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Performance Evaluation of Multiscale Finite Element Modelling in Trabecular Bone Mechanics

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Abstract - Accurate assessment of the mechanical properties of trabecular bone is crucial for osteoporosis diagnosis and treatment. The mechanical behavior of trabecular bone is highly determined by its complex porous microstructure. However, finite element model (FEM) based on high-resolution microstructural geometry is computationally demanding at large spatial scales. Multiscale FEM modelling technique offers a solution, but its application in trabecular bone analysis remains limited. The current work investigates the influence of representative volume element (RVE) size and modelling strategies on the prediction accuracy of multiscale FEM simulations of osteoporotic vertebral trabecular bone. The asymptotic homogenization approach is employed to compute the effective stiffness tensor. The results show that at the RVE scale, the modulus exhibits a power-law relationship with bone volume fraction, and the fitting performance deteriorates as the RVE size decreases. Within the multiscale framework, the effective stiffness tensor is underestimated evidently although all RVE sizes adopted satisfy conventional statistical representativity criteria. This underestimation primarily results from the loss of trabecular connectivity and cooperative load transfer due to RVE partitioning, and becomes more severe as the RVE size decreases. While the oversampling strategy offers slight improvements, it imposes significantly higher computational costs. The findings highlight the critical role of microstructural connectivity in determining mechanical property and provide theoretical guidance for error control in multiscale modelling of trabecular bone.

***Keywords*:** Trabecular bone; Multiscale finite element modelling; Representative volume element; Stiffness tensor

1. Introduction

Trabecular bone, a highly porous network of interconnected rods and plates known as trabeculae, plays a critical role in bone strength and integrity [1]. The deterioration of trabecular bone, characterized by the change in microstructure can result in reduced mechanical property and potential osteoporosis. Accurate assessment of the correlation between trabecular bone's microstructure and the associated mechanical property is crucial for clinical diagnosis and treatments, especially for guiding the personalized design of internal fixation devices and prostheses [2].

Recent advances in imaging and computational technologies have enabled the use of micro-computed tomography (micro-CT) to reconstruct the three-dimensional microstructure of trabecular bone, which can be directly converted into voxel-based finite element models (micro-CT/FEM) [3]. While this approach provides high-resolution representation of trabecular architecture, simulations over large anatomical regions typically involve tens of millions to billions of elements [4], leading to prohibitive computational costs and limiting its feasibility for clinical applications.

To address this issue, multiscale modelling has emerged as an effective strategy that balances computational efficiency with predictive accuracy [5]. This approach constructs high-resolution models in localized regions and transfers their effective mechanical properties to the macroscopic scale, enabling efficient mechanical analysis of complex structures. However, despite its wide application in composite materials, the predictive accuracy assessment of multiscale modelling in trabecular bone remains limited.

In this work, multiscale FEM modelling for the osteoporotic vertebral trabecular bone is carried out. The effects of RVE size and modelling strategies on effective stiffness tensor prediction are investigated. This study aims to provide theoretical guidance for error control in multiscale modelling of trabecular bone.

