

Stock Price Prediction Using Kalman Filter

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Abstract - In this work we study the applicability of Kalman filters in stock prices prediction using the actual observations of stock prices. We investigate the behavior of two state space models where the acceleration or the velocity of the stock price is considered as a zero mean white noise sequence, due to the high fluctuation of the stock market. We propose time varying, time invariant, steady state and Finite Impulse Response form of steady state Kalman filters for each model. We deal with short term prediction, namely daily prediction. The proposed Kalman filters are implemented using historical data of stock price. It was found that the proposed Kalman filters produce reliable predictions. The percent mean absolute error may vary by model and filter; some filters give satisfactory results where the percent mean absolute error in stock price prediction is less than 2%. Furthermore, some filters present relative error less than 1% for 35%-50% of predictions. Finally, the average percent profit can reach 3.5% using the proposed Kalman filters.

Keywords: prediction, Kalman filter, steady state, stock price

1. Introduction

Forecasting deals with the prediction of future values of variables of interest. In the case where the measurements of these variables are collected over time (at regular time intervals), the problem is referred to as time-series forecasting. Forecasting has been applied in a wide range of applications: Operational Research [1], business and economic forecasting [2], supply-chain inventory management [3], economics and finance [4].

The economic and financial time series have attracted attention of scientists, researchers and scholars over the past decades. The analysis and prediction of the change of financial time series can be used in trusty management and decision making for relevant public services and investors. The stock is a financial time series of enormous interesting mainly for investors. The prediction of stock price is of great importance due to the randomness and uncertainty of the stock market.

The financial markets consider the stock price forecast problem from two points of view: the fundamental analysis, which takes into account the underlying factors that affect the actual business and the future prospects of the companies and the technical analysis, which takes into account the stock price movement.

The stock market includes individuals and companies participating in a network of buying and selling stocks. The buying and selling of stocks have high risk, and hence predictions are needed to avoid losses and make profits. Time series analysis is the fundamental method to provide the forecasting.

A common and simple method is to use a Moving Average (MA) model [5]. A popular method implemented in time series analysis is the Autoregressive Integrated Moving Average (ARIMA) model [6]. Various ARIMA models have been used depend on the stock market [6]. Variations of the ARIMA model use clustering time series [7], fuzzy neural networks [8], support vector model [9], ARIMA and Holt Winter time series algorithms [10].

State estimation uses observable data to estimate the unobservable system states. A popular algorithm for this purpose is the well-known Kalman filter [11], [12]. Kalman filter has been used in the prediction of stock price [13], [14].

The novelty of this work concerns: (a) the description of two observable state space models, where the stock price is viewed as a maneuvering system, (b) the design of time varying Kalman filters, where the noise parameters are time varying, (c) the design of time invariant Kalman filters and prediction steady state Kalman filters, where the prediction error covariance is a priori known, (d) the design of the Finite Impulse Response (FIR) form of the prediction steady state Kalman filters, which do not require all the previous observations.

2. State space models

In this section we present two state space models, where the stock price is viewed as a maneuvering system due to the high fluctuation of the stock market.

2.1. Model A – random stock price acceleration

The state consists from two ($n_A = 2$) elements: the stock price $p(k)$ and the rate $r(k)$ of change of the stock price: $x(k) = \begin{bmatrix} p(k) \\ r(k) \end{bmatrix}$. The measurement has one element ($m_A = 1$), the stock price. Due to the high fluctuation of the stock

market, the acceleration $a(k)$ of the stock price can be considered as a zero mean white noise sequence [13], [14]. Then

$$p(k+1) = p(k) + r(k) \cdot T + \frac{1}{2} \cdot a(k) \cdot T^2$$

$$r(k+1) = r(k) + a(k) \cdot T$$

where T is the sampling period. We adopt the value $T = 1$ proposed in [14].

