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Comparison between the Functional Mean and the Deepest Curve in Biomechanical Curves

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***Abstract* -** Functional Data Analysis (FDA) is a branch of statistics that extends the concepts of classical statistics to functions. However, as with classical statistics, where sometimes the mean of a data set is not very representative, the functional mean may not be always the best option in FDA. This is because if the curves are not similar or aligned, this mean may not be very representative. Therefore, the concept of the deepest curve, similar to a functional median, is proposed in this submission. So, we want to reflect on the convenience of using a functional mean or the deepest curve and on which occasions it may be more acceptable to use one versus the other. The analysis is done on a set of data from Biomechanics, where in the literature reviewed, no works have been found that previously discuss this issue. A database of 56 subjects is used to measure the three components of the ground reaction force. Each subject performs six repetitions, so the mean and the deepest curve of these repetitions will be calculated to obtain a representative curve for each subject. Then, starting from these curves, the mean and deepest curves of the entire sample are calculated. In this way, they can be compared visually to find differences. Also, some characteristic values of the curves are calculated for each subject, and a paired t-test is performed to analyze whether there are significant differences between the mean and the deepest curve. This submission concludes that if the subsequent statistical analysis only considers values as maximum or minimum, the use of the mean or the deepest curve is analogous. However, if the entire curve is going to be used, as in FDA, then the deepest curve may be more advisable, especially if the curves present much variability.

***Keywords*:** Functional Data Analysis (FDA), deepest curve, mean curve, biomechanics, Ground Reaction Forces

1. Introduction

Functional Data Analysis (FDA) [1],[2] is a branch of statistics that allows working with continuous functions, offering interesting applications in analyzing human movements or physiological signals [3]. When describing the central values of a set of curves, it is necessary to extend the concepts of mean and median in the functional context. Here, some problems appear that have not been sufficiently studied. In fact, unlike what happens with the mean of a numerical variable, the functional mean corresponds to a curve that can suffer alterations concerning the patterns of the original curves so that it corresponds to an unreal observation, even incompatible with the means of other related variables. This is due to the smoothing and cancellation effect produced by the lags between the curves of the sample, especially in the cases of signals with much variability. Thus, looking for another mathematical concept that allows for obtaining a representative curve when these are very variable between subjects, such as the median in discrete data, is necessary. The equivalent of this functional median is the concept of a deep curve. The concept of deep curve is developed in more detail in [4], [5], [6].

In this submission, we want to compare the functional mean and the deep curve obtained from a set of curves in Biomechanics, specifically, the ground reaction forces of healthy subjects and patients with early knee osteoarthritis with pain. From the literature reviewed in Biomechanics, no works have been found where the deepest curve is used instead of the functional mean. There is only one work in which the deepest curves are used in the BMI measured in children from 2 to 18 years old [7]. Here, they conclude that the concept of the deepest curve helps order curves and detect outliers. However, what is proposed in this submission is to reflect on the extended use of the functional mean since, depending on the type of curves, this mean may not be representative, and it is more appropriate to use the deepest curve since this does consider an actual curve.

2. Methods

The data are part of the OACTIVE H2020 project conducted by the Department of Physical Medicine and Rehabilitation of the Hospital La Fe in Valencia (Spain). The sample comprises 56 subjects (31 healthy and 25 pathological) in whom the ground reaction forces of gait are measured. Table 1 shows the mean and standard deviation by sex for the total sample for different individual characteristics.

Table 1: Mean and standard deviation of age, mass and height by sex for the total sample.

Group	Age (years) mean (sd)	Mass (kg) mean (sd)	Height (mm) mean (sd)
Woman (N=35)	53.4 (6.4)	71.8 (12.8)	1608.6 (55.9)
Men (N=21)	52 (5.1)	81.2 (10.8)	1730.2 (51.5)

All biomechanical measurements were carried out at the Institute of Biomechanics of Valencia using force platforms (Dinascan/IBV). Each trial consisted of walking five steps at a comfortable speed and stepping on one of the platforms. Each subject performs six repetitions with each leg; however, for this work, the data of only one leg is considered, and the left one is chosen randomly.