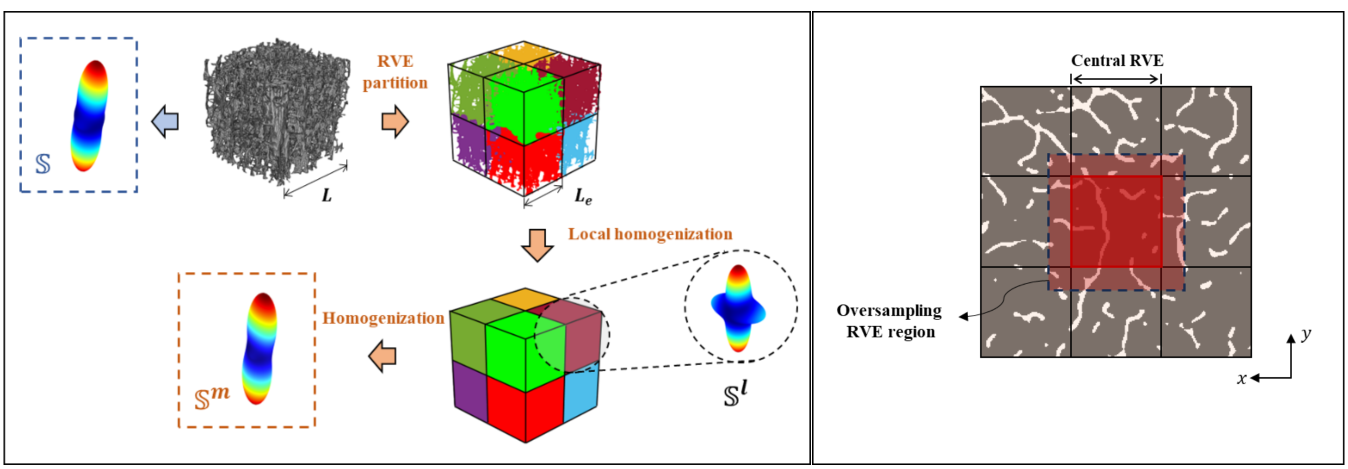
2. Material and methods

Three lumbar spines (numbered by L1-L3) from three cadavers diagnosed with osteoporosis are scanned by micro-CT with an isotropic spatial resolution of 40 μm. The trabecular bone microstructure is reconstructed from the micro-CT images through threshold segmentation and Gaussian smoothing. Twelve cubic trabecular bone samples with an edge length of L =10mm (equivalent to 256 pixels per side) are extracted. Morphometric parameters including volume fraction of bone tissue (BV/TV), structure model index (SMI), trabecular thickness (Tb.Th), and trabecular separation (Tb.Sp) [6] for these samples are quantified using the commercial software Avizo and are summarized in Table 1.

Table 1: Morphometric parameters of the trabecular bone samples.

BV/TV	SMI	Tb.Th (μm)	Tb.Sp (μm)
0.113±0.011	0.447±0.125	119±5.45	958±81.8

A voxel-based micro-CT/FEM model is established by assuming the bone tissue to be a linear elastic material with an elastic modulus of E=10GPa and a Poisson's ratio ν=0.3 [7]. The fourth-order effective stiffness tensor $\mathbb{S}$ for each sample is computed using the asymptotic homogenization method [8], and is regarded as the ground truth reference for subsequent comparison with multiscale predictions. The multiscale modelling framework is illustrated in Fig. 1(a), the entire structure is partitioned into uniformly sized RVEs with edge length L_e. Each RVE is individually homogenized to obtain its local effective stiffness tensor $\mathbb{S}^l$. These local tensors are then assembled into a macroscopic continuum model, which is further homogenized to derive the multiscale effective stiffness tensor $\mathbb{S}^m$. To investigate the influence of partitioning scale, simulations are performed for three RVE sizes: $L_e = 1/2L$, $1/3L$ and $1/4L$.



(a) (b)

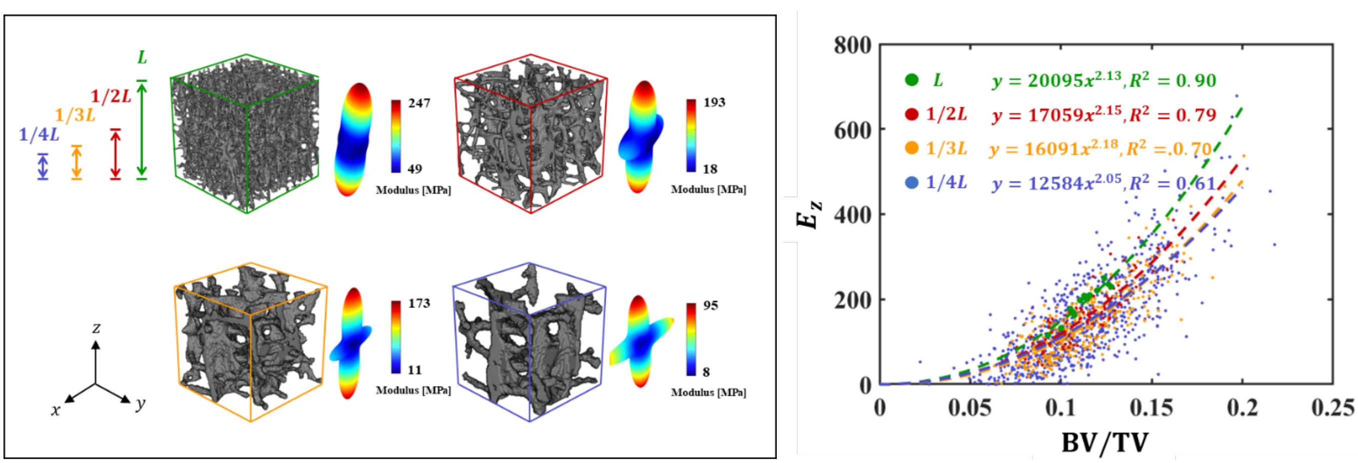
Fig. 1: (a) Schematic of multiscale homogenization framework. (b) Illustration of the oversampling strategy for RVE homogenization.

Standard RVE partitioning disrupts trabecular continuity across neighbouring regions and compromise the cooperative load-bearing characteristics of the microstructure. To mitigate this issue, an oversampling strategy [9] is introduced, as illustrated in Fig. 1(b). Specifically, for each central RVE with an edge length of $1/3L$, a larger envelope domain with an edge length of $1/2L$ is defined, and the homogenization is performed over this expanded region. The resulting effective stiffness tensor is then assigned to the corresponding central RVE. The RVE size used in this strategy is denoted as $L_e = 1/2L^o$.