The linear state space model becomes:

$$x(k+1) = F \cdot x(k) + w(k) \quad (1)$$

$$z(k) = H \cdot x(k) + v(k) \quad (2)$$

For the dynamic part of the model we have: $F = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}$, $H = [1 \ 0]$, $w(k) = \Gamma \cdot a(k)$, $\Gamma = \begin{bmatrix} \frac{1}{2} \cdot T^2 \\ T \end{bmatrix}$. It is worth to note that the model is observable, i.e. all the states can be uniquely determining from the observations. In fact the $(m_A \cdot n_A) \times n_A$ observability matrix $OM = \begin{bmatrix} 1 & 0 \\ 1 & T \end{bmatrix}$ satisfies the observability criterion: $\text{rank}(OM) = n_A$, since $\text{rank}(OM) = 2$.

For the statistical part of the model we have: The acceleration $a(k)$ of the stock price can be regarded as a zero mean Gaussian noise process with variance $q_a(k) > 0$. Then the state noise $w(k)$ is a zero mean Gaussian process with known

covariance $Q(k) = \Gamma \cdot q_a(k) \cdot \Gamma^T = q_a(k) \cdot \begin{bmatrix} \frac{1}{4} \cdot T^4 & \frac{1}{2} T^3 \\ \frac{1}{2} T^3 & T^2 \end{bmatrix}$, where Γ^T denotes the transpose of Γ . The measurement noise

$v(k)$ is a zero mean Gaussian noise process with covariance $R(k)$. The initial state $x(0)$ is a Gaussian random variable with mean x_0 and covariance P_0 . The model A can be time invariant when the state noise covariance and the measurement noise covariance are constant: $Q(k) = Q$ (with $q_a(k) = q_a$) and $R(k) = R$.

2.2. Model B – random stock price velocity

The state consists from one ($n_B = 1$) element: the stock price $p(k)$. The measurement has one element ($m_B = 1$), the stock price. The velocity $v(k)$ of the stock price can be considered as a zero mean white noise sequence. Then

$$p(k+1) = p(k) + v(k) \cdot T$$

where T is the sampling period. We adopt the value $T = 1$ proposed in [14].

The linear state space model becomes:

$$x(k+1) = F \cdot x(k) + w(k) \quad (3)$$

$$z(k) = H \cdot x(k) + v(k) \quad (4)$$

For the dynamic part of the model we have: $F = 1$, $H = 1$, $w(k) = \Gamma \cdot v(k)$. It is worth to note that the model is observable, i.e. all the states can be uniquely determining from the observations. In fact the $(m_B \cdot n_B) \times n_B$ observability matrix $OM = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ satisfies the observability criterion: $\text{rank}(OM) = n_B$, since $\text{rank}(OM) = 1$.

For the statistical part of the model we have: The velocity $v(k)$ of the stock price can be regarded as a zero mean Gaussian noise process with variance $q_v(k) > 0$. Then $w(k)$ is a zero mean Gaussian process with known covariance $Q(k) = T \cdot q_v(k) \cdot T^T = q_v(k) \cdot T^2$. $v(k)$ is a zero mean Gaussian noise process with covariance $R(k)$. The initial state $x(0)$ is a Gaussian random variable with mean x_0 and covariance P_0 . The model B can be time invariant when the state noise covariance and the measurement noise covariance are constant: $Q(k) = Q$ (with $q_v(k) = q_v$) and $R(k) = R$.

3. Kalman filters

Both the models described above, can be considered for short term prediction; in fact by setting $T = 1$, as proposed in [14], we are able to use observations collecting at the end of every day (closing prices) in order to predict the stock price of the next day. Kalman filter [11], [12] computes the state estimation $x(k|k)$ and the estimation error covariance $P(k|k)$ as well as the state prediction $x(k+1|k)$ and the prediction error covariance $P(k+1|k)$, using the Kalman filter gain $K(k)$.

3.1. Time Varying Kalman Filters

In both models, the transition matrix F and the output matrix H are constant, but the state noise covariance $Q(k)$ and the measurement noise covariance $R(k)$ are time varying. Then, the time varying Kalman filter algorithm becomes:

