
$H(k)P(k\vert k-1)H^T(k)$	$nm^2 + nm - \frac{1}{2}m^2 - \frac{1}{2}m$
$H(k)P(k\vert k-1)H^T(k) + R(k)$	$\frac{1}{2}m^2 + \frac{1}{2}m$
$[H(k)P(k\vert k-1)H^T(k) + R(k)]^{-1}$	$\frac{1}{6}(16m^3 - 3m^2 - m)$
$K(k) = P(k\vert k-1)H^T(k) \quad [H(k)P(k\vert k-1)H^T(k) + R(k)]^{-1}$	$2nm^2 - nm$
$h(x(k\vert k-1),k)$	CB_h
$z(k) - h(x(k\vert k-1),k)$	m
$K(k)[z(k) - h(x(k/k-1),k)]$	$2nm - n$
$x(k\vert k) = x(k\vert k-1) + K(k)[z(k) - h(x(k\vert k-1),k)]$	n
$K(k)H(k)P(k\vert k-1)$	$n^2m + nm - \frac{1}{2}n^2 - \frac{1}{2}n$
$P(k\vert k) = P(k\vert k-1) - K(k)H(k)P(k\vert k-1)$	$\frac{1}{2}n^2 + \frac{1}{2}n$
$x(k+1\vert k) = f(x(k\vert k),k)$	CB_f
$F(k)P(k\vert k)$	$2n^3 - n^2$
$F(k)P(k\vert k)F^T(k)$	$n^3 + \frac{1}{2}n^2 - \frac{1}{2}n$
$P(k+1\vert k) = Q(k) + F(k)P(k\vert k)F^T(k)$	$\frac{1}{2}n^2 + \frac{1}{2}n$
$CB_{EKF} = 3n^3 + m + 3n^2m + 2nm + 3nm^2$ $+\frac{1}{6}(16m^3 - 3m^2 - m) + CB_F + CB_H + CB_f + CB_h$	

Table 4: Calculation burden of Extended Kalman Filter gain elimination algorithm.

Matrix Operation	**Calculation Burden**
$\mathrm{F(k)} = \left.\frac{\partial \mathrm{f(x,k)}}{\partial \mathrm{x}}\right\|_{\mathrm{x(k\|k)}}$	$\mathrm{CB_F}$
$\mathrm{H(k)} = \left.\frac{\partial \mathrm{h(x,k)}}{\partial \mathrm{x}}\right\|_{\mathrm{x(k\|k-1)}}$	$\mathrm{CB_H}$
$\mathrm{R^{-1}(k)}$	$\frac{1}{6}(16\mathrm{m}^3 - 3\mathrm{m}^2 - \mathrm{m})$
$\mathrm{H^T(k)R^{-1}(k)}$	$2\mathrm{nm}^2 - \mathrm{nm}$
$\Lambda(\mathrm{k}) = \mathrm{P(k\|k-1)H^T(k)R^{-1}(k)}$	$2\mathrm{n}^2\mathrm{m} - \mathrm{nm}$
$\Lambda(\mathrm{k})\mathrm{H(k)}$	$2\mathrm{n}^2\mathrm{m} - \mathrm{n}^2$
$\mathrm{I} + \Lambda(\mathrm{k})\mathrm{H(k)}$	n
$[\mathrm{I} + \Lambda(\mathrm{k})\mathrm{H(k)}]^{-1}$	$\frac{1}{6}(16\mathrm{n}^3 - 3\mathrm{n}^2 - \mathrm{n})$
$\mathrm{P(k\|k)} = [\mathrm{I} + \Lambda(\mathrm{k})\mathrm{H(k)}]^{-1}\ \mathrm{P(k\|k-1)}$	$\mathrm{n}^3 + \frac{1}{2}\mathrm{n}^2 - \frac{1}{2}\mathrm{n}$
$\mathrm{P(k\|k-1)H^T(k)R^{-1}(k)}$	$2\mathrm{n}^2\mathrm{m} - \mathrm{nm}$
$\mathrm{h(x(k\|k-1),k)}$	$\mathrm{CB_h}$
$\mathrm{z(k) - h(x(k\|k-1),k)}$	m
$\mathrm{P(k\|k)H^T(k)R^{-1}(k)}$ $[\mathrm{z(k) - h(x(k\|k-1),k)}]$	$2\mathrm{nm} - \mathrm{n}$
$\mathrm{x(k\|k) = x(k/k-1) + P(k\|k)H^T(k)R^{-1}(k)[z(k) - h(x(k\|k-1),k)]}$	n
$\mathrm{x(k+1\|k) = f(x(k\|k),k)}$	$\mathrm{CB_f}$
$\mathrm{F(k)P(k\|k)}$	$2\mathrm{n}^3 - \mathrm{n}^2$
$\mathrm{F(k)P(k\|k)F^T(k)}$	$\mathrm{n}^3 + \frac{1}{2}\mathrm{n}^2 - \frac{1}{2}\mathrm{n}$
$\mathrm{P(k+1\|k) = Q(k) + F(k)P(k\|k)F^T(k)}$	$\frac{1}{2}\mathrm{n}^2 + \frac{1}{2}\mathrm{n}$
$\mathrm{CB_{EKFge}} = \frac{1}{2}(8\mathrm{n}^3 - \mathrm{n}^2 + \mathrm{n}) + \mathrm{m} + 6\mathrm{n}^2\mathrm{m} - \mathrm{nm} + 2\mathrm{nm}^2$ $+\frac{1}{6}(16\mathrm{n}^3 - 3\mathrm{n}^2 - \mathrm{n}) + \frac{1}{6}(16\mathrm{m}^3 - 3\mathrm{m}^2 - \mathrm{m}) + \mathrm{CB_F} + \mathrm{CB_H} + \mathrm{CB_f} + \mathrm{CB_h}$	

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