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Swift Statistical Fingerprinting Supporting Electronic Warfare Signal Characterization: A Music Processing Approach

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Abstract **-** In electronic warfare, distinguishing adversarial from friendly signals is a persistent challenge, especially when no reference templates exist. This work explores a blind signal processing approach, combining latent dictionary learning with Markov chain modeling, to rapidly characterize unknown radio-frequency signals using frequency state transitions. To test the method, we used musical excerpts (Brahms' Violin Concerto) as proxies for radio frequency waveforms, analyzing both clean and corrupted versions. First, we decomposed the audio signals into time-frequency spectra using the Fourier transform, then construct overcomplete spectral dictionaries via nonnegative matrix factorization. By applying sparse coding using the least absolute shrinkage and selection operator and Markovian analysis, we derived transition matrices that served as statistical, frequency state transition fingerprints. These fingerprints revealed clear differences between clean and distorted signals—such as gaps in transition probabilities (e.g., [898–1021] Hz) caused by simulated violin playing errors. While nonnegative matrix factorization and Markov modeling successfully highlighted corruption patterns, simplex-based endmember extraction proved less discriminative, likely due to its inherent lack of eigenmode generation. The results suggest that temporal-spectral fingerprinting—without relying on neural networks—could enable fast, efficient signal classification in electronic warfare, particularly for signals of unknown origin. This approach may serve as a preprocessing step for more complex radio frequency analysis pipelines.

Keywords**:** blind signal processing, frequency transitions, least absolute shrinkage and selection operator, nonnegative matrix factorization, Markov chains, signal dictionaries, signal fingerprinting, simplex-based endmember extraction

1. Introduction

The ever-evolving landscape of electronic warfare necessitates sophisticated methodologies for characterizing electromagnetic signals that emanate from adversaries. This is especially true given how global defense strategies have increasingly relied on geo-intelligence to inform tactical actions producing an imperative to effectively categorize these signals. The complexity of this problem is exacerbated in environments where prior knowledge is limited, making it challenging to distinguish between benign signals, which pose no threat, and malicious signals, which could potentially inflict significant harm. Electromagnetic signals themselves can be intricate and multifaceted, often obscured by noise and interference that complicate their analysis. The development of fast and robust signal assessment methodologies is therefore critical. Such algorithms offer a quick assessment of signals before being processed by more complicated algorithms directed toward intensive information extraction.

Signal fingerprinting is not a new area but is an endeavor experiencing heightened research and interest due to the dire need to robustly identify transmitters producing radio frequency emanations [1]. State-of-the-art methods for signal fingerprinting have focused on machine learning techniques to gain intelligence about emitter sources [2]. In particular, focus has been placed on using the transient structure of high frequency signals to gain insight into signal generating sources especially in such domains as wireless communication [3,4]. It is the high frequency, transient signal domain and how it changes that spawns this research.

In response to the previously mentioned challenges, a statistical algorithm utilizing nonnegative matrix factorization (NMF) and classical Markovian analysis has been developed to enhance the characterization of high fluctuating, electromagnetic signals. Markovian models are particularly suited for this application as they allow for the probabilistic modeling of sequential data. By analyzing the frequency state changes of a signal over time, these models can provide insights into the underlying structure and characteristics of high frequency, temporal signals. The approach developed in this work draws inspiration from music signal processing but is firmly directed towards the exploitation of NMF and Markovian models as a form of intelligence data analysis dedicated to geo-intelligence signal differentiation and classification [5]. This work takes the unique approach of using acoustic/music signals as a proxy for radio frequency signal emanations. It is the adoption of both acoustic/music signal processing and a Markovian methodology that represents a form of robust signal fingerprinting for modeling the temporal sequential nature of information. This in turn supports the characterization of geo-intelligence electromagnetic signals.

This paper's structural format is as follows. The type of data used in the development of the statistical algorithm and model for high frequency fluctuating signals is first provided. This is followed by a brief explication of the statistical methodology which converts these signals into matrix models quantifying the transition relationships between frequency states embedded in the signals. Statistical model results are then provided which seek to demonstrate how the models capture differences in how signal frequencies transition in different acoustic/music signals. Characterization of electromagnetic geo-intelligence signals is a perpetually challenging task warranting a need for nontraditional and esoteric methods to address defense force-adversarial differentiation of signal structure. Conclusions are provided which seek to show the applicability of the method to geo-intelligence based on the exploitation of acoustic/music signals as adequate and useful proxies for real geo-intelligence signals.