<p><i>Time Varying Kalman Filter (TVKF)</i></p> $K(k) = P(k k-1) \cdot H^T(k) \cdot [H(k) \cdot P(k k-1) \cdot H^T(k) + R(k)]^{-1}$ $x(k k) = [I - K(k) \cdot H(k)] \cdot x(k k-1) + K(k) \cdot z(k)$ $P(k k) = [I - K(k) \cdot H(k)] \cdot P(k k-1)$ $x(k+1 k) = F(k) \cdot x(k k)$ $P(k+1 k) = Q(k) + F(k) \cdot P(k k) \cdot F^T(k)$ <p>for $k = 0, 1, \dots$, with initial conditions $x(0 -1) = x_0, P(0 -1) = P_0$</p>
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Model A. The measurement noise covariance is time varying: in fact $R(k)$ is the variance of N last observations (prices). A reasonable choice is to set $N = 5$ in order to take into account the last week period observations. If less than N observations are available, then $R(k)$ is the variance of the available observations. The state noise covariance is time varying: $Q(k)$ depends on $q_a(k)$, which is the acceleration variance of N last rate differences (successive rate differences). A reasonable choice is to set $N = 5$ in order to take into account the last week period observations. If less than N observations are available, then $Q(k)$ takes into account the available observations.

Model B. The measurement noise covariance is time varying: in fact $R(k)$ is the variance of N last observations (prices). A reasonable choice is to set $N = 5$ in order to take into account the last week period observations. If less than N observations are available, then $R(k)$ is the variance of the available observations. The state noise covariance is time varying: $Q(k)$ depends on $q_v(k)$, which is the velocity variance of N last rate differences (successive rate differences). A reasonable choice is to set $N = 5$ in order to take into account the last week period observations. If less than N observations are available, then $Q(k)$ takes into account the available observations.

3.2. Time Invariant Kalman Filters

In both models, the transition matrix F and the output matrix H are constant. In addition, the acceleration variance q_a of model A and the velocity variance q_v of model B are constant. Then, the state and measurement noise covariances are time invariant: $Q(k) = Q$ and $R(k) = R$. Then, the time invariant Kalman filter algorithm becomes:

<p><i>Time Invariant Kalman Filter (TIKF)</i></p> $K(k) = P(k k-1) \cdot H^T \cdot [H \cdot P(k k-1) \cdot H^T + R]^{-1}$ $x(k k) = [I - K(k) \cdot H] \cdot x(k k-1) + K(k) \cdot z(k)$ $P(k k) = [I - K(k) \cdot H] \cdot P(k k-1)$ $x(k+1 k) = F \cdot x(k k)$ $P(k+1 k) = Q + F \cdot P(k k) \cdot F^T$ <p>for $k = 0, 1, \dots$, with initial conditions $x(0 -1) = x_0, P(0 -1) = P_0$</p>
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Model A. The constant state noise covariance Q can be determined setting $T = 1$, as proposed in [14] and setting the acceleration variance q_a as the mean of the acceleration variance of a previous period (one year, for example). We are able to adopt the value $q_a = 1$ proposed in [14], or setting $q_a = 0.1$. Then $Q = q_a \cdot \begin{bmatrix} \frac{1}{4} \cdot T^4 & \frac{1}{2} T^3 \\ \frac{1}{2} T^3 & T^2 \end{bmatrix} = q_a \cdot \begin{bmatrix} \frac{1}{4} & \frac{1}{2} \\ \frac{1}{2} & 1 \end{bmatrix}$. The constant measurement noise covariance R can be determined as the mean of the stock price variance of a previous period (one year, for example). We are able to adopt the value $R = 1$ proposed in [14], or setting $R = 0.1$.

Model B. The constant state noise covariance Q can be determined setting $T = 1$, as proposed in [14] and setting the velocity variance q_v as the mean of the velocity variance of a previous period (one year, for example). We are able to adopt the value $q_v = 1$ proposed in [14], or setting $q_v = 0.1$. Then $Q = q_v \cdot T^2 = q_v$. The constant measurement noise covariance R can be determined as the mean of the stock price variance of a previous period (one year, for example). We are able to adopt the value $R = 1$ proposed in [14], or setting $R = 0.1$.

