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Extended Information Filter and Extended Kalman Filter Comparison: Selection of the Faster Filter

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Abstract **-** A comparison study is presented between the Extended Kalman filter and the Extended Information filter, which are equivalent with respect to their behavior, since they produce the same estimations. The computational requirements of Extended Kalman filter and Extended Information filter are determined and a method is proposed to a-priori (before the filters' implementation) decide which filter is the faster one.

Keywords**:** extended Kalman filter, extended Information filter, nonlinear estimation

1. Introduction

The importance of nonlinear estimation in many fields of science and engineering is undoubtable. The parameter identification problem is a common nonlinear estimation problem, for example, nonlinear estimation techniques are implemented in damping parameter estimation and estimation of the parameters in a macroscopic freeway traffic model [1]. Nonlinear estimation is required in various application areas, as in demodulation of angle-modulated signals [2], estimation of frequency and phase of the frequency demodulation model [3], simultaneous localization and mapping (SLAM) problem [3], object detection/tracking [4]-[5].

The nonlinear estimation problem is associated with nonlinear discrete time systems described by the following state space equations:

$$x(k+1) = f(x(k), k) + w(k) \tag{1}$$

$$z(k) = h(x(k), k) + v(k) \tag{2}$$

where $x(k)$ is the $n \times 1$ state vector, $z(k)$ is the $m \times 1$ measurement vector, $u(k)$ is the input, $f(x(k), u(k))$ is the state nonlinear vector function, $h\big(x(k)\big)$ is the observation nonlinear vector function, $w(k)$ is the $n \times 1$ state noise and $v(k)$ is the $m \times 1$ measurement noise at time $k \geq 0$. The statistical model expresses the nature of the state and the measurements. The basic assumption is that the state noise and the measurement noise are Gaussian white noises with known covariances $Q(k)$ of dimension $n \times n$ and $R(k)$ of dimension $m \times m$, respectively. The following assumptions also hold: (a) the initial value of the state $x(0)$ is a Gaussian random variable with mean x_0 and covariance P_0; (b) the stochastic processes $\{w(k)\}$, $\{v(k)\}$ and the random variable $x(0)$ are independent.

The **Extended Kalman filter (EKF)** [1], [2], [6], [7], [8] is the most well-known algorithm that solves the nonlinear estimation problem, extending the ideas of linear Kalman filter to nonlinear estimation problems. The traditional Extended Kalman filter implements the linearization of the nonlinear system using the first order term of Taylor series. The Extended Kalman filter computes the state estimation and the corresponding estimation error covariance matrix as well as the state prediction and the corresponding prediction error covariance matrix. There are a many variations of the traditional Extended Kalman filter, which depend on the derivation technique implemented and the required assumptions. The second order Extended Kalman filter [9] can be derived by taking into account two terms in the Taylor series expansions of the state and observation nonlinear functions; then an improved filter is derived at the expense of an increasing computational burden [1]. The iterated Extended Kalman filter [2] uses the idea that once an estimate is calculated via the Extended Kalman filter, then the observation nonlinear function can be linearized about the estimate rather than the prediction;

generally this should be an improved estimate. There is a class of extended Kalman filter algorithms that can be applied to nonlinear systems. The Square Root Extended Kalman filter [10] uses the square root of the estimation error covariance matrix. The Adaptive Extended Kalman filter [11] is an adaptive version of the Extended Kalman Filter. The Rhythmic Extended Kalman filter [12] is developed to estimate the pose, learn an individualized model of periodic movement over time, and use the learned model to improve pose estimation. The Fuzzy Extended Kalman filter [13] uses hybrid estimator based on combination of Extended Kalman Filter and fuzzy inference system. The **Extended Information filter (EIF)** [2], [14] uses of the inverse of the error covariance matrix and is a notable nonlinear estimation algorithm, due to the fact that its concept has been used to derive other estimation algorithms. The cubature Information filter [3] is derived from an Extended Information filter and a cubature Kalman filter, which is the closest known approximation to the Bayesian filter that could be designed in a nonlinear setting under the Gaussian assumption. The sparse Extended Information filter [15] is derived using the Extended Information filter and assuming the availability of the signal sparse property.