2. Data Signal Collection

Music signals were used as proxies for geo-intelligence, electromagnetic signals in this work. In particular, the Brahms' Violin Concerto in D major was used as the basis of statistical algorithm development. Two uncorrupted music snippets from the concerto featuring only the violin were extracted. Their time-frequency structure in the form of musical scores is shown in Fig. 1a) and 1b). These musical renditions were played at the tempo of allegro giocoso, ma non troppo vivace of 95 beats per minute and 57 beats per minute respectively for the time interval of 12 seconds and 8 seconds for the first and second music snippets respectively. Corrupted versions of these music snippets were produced by playing notes with a significantly different tempo and with slight frequency distortion. Corrupted versions of the first music snippet was played at the significantly slower tempo of approximately 63 beats per minute. The corrupted version of the second music snippet was played at a slower tempo but notated with a 4/4 time signature and tempo of 194 beats per minute instead of the traditional 3/4 time signature. The corrupted version of the first music snippet was 26 seconds in length whereas the corrupted version of the second music snippet was 22 seconds in length. The violin used was a traditional concert violin tuned to A=440 Hz where the top of the instrument was made of spruce wood and the sides and back made of flame maple wood. The strings were made of metal and a standard bow strung with white horse hair was used. The music snippets were recorded with a digital recorder with a sampling frequency of 48 kHz and captured in the form of M4A media files. Data files were snipped to include the signal portion containing the actual music notes.

a)

b)

c)

d)

Figure 1: Music scores showing music signal snippets played by a single violin. The musical key is D major. a) The first music snippet, from the beginning of the concerto, was 12 s long. b) The second music snippet, representing a musical bridge, is 8 s long. c) The third music snippet, a corrupted version of the first snippet, is 26 s long. d) The third music snippet, a corrupted version of the second snippet, is 22 s long.

3. Matrix Factorization Analysis and Markov Modeling Methodology

The approach used to distill the high fluctuation music data signals into Markovian transition matrix fingerprints, which capture the temporal change in eigenfrequency structure in data signals over time, is as follows. For each of the four acoustic/music data signals, the analysis begins with the computation of the time-windowed Fourier transform of the signal to obtain the time dependent power spectrum V(f,t). Here f and t quantify frequency bands and the time interval over which the Fourier transform was computed respectively. The time dependent power spectrum was calculated by taking the signals and dividing them into data segments containing 10,000 points. The Welch method for estimation of power spectra over the data segments was then applied via the application of the Fourier transform to each signal segment using a 50% overlapping time window segment containing 512 temporal samples. A Hanning window was applied over the time window subsegments and the spectral estimates were interpolated to 1024 points in length. The average periodogram obtained in this manner produced a smooth estimate of a signal's power spectral density (PSD) for different data segments by mitigation of variance [6]. This enhanced reliability of the power spectrum over time is necessary for next part of the algorithm.

The m x n, time dependent power spectral matrix for each signal, V(f,t) with m frequency estimates and t time intervals with 0.22 s resolution, was factored into weight coefficient and eigenvector matrices using four distinct NMF algorithms. These were the NMF-ALS (Alternating Least Squares) [7], NMF-BETA (Beta Divergence) [8], NMF-TITAN (Tensor-based NMF) [9], and the NMF-PALM (Proximal Alternating Linearized Minimization) [10]. Nonnegative matrix factorization is a method that factors a matrix into the product of two nonnegative matrices allowing for reduction in dimensionality of the information contained in the original data matrix [11]. Applied here, each algorithm is designed to decompose the time dependent spectrogram matrix V(f,t) into a nonnegative array of basis vectors and weights, facilitating greater interpretability of the underlying signal structure.

All NMF algorithms minimize the cost function of the spectral data and the matrix factorization results by employing a variety of different algorithms and constraints. The NMF-ALS employs alternating minimization of the Frobenius or Euclidean norm. The NMF-Beta minimizes the beta divergence of the spectral data and the matrix factorization output. The NMF-PALM is an NMF method using an alternating matrix factor minimization technique which employs sparsity for the eigenvector matrix and smoothness for the coefficient matrix. The NMF-TITAN is a sparse NMF method which employs the block-coordinate update method which relies on the maximization-minimization framework while embedding an inertial force to each step of the block updates.

The span of these matrix factorizations was independently assessed through a covariance and correlation-based, virtual dimension analysis, allowing estimation of the appropriate dimensionality of the matrix eigenvectors. This analysis influences the algorithm’s capacity to distill essential signal features [12]. The virtual dimension of 9 was used for all NMF algorithms and was found appropriate for decomposition of the spectral data for all NMF algorithms. A diagram of the NMF process is shown in Fig. 2a)-b).