3.3. Prediction Steady State Kalman Filters

We consider the time invariant models and the steady state case where the estimation error covariance, the prediction error covariance and the Kalman filter gain remain constant. In fact, the steady state prediction error covariance P_p satisfies the algebraic Riccati Equation:

$$P_p = Q + F \cdot P_p \cdot F^T - F \cdot P_p \cdot H^T \cdot [H \cdot P_p \cdot H^T + R]^{-1} \cdot H \cdot P_p \cdot F^T \quad (5)$$

Then the steady state Kalman filter gain K is:

$$K = P_p \cdot H^T \cdot [H \cdot P_p \cdot H^T + R]^{-1} \quad (6)$$

Combining the time invariant Kalman filter equations and assuming the steady state Kalman filter gain K , the Prediction Steady State Kalman Filter is:

Prediction Steady State Kalman Filter (PSSKF)
 $x(k+1|k) = C_x \cdot x(k|k-1) + C_z \cdot z(k)$
for $k = 0, 1, \dots$, with initial condition $x(0|-1) = x_0$
where $C_x = F \cdot [I - K \cdot H]$, $C_z = F \cdot K$

It is worth to note that the coefficients C_x, C_z are a-priori computed, i.e. before the filter implementation, by first off-line solving the corresponding algebraic Riccati Equation [12]. It is also important to note that the state prediction computation requires the current observation and the previous state prediction.

Model A. In the case where $T = 1, q_a = 1, R = 1$ as proposed in [14], the steady state prediction error covariance is: $P_p = \begin{bmatrix} 3 & 2 \\ 2 & 2 \end{bmatrix}$. Then, the steady state Kalman filter gain K is: $K = P_p \cdot H^T \cdot [H \cdot P_p \cdot H^T + R]^{-1} = \begin{bmatrix} 0.75 \\ 0.50 \end{bmatrix}$. The prediction steady state Kalman filter coefficients are: $C_x = F \cdot [I - K \cdot H] = \begin{bmatrix} -0.25 & 1 \\ -0.50 & 1 \end{bmatrix}$, $C_z = F \cdot K = \begin{bmatrix} 1.25 \\ 0.50 \end{bmatrix}$

Model B. In the case where $T = 1, q_v = 1, R = 1$ as proposed in [14], the steady state prediction error covariance is: $P_p = \frac{\sqrt{5}+1}{2}$. Then, the steady state Kalman filter gain K is: $K = P_p \cdot [P_p + 1]^{-1} = \frac{\sqrt{5}-1}{2}$. The prediction steady state Kalman filter coefficients are: $C_x = [1 - K] = \frac{3-\sqrt{5}}{2} = 0.3820$, $C_z = K = \frac{\sqrt{5}-1}{2} = 0.6180$.

It is worth to note that: $P_p = \frac{\sqrt{5}+1}{2} = 1.6180 = \text{goldensection}$, $C_z = \frac{\sqrt{5}-1}{2} = 0.6180 = \text{goldensection} - 1$, $C_x = 1 - C_z$.

3.4. FIR form of Prediction Steady State Kalman Filters

From the prediction equation of the Prediction Steady State Kalman Filter we get:

$$\begin{aligned} x(1|0) &= C_x \cdot x(0|-1) + C_z \cdot z(0) = C_x \cdot x_0 + C_z \cdot z(0) \\ x(2|1) &= C_x \cdot x(1|0) + C_z \cdot z(1) = C_x^2 \cdot x_0 + C_x \cdot C_z \cdot z(0) + C_z \cdot z(1) \\ &\dots \\ x(k+1|k) &= C_x^{k+1} \cdot x_0 + C_x^k \cdot C_z \cdot z(1) + \dots + C_z \cdot z(k) \end{aligned}$$

It is known [15] that if the spectral radius $\rho(M)$ of a matrix M is less than 1, then the powers of the matrix can be expected to converge to zero. Then, there exists a positive integer L , such that $\|M^L\| < \varepsilon$, where ε is the convergence criterion and $\|M\|$ denotes the norm-2 of M . Hence, due to computational accuracy, there exists some positive integer L , such that: $M^L \neq 0, M^{L+1} = 0$.