Thus, the three components of ground reaction force are measured at a sampling frequency of 100 Hz: anteroposterior F_x, vertical F_y, and mediolateral F_z. The raw curves are normalized linearly in time to express them from 0 to 100% of stance time. The amplitude of the forces is normalized by dividing by the weight of each subject. In addition, since the anteroposterior and mediolateral forces are small, the three forces are multiplied by 100 for better visualization.

Since each subject has six repetitions, a single representative curve for each subject is obtained. On the one hand, the mean curve is obtained, and on the other hand, the deepest curve of these six repetitions. Subsequently, the mean curve and the deepest curve of the 56 curves previously obtained are calculated. The depth function used is the h-mode method.

In order not only to visually compare these mean and the deepest curves, specific characteristic values of the forces are compared for the deepest and mean curves. Thus, the minimum and maximum values chosen for each force are those shown in Figure 1. In this way, the maximum and minimum for each force are extracted for the 56 mean and the deepest curves, and a paired t-test is performed to see if there are significant differences between both curves.

3. Results

Figure 1 shows all the curves of the three force components for the 56 subjects (grey) and the functional mean (black). Also in red are the maximums and minimums chosen for each force.

Table 2 shows the mean values and standard deviation of the maximums and minimums obtained from the mean function and the deepest curve marked in Figure 1 for the sample of 56 subjects. A paired t-test is performed with these maximums and minimums, obtaining the p-values shown in Table 2. Significant differences are only found between the mean and the deepest curves in mediolateral force, with a p-value for the maximum values of 0.02, while for the minimum, it is 0.08.

Finally, Figure 2 shows the anteroposterior, vertical, and mediolateral force's mean (red) and the deepest curve (black). The anteroposterior force is where fewer differences are observed between both curves. Subsequently, in the vertical force, notable differences are observed in the first maximum and slightly more in the minimum. Finally, the differences between the mean and the deepest curve are most evident in the mediolateral force.