Extended Kalman filter (EKF) and Extended Information filter (EIF) have been successfully implemented with a variety of challenging real world problems, such as damping parameter estimation [1], demodulation of angle-modulated signals [2], simultaneous localization and mapping (SLAM) problem [3], problem to estimate the position, velocity and constant ballistic coefficient of a body as it re-enters the atmosphere at a very high altitude at a very high velocity [4], underwater object detection/tracking [5], tracking and navigation applications [6], Simultaneous Localization and Mapping (SLAM) applications [16], localization of humanoid robots [17], online optimization of an uncertain biochemical reactor [18], target tracking with dynamic quantization via communication channels [19], the Gyroless Star Tracker [20].

In practice, Extended Kalman filter is more popular than the Extended Information filter. In this paper, a comparison study is presented between Extended Kalman filter and Extended Information filter, in order to show the potential advantages of Extended Information filter. The novelty of the paper concerns: (a) the calculation of the computational burdens of the Extended Kalman filter and the Extended Information Filter, (b) the development of a method for selecting the faster filter.

2. Extended Kalman Filter (EKF) and Extended Information Filter (EIF)

Extended Kalman filter equations result from truncating a Taylor series expansion of the nonlinear functions $f(x(k), u(k))$ and $h(x(k))$ after the linear terms, using the Jacobians $F(k)$ and $H(k)$ of the functions $f(x(k), u(k))$ and $h(x(k))$, respectively.

Extended Kalman filter (EKF) computes iteratively the Kalman filter gain $K(k)$, the state estimation $x(k|k)$ and the estimation error covariance matrix $P(k|k)$, the state prediction $x(k+1|k)$ and the prediction error covariance matrix $P(k+1|k)$ using measurements and inputs till time k.

The Extended Kalman filter (EKF) is summarized in the following:

Extended Kalman Filter (EKF)

$$H(k) = \left.\frac{\partial h(x(k))}{\partial x}\right|_{x(k|k-1)}$$
$$K(k) = P(k|k-1)H^T(k)[H(k)P(k|k-1)H^T(k) + R(k)]^{-1}$$
$$x(k|k) = x(k|k-1) + K(k)[z(k) - h(x(k|k-1))]$$
$$P(k|k) = [I - K(k)H(k)]P(k|k-1)$$
$$F(k) = \left.\frac{\partial f(x(k), u(k))}{\partial x}\right|_{x(k|k)}$$
$$x(k+1|k) = f(x(k|k), u(k))$$
$$P(k+1|k) = Q(k) + F(k)P(k|k)F^T(k)$$
for $k = 0,1, \ldots,$ with initial conditions $x(0|-1) = x_0$, $P(0|-1) = P_0$

I is the identity matrix. Note that the existence of the inverse of the matrices in the Kalman filter gain equation is ensured assuming that every covariance matrix R(k) is positive definite; this has the significance that no measurement is exact.

The Extended Information filter equations result from Extended Kalman filter equations using of the inverse of the error covariance matrix and the Matrix Inversion Lemma. In fact, the Extended Information filter uses the quantities:

$$y(k|k) = P^{-1}(k|k)x(k|k) \tag{3}$$

$$S(k|k) = P^{-1}(k|k)(k) \tag{4}$$

$$y(k|k-1) = P^{-1}(k|k-1)x(k|k-1) \tag{5}$$

$$S(k|k-1) = P^{-1}(k|k-1) \tag{6}$$

The Extended Information filter (EIF) computes iteratively

- the information state estimation $y(k|k)$ and the estimation information matrix $S(k|k)$
- the state estimation $x(k|k)$ and the estimation error covariance matrix $P(k|k)$
- the Kalman filter gain $K(k)$
- the information state prediction $y(k+1|k)$ and the prediction information matrix $S(k+1|k)$
- the state prediction $x(k+1|k)$ and the prediction error covariance matrix $P(k+1|k)$

using measurements and inputs till time k.