If the coefficient $C_x = F \cdot [I - K \cdot H]$ has this property, then $x(L+1|L) = C_x^{L+1} \cdot x_0 + C_x^L \cdot C_z \cdot z(1) + \dots + C_z \cdot z(L)$ and $x(L+1|L) = C_x^L \cdot C_z \cdot z(1) + \dots + C_z \cdot z(L)$

$$x(L+2|L+1) = C_x^L \cdot C_z \cdot z(2) + \dots + C_z \cdot z(L+1)$$

...

$$x(k+1|k) = C_x^L \cdot C_z \cdot z(k-L) + \dots + C_z \cdot z(k)$$

Assuming that $z(k) = 0, k < 0$, the Finite Impulse Response (FIR) form of the prediction steady state Kalman filter is:

FIR Prediction Steady State Kalman Filter (FIRPSSKF)

$$x(k+1|k) = \sum_{i=0}^L C_x^i \cdot C_z \cdot z(k-i)$$

for $k = 1, 2, \dots$

$$\text{where } C_x = F \cdot [I - K \cdot H], C_z = F \cdot K \text{ and } \|C_x^L\| \geq \varepsilon, \|C_x^{L+1}\| < \varepsilon$$

It is worth to note that the computation of the prediction is not done iteratively, since the knowledge of the previous estimate is not required; in fact the computation of the prediction requires the knowledge of the last $L+1$ observations. It is obvious that L depends on the desired convergence criterion ε . A reasonable choice is to set $\varepsilon = 10^{-2}$ since the stock prices appear with two decimal places.

Model A. From the model A parameters we get $F = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}, H = [1 \ 0], Q = q_a \cdot \begin{bmatrix} \frac{1}{4} \cdot T^4 & \frac{1}{2} T^3 \\ \frac{1}{2} T^3 & T^2 \end{bmatrix}, q_a > 0, R > 0$. In the case where $T = 1, q_a = 1, R = 1$ as proposed in [14], the coefficient $C_x = \begin{bmatrix} -0.25 & 1 \\ -0.50 & 1 \end{bmatrix}$ has the property $\rho(C_x) = 0.5 < 1$. Then $L = 7$ for $\varepsilon = 10^{-2}$.

Model B. From the model B parameters we get $C_x = F \cdot [I - K \cdot H] = 1 - K = 1 - \frac{P_p}{P_p + R} = \frac{R}{P_p + R}$. Consider that P_p and R are positive definite, as covariances, without loss of generality. Then $0 < C_x < 1$ and $\rho(C_x) = C_x$. Hence, C_x has always the property $\rho(C_x) < 1$. In the case where $T = 1, q_v = 1, R = 1$ as proposed in [14], the coefficient $C_x = \frac{3-\sqrt{5}}{2} = 0.3820$ has the property $\rho(C_x) = \frac{3-\sqrt{5}}{2} = 0.3820 < 1$. Then $L = 4$ for $\varepsilon = 10^{-2}$.

Table 1 summarizes the proposed state space models and Kalman filters for stock price.

Table 1: State space models and Kalman filters for stock price prediction.

model	Kalman Filter	F	H	Q	R
model A	TVKF-A	$F = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}$	$H = [1 \ 0]$	$Q(k) = q_a(k) \cdot \begin{bmatrix} \frac{1}{4} \cdot T^4 & \frac{1}{2} T^3 \\ \frac{1}{2} T^3 & T^2 \end{bmatrix}$	$R(k)$
	TIKF-A, PSSKF-A, FIRPSSKF-A	$F = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}$	$H = [1 \ 0]$	$Q = q_a \cdot \begin{bmatrix} \frac{1}{4} \cdot T^4 & \frac{1}{2} T^3 \\ \frac{1}{2} T^3 & T^2 \end{bmatrix}$	R
model B	TVKF-B	$F = 1$	$H = 1$	$Q(k) = q_v(k) \cdot T^2$	$R(k)$
	TIKF-B, PSSKF-B, FIRPSSKF-B	$F = 1$	$H = 1$	$Q = q_v \cdot T^2$	R

4. Simulation results

The experimental data used concerns:

(a) data of the order of one month: $N = 27$ stock closing price historical data (September 22, 2014 to November 4, 2014) of the stock Changbaishan taken from [14],

(b) data of the order of one year: $N = 250$ stock closing price historical data (January 3, 2023 to December 29, 2023) of the stock Ford Motor Company (F) taken from [16].

The stock price observations are: $z(0), z(1), \dots, z(N-1)$. The first observation $z(0)$ is used for the initial condition $x(0|-1)$ determination. Then, the Kalman filter predictions $x(1|0), x(2|1), x(3|2), \dots, x(N-1|N-2)$ are compared to the rest of the observations $z(1), \dots, z(N-1)$.