3. Results and discussions

The trabecular bone samples exhibit pronounced structural and mechanical anisotropy. Typical microstructures and their elastic surfaces at different scales, from the full-size sample ($L_e = L$) to various RVE sizes ($L_e = 1/2L, 1/3L, 1/4L$), are shown in Fig. 2(a). The principal moduli (E_x, E_y, E_z) and shear moduli (G_{xy}, G_{xz}, G_{yz}) are derived from the local effective stiffness tensor $\mathbb{S}^l$, and all exhibit power-law relationships with the bone volume fraction [10]. As the Z direction aligns with the axial direction of the vertebrae, the trabeculae exhibit a strong preferential alignment along this direction, resulting in a significantly higher modulus E_z compared to the other directions. As shown in Fig. 2(b), the relationship between E_z and bone volume fraction varies with RVE size. As the RVE size decreases, both bone volume fraction and E_z exhibit increased variability, and the quality of the power-law fitting deteriorates substantially, with the coefficient of determination R^2 decreasing from 0.90 ($L_e = L$) to 0.61 ($L_e = 1/4L$).

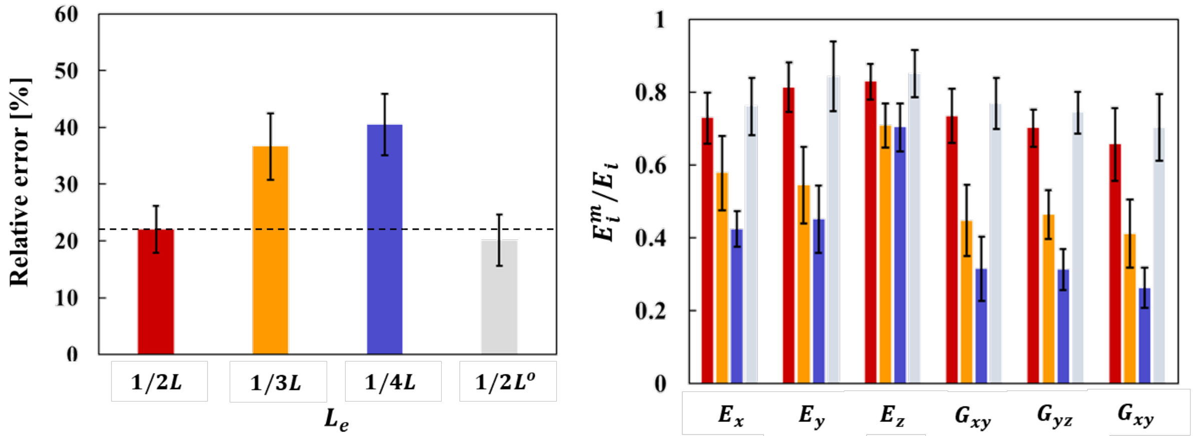
According to the statistical representativity criterion proposed by Podshivalov et al. [4], the edge length of an RVE should satisfy $L_e \geq 2$(Tb.Th+Tb.Sp). In this study, the full-size samples have edge lengths of approximately $L \approx 10$(Tb.Th+Tb.Sp), ensuring that even the smallest RVE ($L_e = 1/4L$) meets this criterion. However, the results indicate that RVE size still has a considerable impact on the mechanical response. Larger RVEs better preserve the continuity and directional connectivity of the trabecular microstructure, thus exhibiting stronger statistical representativity and enabling more stable structure–property relationships.



(a) (b)

Fig. 2: (a) Typical microstructures and elastic surfaces of RVEs at different scales. (b) Power-law relationship between modulus and bone volume fraction (BV/TV).

To quantify the deviation between the multiscale homogenized stiffness tensor $\mathbb{S}^m$ and the ground-truth stiffness tensor $\mathbb{S}$, the relative error is defined as $\| \mathbb{S}^m - \mathbb{S} \| / \mathbb{S}$, where $\| \cdot \|$ denotes the L2 norm. The relative errors under different RVE sizes are presented in Fig. 3(a). As the RVE size decreases, the relative error increases. Among all cases, the oversampling strategy ($L_e = 1/2L^o$, containing 27 RVEs) yields the lowest error of 20%, while the non-overlapping counterpart with the same RVE size ($L_e = 1/2L$, containing only 4 RVEs) results in a slightly higher relative error of 22%. Although the oversampling strategy provides a modest improvement in accuracy, it comes at the cost of significantly increased computational effort, making it less efficient overall. Fig. 3(b) shows the ratio between multiscale predicted modulus E_i^m and the corresponding ground truth modulus E_i. All moduli are reduced, indicating that the discrepancy in the effective stiffness tensor primarily arises from the underestimation of moduli. Compared to the axial modulus E_z, the transverse moduli (E_x, E_y) and shear moduli are more sensitive to changes in RVE size, with their reductions becoming increasingly pronounced as the RVE size decreases.