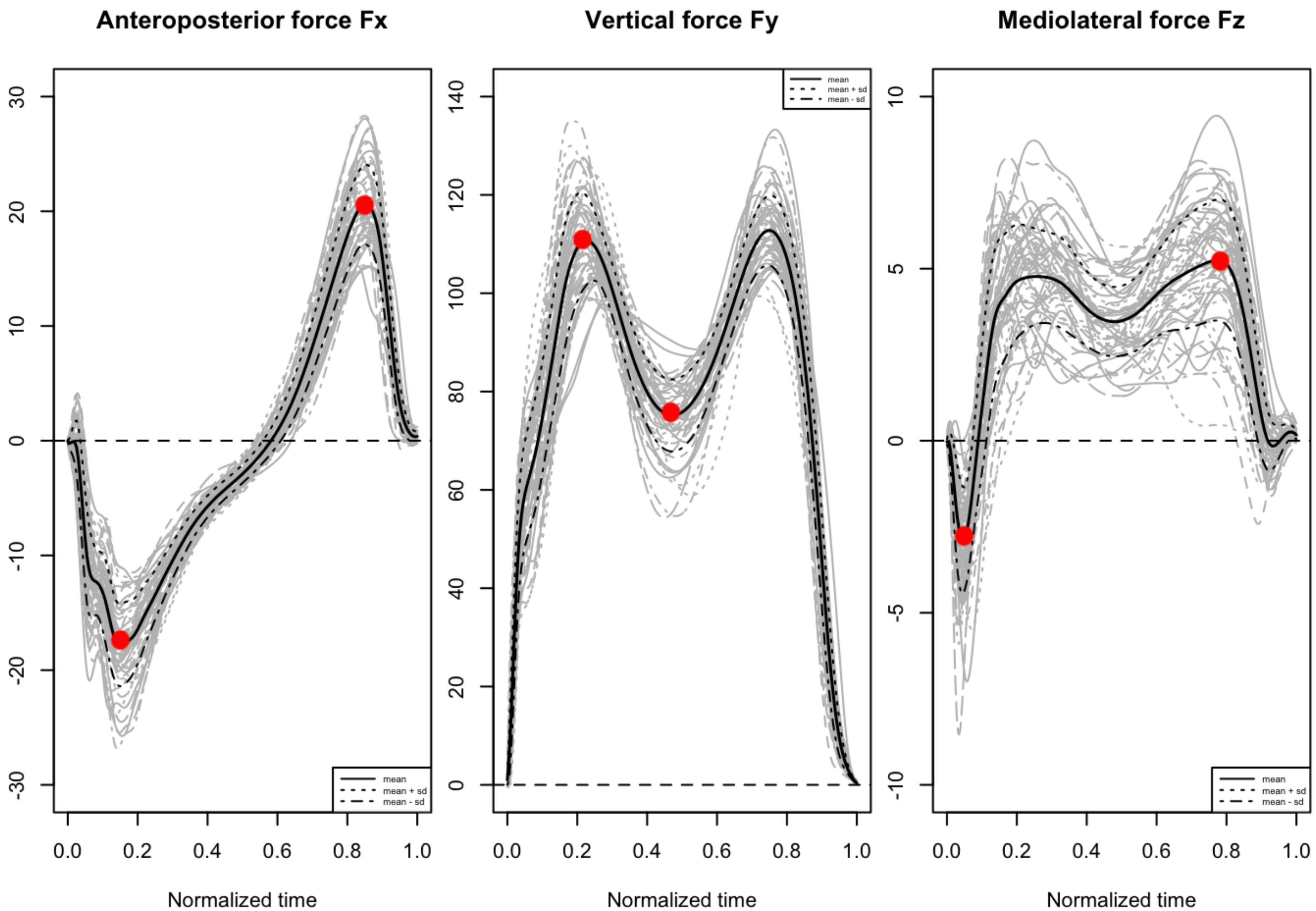

Fig. 1: Anteroposterior (left), vertical (middle), and mediolateral (right) force curves of the entire sample (grey), together with the mean plus or minus one standard deviation. The red dots show the maximums and minimums considered in each curve to make a comparison between the mean and the deepest curve.

Table 2: Mean and standard deviation of the maximum and minimum values of the mean and deepest curve for each subject, obtained from the 6 repetitions available for each subject. The table also shows the p-value of the paired t-test.

FORCE	MAXIMUM VALUE			MINIMUM VALUE		
	Functional mean mean (sd)	Deepest curve mean (sd)	p-value paired t-test	Functional mean mean (sd)	Deepest curve mean (sd)	p-value paired t-test
Anteroposterior F_x	20.8 (3.4)	20.8 (3.6)	0.866	-18 (3.6)	-18.1 (3.5)	0.473
Vertical F_y	112.6 (8.6)	113.1 (8.8)	0.224	74.3 (7.1)	74 (7.2)	0.197
Mediolateral F_z	5.6 (1.5)	5.8 (1.5)	0.02	-3.3 (1.7)	-3.5 (1.9)	0.08