All the proposed Kalman filters (time varying, time invariant, prediction steady state and FIR form of prediction steady state Kalman filters) were implemented for both models A and B, with sampling period: $T = 1$. Time varying models take into account the last 5 observations in order to determine (a) the time varying measurement noise covariance and (b) the time varying state noise covariance, using the acceleration variance of model A and the velocity variance of model B. Time invariant models use the constant measurement noise covariance: $R = 1$ and the constant state noise covariance Q , using the acceleration variance $q_a = 1$ of model A and the velocity variance $q_v = 1$ of model B. The initial conditions are $x_0 = \begin{bmatrix} z(0) \\ 0 \end{bmatrix}$, $P_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ for model A and $x_0 = z(0)$, $P_0 = 1$ for model B.

4.1. Prediction

Figure 1 depicts the stock price (observations), as well as the daily stock price prediction using the Prediction Steady State Kalman Filter for both models A and B, for month and year data.

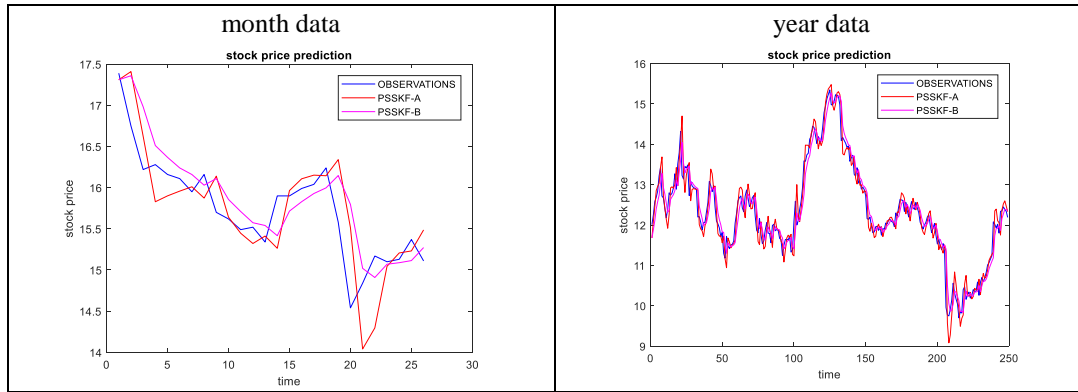


Fig. 1: Stock price prediction using PSSKF.

4.2. Prediction errors

The following error metrics were calculated for the daily stock price prediction using all the proposed filters:

$$\text{Mean Bias Error (MBE)} = \frac{1}{N} \sum_{k=1}^N e(k) \quad (7)$$

$$\% \text{ Mean Absolute Error (\%MAE)} = \frac{1}{N} \sum_{k=1}^N \frac{|e(k)|}{z(k)} \cdot 100 \quad (8)$$

$$\% \text{ Root Mean Squared Error (\%RMSE)} = \sqrt{\frac{1}{N} \sum_{k=1}^N \frac{e^2(k)}{z^2(k)}} \cdot 100 \quad (9)$$

where $e(k) = x(k+1|k) - z(k)$ and N is the number of observations.

Table 2 presents MBE, %MAE, %RMSE for the daily stock price prediction for month and year data.

Table 2: Prediction errors using TVKF, TIKF, PSSKF, FIRPSSKF.

Kalman Filter	month data			year data		
	MBE	%MAE	%RMSE	MBE	%MAE	%RMSE
TVKF-A	0.0503	2.5993	3.3173	0.0115	2.0716	2.8745
TIKF-A	0.0013	2.0258	2.7839	0.0011	1.9559	2.6884
PSSKF-A	0.0028	2.0296	2.7878	0.0009	1.9551	2.6879
FIRPSSKF-A	0.5901	6.1394	12.7158	-0.0354	2.4207	4.3257
TVKF-B	0.2305	2.3885	3.1308	-0.0077	2.1854	3.0414
TIKF-B	0.1331	1.8258	2.5269	-0.0038	1.8479	2.4997
PSSKF-B	0.1331	1.8256	2.5268	-0.0037	1.8475	2.4992
FIRPSSKF-B	-0.3806	3.4005	8.1679	-0.1306	2.2002	3.8019

4.3. Prediction statistics

Table 3 presents the percent Mean Absolute Error (%MAE) statistics for the daily stock price prediction using all the proposed Kalman filters; namely the percent prediction cases with respect to the error ranges, for month and year data.