(a) (b)

Fig. 3: (a) Relative error of stiffness tensor computed by the multiscale model with different RVE sizes. (b) Ratio of multiscale predicted moduli to ground truth moduli.

The underlying mechanism of moduli reduction is illustrated in Fig. 5. In the entire structure, trabeculae form continuous load-bearing paths that traverse multiple regions, enabling long-range mechanical cooperation. However, in multiscale modelling, the RVE partition disrupts the inter-RVE trabeculae connection, thus impairs the original load transfer pathways and compromises mechanical connectivity. Consequently, load transfer is redistributed within each RVE, ultimately leading to an underestimation of mechanical property. As the RVE size decreases, the probability of trabecular discontinuity increases, resulting in a more pronounced reduction in modulus. This underestimation is not caused by a lack of statistical representativeness in the traditional sense, but rather by the loss of structural connectivity and inter-RVE interactions due to RVE partitioning. Although the oversampling strategy introduces additional connection information from surrounding regions, it provides only limited improvement. Unlike reinforced composites with randomly distributed inclusions, trabecular bone is a highly complex and interconnected network, even minor local structure change can lead to a server global reorganization of load transmission paths.

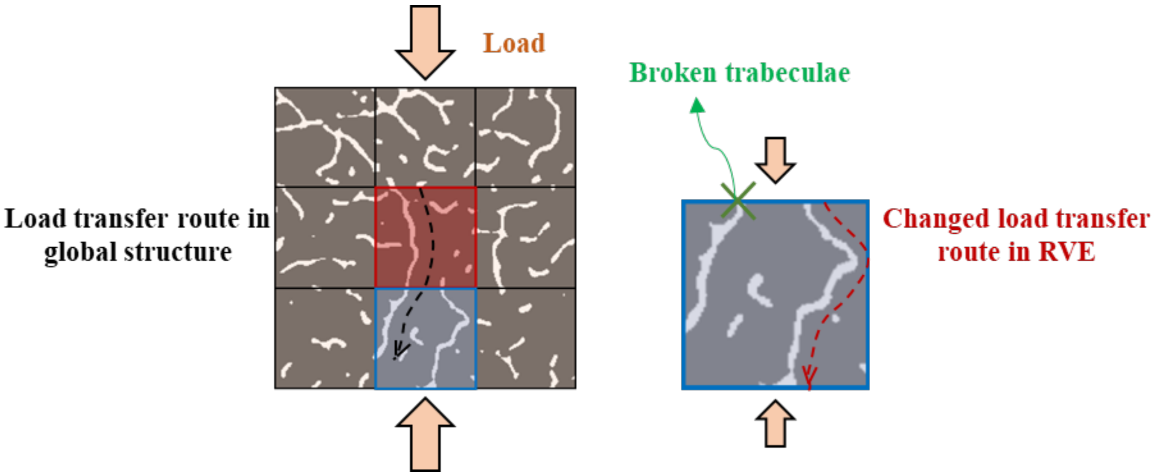


Fig. 4: Structural discontinuity and altered load path induced by RVE partitioning.

Therefore, to further improve the accuracy of multiscale predictions while maintaining computational efficiency, future work could explore the use of empirical correction strategies. For example, regression-based models linking local BV/TV and predicted moduli may be employed to calibrate the multiscale homogenization results, enabling more accurate estimation of the macroscopic mechanical property.

4. Conclusion

The current study focuses on evaluating multiscale FEM modelling accuracy of osteoporotic vertebral trabecular bone. A voxel-based FEM model is constructed by reconstructing the microstructure from micro-CT images. A multiscale homogenization framework is used to compute the effective stiffness tensor and assess the associated prediction errors. The main conclusions are as follows:

(1) The elastic moduli of local RVE follow a power-law relationship with bone volume fraction, and the fitting accuracy decreases as the RVE size decreases.

(2) The effective stiffness tensor of the full-size microstructure obtained by multiscale FEM prediction is evidently underestimated while all RVE sizes adopted satisfy conventional statistical representativity criteria. This underestimation primarily results from the loss of trabecular connectivity and cooperative load transfer due to RVE partitioning, and becomes more severe as the RVE size decreases.

(3) The oversampling strategy provides slight mitigation of the underestimation error but significantly increases computational cost. Given the high sensitivity of mechanical behavior to local microstructural disruptions, it is recommended that future studies adopt the largest feasible RVE size within acceptable computational limits to preserve structural connectivity. Additionally, empirical correction models may be considered to further improve prediction accuracy.

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