The Extended Information Filter (EIF) is summarized in the following:

Extended Information Filter (EIF)

$$H(k) = \left.\frac{\partial h(x(k),k)}{\partial x}\right|_{x(k|k-1)}$$

$$y(k|k) = y(k|k-1) + H^T(k)R^{-1}(k|k)\left[z(k) - h\big(x(k|k-1)\big) + H(k)x(k|k-1)\right]$$

$$S(k|k) = S(k|k-1) + H^T(k)R^{-1}(k|k)H(k)$$

$$P(k|k) = S^{-1}(k|k)$$

$$x(k|k) = S^{-1}(k|k)y(k|k)$$

$$K(k) = P(k|k)H^T(k)R^{-1}(k|k)$$

$$F(k) = \left.\frac{\partial f(x(k),u(k))}{\partial x}\right|_{x(k|k)}$$

$$x(k+1|k) = f(x(k|k),u(k))$$

$$P(k+1|k) = Q(k) + F(k)P(k|k)F^T(k)$$

$$S(k+1|k) = P^{-1}(k+1|k)$$

$$y(k+1|k) = S(k+1|k)x(k+1|k)$$

for $k = 0,1,\ldots,$ with initial conditions $y(0|-1) = P^{-1}(0|-1)x(0|-1) = P_0^{-1}x_0, S(0|-1) = P^{-1}(0|-1) = P_0^{-1}$

Note that the existence of the inverse matrices that appear in Extended Information filter equations is guaranteed in the case where the measurements noise covariances $R(k)$ are positive definite; this happens in the case where no measurement is exact. Also, the initial condition P_0 has to be nonsingular.

Remark. The Kalman filter gain calculation can be omitted, due to the fact that it is not required in any other calculations. In this case the Extended Information Filter gain omitted (EIFgo) algorithm is derived.

3. Selection of the faster filter

It is established that the Extended Information filter algorithm equations have been derived by the Extended Kalman filter equations. Thus the Extended Kalman filter and the Extended Information filter are equivalent filters with respect to their behavior, since they calculate theoretically the same estimates. Both filters are iterative algorithms; then, it is reasonable to assume that the filters compute the estimation $x(k|k)$ executing the same number of iterations. Thus, in order to compare the algorithms with respect to their computational time, we have to compare their per step (iteration) calculation burdens (CB) required for the on-line calculations; the calculation burden of the off-line calculations (initialization process for the Extended Information filter) is not taken into account.

Scalar operations are involved in matrix manipulation operations, which are needed for the implementation of the filtering algorithms. Table 1 summarizes the calculation burden of needed matrix operations; a symmetric matrix is denoted by S. The details are given in [21].

Table 1: Calculation burden of matrix operations.

Matrix Operation	Matrix Dimensions	Calculation Burden
$C = A + B$	$(n \times m) + (n \times m)$	nm
$S = A + B$	$(n \times n) + (n \times n)$	$\frac{1}{2}n^2 + \frac{1}{2}n$
$B = I + A$	$(n \times n) + (n \times n)$	n
$C = A \cdot B$	$(n \times m) \cdot (m \times \ell)$	$2nm\ell - n\ell$
$S = A \cdot B$	$(n \times m) \cdot (m \times n)$	$n^2m + nm - \frac{1}{2}n^2 - \frac{1}{2}n$
$B = A^{-1}$	$n \times n$	$\begin{cases} \frac{1}{6}(16n^3 - 3n^2 - n) & n \geq 2 \\ 1 & n = 1 \end{cases}$

The per iteration calculation burden of EKF and EIF are analytically calculated in the Tables 2 and 3, respectively.

Table 2: Calculation burden of Extended Kalman Filter (EKF).

