# A Computational Simulation of the Urine Output Flow Rate 

Poupak Kermani (M.Sc.A)<br>CAE Inc<br>8585 Cote de Liesse, Montreal, Canada<br>poupak.kermani@cae.com


#### Abstract

This paper presents a mathematical model, computing the rate of the urine flow that travels between the Kidney's pelvis and the bladder. This model predicts that a total of 1.8 liter of urine is disposed from two Kidneys every 24 hours. The simulation result yields toward the nominal quantity of 1.5 liter of urine disposed by a healthy adult with normal Kidney function.


Keywords: Ureteral Peristalsis, Urine Output Rate

## Introduction

The ureter is a smooth tubular muscle, which transports urine from the Kidney's pelvis to the bladder. The passage of urine through the ureter's lumen is regulated by the rhythmic contractions of the ureter walls, which pushes the bolus of urine through its lumen [Ref 3]. This phenomena, which is called the Peristalsis, exercises consistently luminal pressure which occurs behind the urine bolus during its passage through the ureter [Ref 1].

The muscular contraction force which travels through the ureter's walls and regulates the transport of the urine bolus, is difficult to measure experimentally. The peristaltic movement of the ureter is a complex phenomenon, which is difficult to replicate accurately with a computational model.

The purpose of this paper is to present a mathematical model which is based on the difference of pressure created inside the ureter's lumen during the ureteral peristalsis. This model allows to compute the urine output flow rate in a healthy person.

## Simulation Method

The ureter is prototyped as a cylinder with circular cross section. The ureter is considered to be positioned horizontally. One end of the ureter is connected to the Kidney's pelvis and the other end to the bladder. The urine is considered to be an incompressible fluid.

It is assumed that the conduction velocity of urine is governed by the Peristalsis. The velocity of the bolus of urine is considered to be at its maximum value at the pyelo-ureteral junction (UPJ). This velocity decreases to zero once the bolus of urine reaches the junction between the ureter and the bladder [Ref 3].

## Mathematical Model

The volumetric flow of the bolus of urine through the ureter's lumen is derived using the Poiseuille's law, in concert with the Bernoulli's equation:

$$
\begin{equation*}
\Delta P=P_{2}-P_{1}=\frac{1}{2} \rho V^{2}=\frac{8 \mu \mathrm{LQ}}{\pi R^{4}} \tag{1}
\end{equation*}
$$

Where:
$\Delta \mathrm{p}$ is the pressure difference between the two ends of the ureter $\mu$ is the dynamic viscosity of urine at $37^{\circ} \mathrm{C}$

L is the length of the ureter
Q is the volumetric flow rate of a bolus of urine
R is the radius of the Ureter
$\rho$ is the density of Urine at $37^{\circ} \mathrm{C}$
V : is the velocity of the bolus of Urine, initiated at UPJ
The diagram below shows a Matlab simulation representation of Equation (1), used to compute the urine output rate flow through the ureter's lumen. For this simulation, the ureter is considered to be of 70 cm long with a circular cross section of 3 mm in radius [Ref 2]. The dynamic viscosity of the urine at $37^{\circ} \mathrm{C}$ is considered to be equal to 0.000824 Pa . Sec [Ref 4]. The density of urine at $37^{\circ} \mathrm{C}$ is considered to be equal to $993.36 \mathrm{~kg} / \mathrm{m}^{3}$ [Ref 4]. The computed velocity of the bolus of urine is estimated to be equal to $4.8 \mathrm{~cm} / \mathrm{sec}[\operatorname{Ref} 3]$.


Fig. 1: Matlab representation of the mathematical model to compute the urine output flow rate using Poiseuille's law and Bernoulli's equation.

## Result

Figure 2 shows the computed urine output rate during one cycle of uureteral peristalsis. Each cycle of peristalsis which travels across the ureteral walls, will transport $0.053 \mathrm{ml} / \mathrm{sec}$ of urine, through the ureter's lumen. When repeating the ureteral peristalsis' cycle during 24 hours, a total volume of 915.84 milliliter of urine will be passed from the kidney's pelvis to the bladder. A person with normal kidney function should dispose of 1.5 liter of urine every 24 hours. This model predicts a total of 1.8 liter of urine disposition from two Kidneys every 24 hours. The simulation result of this study yields toward the urine disposition of a healthy adult with two normal Kidney function.


Figure 2: Computed Urine Volumetric Flow Rate

## Conclusion

The ureteral contraction force, leading the passage of urine through the ureter's lumen, is difficult to measure experimentally, and quite complex to be modelled mathematically. The present study has focused on the foot print of the ureteral peristalsis, which is the pressure difference inside the ureter's lumen behind every movement of the bolus of urine. The mathematical model based on Poiseuille's law and Bernoulli's equation allowed the calculation of the urine output flow rate for a normal kidney function.

## References

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