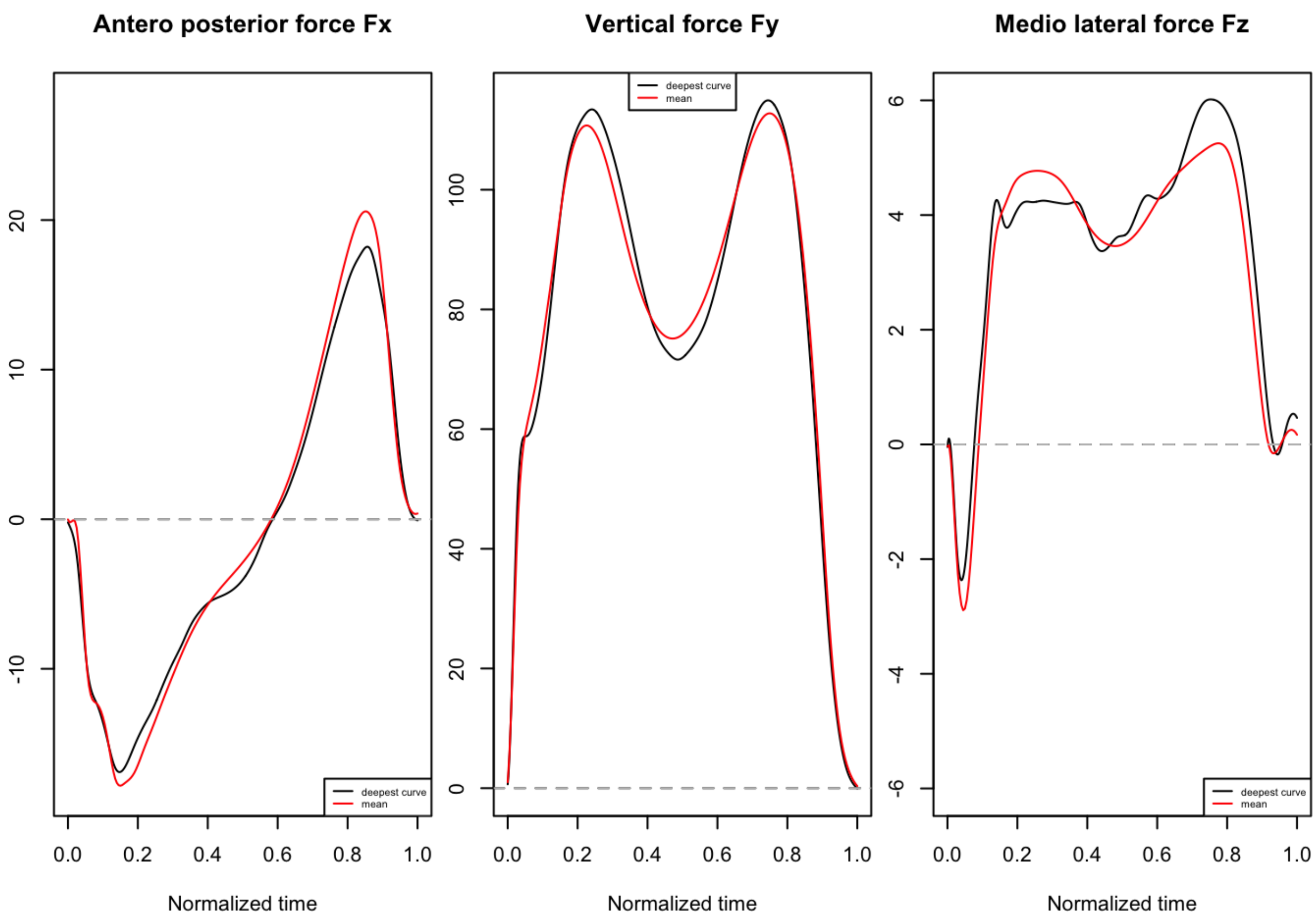

Fig. 2: Functional mean (red) and deepest curve (black) of the sample for anteroposterior (left), vertical (middle) and mediolateral (right) force.

4. Discussion

Visually, it can be observed in Figure 2 how the deepest curve is so different from the mean curve in mediolateral force. These differences are not as evident in the other two forces, where, although the amplitude of the extremes may vary, as in the vertical force, the shape of the curve is similar. This significant difference in mediolateral force may be due to the great variability that this force presents, which may result in a fictitious curve that is not real for any subject when considering the mean function.

In any case, the differences between maximums and minimums of the curve for each subject are not as evident, as can be seen from the p-values of the paired t-test in Table 2, except again for mediolateral force. Therefore, the conclusion drawn from this submission is that if what is wanted to be done is a classic statistical analysis that considers maximums or minimums, the mean curve is robust, especially for the analysed case of vertical or anteroposterior force. This may be because, in discrete data, the mean corresponds to a possible and probable observation. However, in the case of functional data, the mean curve may not represent a real or even possible observation since the curves have high dimensionality and the observations are not independent, apart from the problem of cancellation of the mean associated with the lags between curves.

So, if the objective is to use Functional Data Analysis, where all the information in the curve is used, then the mean or the deepest curve choice is decisive.

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