Matrix operation	Calculation burden
$H(k) = \left.\frac{\partial h(x(k))}{\partial x}\right\vert_{x(k\vert k-1)}$	CB_H
$H(k)P(k\vert k-1)$	$2n^2m - nm$
$H(k)P(k\vert k-1)H^T(k)$	$nm^2 + nm - \frac{1}{2}m^2 - \frac{1}{2}m$
$H(k)P(k\vert k-1)H^T(k) + R(k)$	$\frac{1}{2}m^2 + \frac{1}{2}m$
$[H(k)P(k\vert k-1)H^T(k) + R(k)]^{-1}$	$\frac{1}{6}(16m^3 - 3m^2 - m)$
$K(k) = P(k\vert k-1)H^T(k)[H(k)P(k\vert k-1)H^T(k) + R(k)]^{-1}$	$2nm^2 - nm$
$h(x(k\vert k-1))$	CB_h
$z(k) - h(x(k\vert k-1))$	m
$K(k)[z(k) - h(x(k/k-1))]$	$2nm - n$
$x(k\vert k) = x(k\vert k-1) + K(k)[z(k) - h(x(k\vert k-1))]$	n
$K(k)H(k)P(k\vert k-1)$	$n^2m + nm - \frac{1}{2}n^2 - \frac{1}{2}n$
$P(k\vert k) = P(k\vert k-1) - K(k)H(k)P(k\vert k-1)$	$\frac{1}{2}n^2 + \frac{1}{2}n$
$F(k) = \left.\frac{\partial f(x(k), u(k))}{\partial x}\right\vert_{x(k\vert k)}$	CB_F
$x(k+1\vert k) = f(x(k\vert k), u(k))$	CB_f
$F(k)P(k\vert k)$	$2n^3 - n^2$
$F(k)P(k\vert k)F^T(k)$	$n^3 + \frac{1}{2}n^2 - \frac{1}{2}n$
$P(k+1\vert k) = Q(k) + F(k)P(k\vert k)F^T(k)$	$\frac{1}{2}n^2 + \frac{1}{2}n$
$n \geq 2, m \geq 2$	$CB_{EKF} = 3n^3 + 3n^2m + 2nm + 3nm^2 + \frac{1}{6}(16m^3 - 3m^2 + 5m) + CB_F + CB_H + CB_f + CB_h$
$n = 1, m \geq 2$	$CB_{EKF} = \frac{1}{6}(16m^3 + 15m^2 + 35m + 18) + CB_F + CB_H + CB_f + CB_h$
$n \geq 2, m = 1$	$CB_{EKF} = 3n^3 + 3n^2 + 5n + 2 + CB_F + CB_H + CB_f + CB_h$
$n = 1, m = 1$	$CB_{EKF} = 13 + CB_F + CB_H + CB_f + CB_h$

Table 3: Calculation burden of Extended Information Filter (EIF).

Matrix operation	Calculation burden
$H(k)=\left.\frac{\partial h(x(k))}{\partial x}\right\vert_{x(k\vert k-1)}$	CB_H
$h(x(k\vert k-1))$	CB_h
$H(k)x(k\vert k-1)$	$2nm-m$
$z(k)-h(x(k\vert k-1))$	m
$z(k)-h\big(x(k\vert k-1)\big)+H(k)x(k\vert k-1)$	m
$R^{-1}(k\vert k)$	$\frac{1}{6}(16m^3-3m^2-m)$
$H^T(k)R^{-1}(k\vert k)$	$2nm^2-nm$
$H^T(k)R^{-1}(k\vert k)\big[z(k)-h\big(x(k\vert k-1)\big)+H(k)x(k\vert k-1)\big]$	$2nm-n$
$y(k\vert k)=y(k\vert k-1)+H^T(k)R^{-1}(k\vert k)\big[z(k)-h\big(x(k\vert k-1)\big)+H(k)x(k\vert k-1)\big]$	n
$H^T(k)R^{-1}(k\vert k)H(k)$	$n^2m+nm-\frac{1}{2}n^2-\frac{1}{2}n$
$S(k\vert k)=S(k\vert k-1)+H^T(k)R^{-1}(k\vert k)H(k)$	$\frac{1}{2}n^2+\frac{1}{2}n$
$P(k\vert k)=S^{-1}(k\vert k)$	$\frac{1}{6}(16n^3-3n^2-n)$
$x(k\vert k)=S^{-1}(k\vert k)y(k\vert k)$	$2n^2-n$
$K(k)=P(k\vert k)H^T(k)R^{-1}(k\vert k)$	$2n^2m-nm$
$F(k)=\left.\frac{\partial f(x(k),u(k))}{\partial x}\right\vert_{x(k\vert k)}$	CB_F
$x(k+1\vert k)=f(x(k\vert k),u(k))$	CB_f
$F(k)P(k\vert k)$	$2n^3-n^2$
$F(k)P(k\vert k)F^T(k)$	$n^3+\frac{1}{2}n^2-\frac{1}{2}n$
$P(k+1\vert k)=Q(k)+F(k)P(k\vert k)F^T(k)$	$\frac{1}{2}n^2+\frac{1}{2}n$
$S(k+1\vert k)=P^{-1}(k+1\vert k)$	$\frac{1}{6}(16n^3-3n^2-n)$
$y(k+1\vert k)=S(k+1\vert k)x(k+1\vert k)$	$2n^2-n$