Table 3: Percent Mean Absolute Error (%MAE) statistics.

Kalman Filter	month data					year data				
	[0-1)	[1-2)	[2-3)	[3-4)	>=4	[0-1)	[1-2)	[2-3)	[3-4)	>=4
TVKF-A	30.7692	19.2308	7.6923	15.3846	26.9231	35.7430	24.0964	19.2771	7.6305	13.2530
TIKF-A	50.0000	11.5385	15.3846	3.8462	19.2308	34.1365	31.7269	14.8594	9.2369	10.0402
PSSKF-A	50.0000	11.5385	15.3846	3.8462	19.2308	34.1365	31.7269	14.8594	9.2369	10.0402
FIRPSSKF-A	23.0769	26.9231	11.5385	3.8462	34.6154	33.7349	23.2932	20.8835	8.4337	13.6546
TVKF-B	23.0769	38.4615	11.5385	15.3846	11.5385	35.7430	21.2851	17.2691	10.4418	15.2610
TIKF-B	26.9231	50.0000	3.8462	11.5358	7.6923	36.9478	24.8996	21.2851	8.4337	8.4337
PSSKF-B	26.9231	50.0000	3.8462	11.5358	7.6923	36.9478	24.8996	21.2851	8.4337	8.4337
FIRPSSKF-B	38.4615	30.7692	15.3846	3.8462	11.5385	29.7189	32.9317	16.4659	8.8353	12.0482

4.4. Profits/Losses

Table 4 presents the percent profits/losses statistics for the daily stock price prediction using all the proposed Kalman filters, for month and year data.

Table 4: Daily percent profits/losses.

Kalman Filter	month data		year data	
	max	mean	max	mean
TVKF-A	7.4881	0.3023	13.0947	0.0988
TIKF-A	6.7000	0.0016	13.8379	0.0089
PSSKF-A	6.7000	0.0210	13.8379	0.0076
FIRPSSKF-A	49.1530	3.4892	40.4286	-0.2880
TVKF-B	9.2148	1.4932	15.8240	0.0090
TIKF-B	8.6432	0.8632	14.5501	0.0111
PSSKF-B	8.6432	0.8630	14.5501	0.0114
FIRPSSKF-B	7.7644	-2.2073	13.5991	-1.0233

5. Conclusions

Kalman filters are applicable in stock prices prediction using the actual observations of stock prices. Two state space models are proposed, where the acceleration or the velocity of the stock price is considered as a zero mean white noise sequence. The models are fully defined and observable. For each model, the following types of Kalman Filter (KF) are derived: time varying KF, time invariant KF, steady state KF, FIR form of steady state KF. The FIR form of the steady state KF requires the knowledge only of a well-defined subset of previous time observations to calculate the prediction.

The proposed Kalman filters are easily programmable and do not require training (as methods using ARIMA models), since they do not depend on the type of stock or company. They could be combined with other methods for correcting and improving the prediction results.

The proposed Kalman filters were applied for short term prediction, namely daily prediction. The proposed Kalman filters are implemented using historical data of stock price. It was found that the proposed Kalman filters produce reliable predictions and have a satisfactory behavior. The percent mean absolute error may vary by model and filter; some Kalman filters give satisfactory results where the percent mean absolute error in stock price prediction is less than of 2%. Furthermore, some Kalman filters present relative error less than 1% for 35%-50% of predictions. Finally, the average percent profit can reach 3.5% using the proposed Kalman filters.

A subject of future work may be the very short term prediction, for example prediction per hour with respect to the sampling period of observations collection. Another subject of future work may be the long term prediction, for example average prediction per one week period (5 days); we propose to process blocks of measurements of time window corresponding for example to one week period in order to predict the average of stock price.

Nomenclature list

F	transition matrix
H	output matrix
P _p	steady state prediction error covariance
Q	state noise covariance
R	measurement noise covariance
v	measurement noise
w	state noise
x	state
z	measurement

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