$n\geq 2, m\geq 2$	$CB_{EIF}=\frac{1}{3}(25n^3+9n^2-7n)+3n^2m+3nm+2nm^2+\frac{1}{6}(16m^3-3m^2+5m)+CB_F+CB_H+CB_f+CB_h$
$n=1, m\geq 2$	$CB_{EIF}=\frac{1}{6}(16m^3+9m^2+41m+42)+CB_F+CB_H+CB_f+CB_h$
$n\geq 2, m=1$	$CB_{EIF}=\frac{1}{3}(25n^3+18n^2+8n+6)+CB_F+CB_H+CB_f+CB_h$
$n=1, m=1$	$CB_{EIF}=17+CB_F+CB_H+CB_f+CB_h$
$n\geq 2, m\geq 2$	$CB_{EIFgo}=\frac{1}{3}(25n^3+9n^2-7n)+n^2m+4nm+2nm^2+\frac{1}{6}(16m^3-3m^2+5m)+CB_F+CB_H+CB_f+CB_h$
$n=1, m\geq 2$	$CB_{EIgoF}=\frac{1}{6}(16m^3+9m^2+35m+42)+CB_F+CB_H+CB_f+CB_h$
$n\geq 2, m=1$	$CB_{EIFgo}=\frac{1}{3}(25n^3+12n^2+11n+6)+CB_F+CB_H+CB_f+CB_h$
$n=1, m=1$	$CB_{EIFgo}=16+CB_F+CB_H+CB_f+CB_h$

The per iteration calculation burden of EKF and EIF are summarized in Table 4.

Table 4: Per iteration calculation burden of EKF and EIF.

Model dimensions	Filter	Calculation burden
$n \geq 2$ $m \geq 2$	EKF	$CB_{EKF} = 3n^3 + 3n^2m + 2nm + 3nm^2 + \frac{1}{6}(16m^3 - 3m^2 + 5m) + CB_F + CB_H + CB_f + CB_h$
	EIF	$CB_{EIF} = \frac{1}{3}(25n^3 + 9n^2 - 7n) + 3n^2m + 3nm + 2nm^2 + \frac{1}{6}(16m^3 - 3m^2 + 5m) + CB_F + CB_H + CB_f + CB_h$
	EIF gain omitted	$CB_{EIFgo} = \frac{1}{3}(25n^3 + 9n^2 - 7n) + n^2m + 4nm + 2nm^2 + \frac{1}{6}(16m^3 - 3m^2 + 5m) + CB_F + CB_H + CB_f + CB_h$
$n = 1$ $m \geq 2$	EKF	$CB_{EKF} = \frac{1}{6}(16m^3 + 15m^2 + 35m + 18) + CB_F + CB_H + CB_f + CB_h$
	EIF	$CB_{EIF} = \frac{1}{6}(16m^3 + 9m^2 + 41m + 42) + CB_F + CB_H + CB_f + CB_h$
	EIF gain omitted	$CB_{EIgoF} = \frac{1}{6}(16m^3 + 9m^2 + 35m + 42) + CB_F + CB_H + CB_f + CB_h$
$n \geq 2$ $m = 1$	EKF	$CB_{EKF} = 3n^3 + 3n^2 + 5n + 2 + CB_F + CB_H + CB_f + CB_h$
	EIF	$CB_{EIF} = \frac{1}{3}(25n^3 + 18n^2 + 8n + 6) + CB_F + CB_H + CB_f + CB_h$
	EIF gain omitted	$CB_{EIFgo} = \frac{1}{3}(25n^3 + 12n^2 + 11n + 6) + CB_F + CB_H + CB_f + CB_h$
$n = 1$ $m = 1$	EKF	$CB_{EKF} = 13 + CB_F + CB_H + CB_f + CB_h$
	EIF	$CB_{EIF} = 17 + CB_F + CB_H + CB_f + CB_h$
	EIF gain omitted	$CB_{EIFgo} = 16 + CB_F + CB_H + CB_f + CB_h$

From Table 4, a method is developed for selecting the faster filter:

A. In the general case, where **the Kalman filter gain calculation is included**, we get:
i) $n = 1, m = 1$: $CB_{EKF} < CB_{EIF}$
ii) $n \geq 2, m = 1$: $CB_{EKF} < CB_{EIF}$
iii) $n = 1, m \geq 2$: $\begin{cases} CB_{EKF} < CB_{EIF}, m = 2 \\ CB_{EIF} < CB_{EKF}, m \geq 3 \end{cases}$
iv) $n \geq 2, m \geq 2$: $CB_{EKF} - CB_{EIF} = n\left\{m^2 - m - \frac{1}{3}(16n^2 + 9n - 7)\right\}$
Then $CB_{EIF} < CB_{EKF}$ when

$$m > \frac{3 + \sqrt{192n^2 + 108n - 75}}{6} \tag{7}$$

Hence, for multidimensional systems, where $n \geq 2, m \geq 2$, the following rule is derived:

$$if\ m > \frac{3+\sqrt{192n^2+108n-75}}{6}\ then\ EIF\ is\ faster\ than\ EKF$$
$$else\ EKF\ is\ faster\ than\ EIF$$

B. In the special case, where **the Kalman filter gain calculation is omitted**, we get:

i) $n = 1, m = 1$: $CB_{EKF} < CB_{EIFgo}$

ii) $n \geq 2, m = 1$: $CB_{EKFgo} < CB_{EIF}$

iii) $n = 1, m \geq 2$: $\begin{cases} CB_{EIFgo} = CB_{EKF}, m = 2 \\ CB_{EIFgo} < CB_{EKF}, m \geq 3 \end{cases}$

iv) $n \geq 2, m \geq 2$: $CB_{EKF} - CB_{EIFgo} = n\left\{m^2 + 2m(n-1) - \frac{1}{3}(16n^2 + 9n - 7)\right\}$

Then $CB_{EIFgo} < CB_{EKF}$ when

$$m > \frac{-6(n-1) + \sqrt{228n^2 + 36n - 48}}{6} \tag{8}$$

Hence, for multidimensional systems, where $n \geq 2, m \geq 2$, the following rule is derived:

$$if\ m > \frac{-6(n-1)+\sqrt{228n^2+36n-48}}{6}\ then\ EIFgo\ is\ faster\ than\ EKF$$
$$else\ EKF\ is\ faster\ than\ EIFgo$$

For multidimensional systems, where $n \geq 2, m \geq 2$, the areas depending on the system dimensions, where EKF or the EIF is faster, are shown in Figure 1.

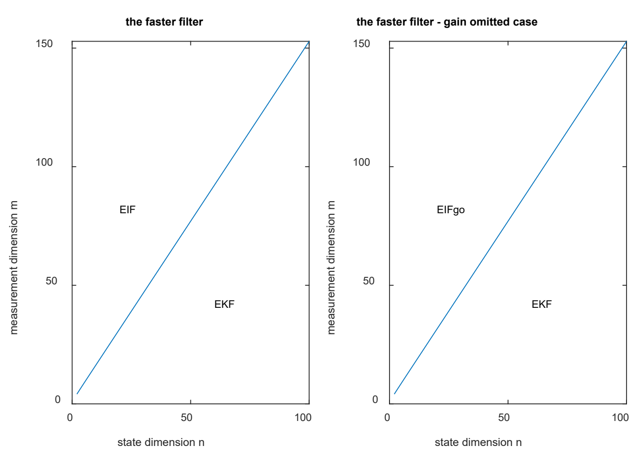

Fig. 1: EKF vs EIF: selection of the faster filter for multidimensional systems.

4 Conclusions

Nonlinear estimation plays a significant role in many fields of science and engineering. The computational requirements of Extended Kalman filter and Extended Information filter are determined. A method is proposed to a-priori (before the filters' implementation) decide which filter is the faster one. The decision is based on the knowledge of the state and measurement dimensions. The potential advantage of Extended Information filter was shown due to the fact that

Extended Information filter may be faster than Extended Kalman filter, depending on the known state and measurement dimensions.

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