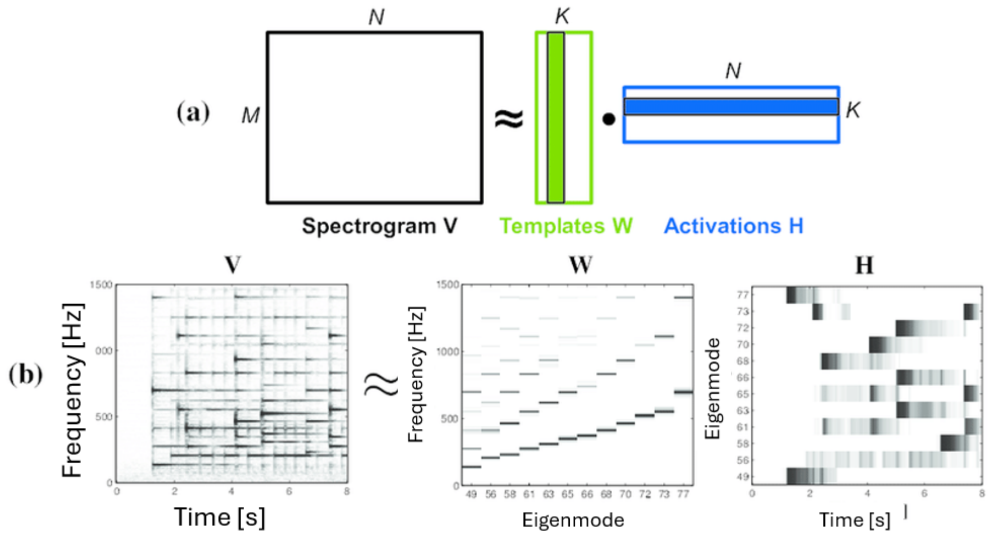


Figure 2: Nonnegative matrix factorization conceptual diagram for matrix factorization of the time dependent spectral data matrix V or time dependent spectrogram. a) Mathematical conceptual diagram of the matrix factorization of V into eigenmode W and coefficient weight H matrices. b) Data conceptual diagram and matrix factorization depicting possible structure of eigenvector and coefficient matrices.

An overcomplete spectral encyclopedia of NMF eigenvectors was constructed for the complete spectral data matrix of a signal using the aforementioned NMF eigenvector dictionaries. This encyclopedia was generated by aggregating the eigenvectors of each NMF algorithm-based eigenvector dictionary obtained from all four NMF techniques into a m X N matrix where m rows delineate the frequency estimates and N <<t columns. The value of N is the number of eigenvectors emanating from all 4 NMF eigenvector dictionaries which lie horizontally abreast to each other in the eigenvector encyclopedia matrix. The overcomplete or redundant nature of the spectral encyclopedia provides a rich basis function span appropriate to the decomposition of the original spectral data.

The most significant NMF eigenvector for each spectrum associated with each 0.22 s time interval in the spectral data matrix was identified from the eigenvector encyclopedia using the Least Absolute Shrinkage and Selection Operator (LASSO) [13]. This technique was utilized to estimate the sparse representation of the NMF eigenvectors appropriate to each spectrum at time interval t. The weights of all eigenvectors in the encyclopedia were calculated and the eigenvector with highest weight selected for each time interval. Over the complete spectral data time span, the mode NMF eigenvector was found (which is a member of one and only one of the NMF dictionaries contained in the encyclopedia). This process was performed for each of the four music/acoustic signals creating a single optimal NMF eigenvector dictionary for each data signal which is not necessarily the same for all.

Once the optimal NMF eigenvector dictionary was established for each music/acoustic signal, this single dictionary (which in general is different for each signal) and its coefficient weight matrix was used in establishing eigenfrequencies which change over the time intervals. For each time interval, the maximum value in the NMF coefficient weight matrix associated with the optimal NMF dictionary was found. This maximum value is related in an one-to-one fashion with a specific eigenvector in the optimal NMF dictionary, allowing designation of a characteristic NMF eigenvector from the optimal NMF eigenvector dictionary. This step simply designates the optimal eigenvector for each spectral time interval creating a clear linkage between the time-domain features and their eigenspectral representations. Maximum eigenvector time series for musical snippet 1 and its corrupted version are shown in Fig. 3a) and b) respectively as examples.

From the maximum eigenvector label time series, the physical maximum frequency values at each spectral time interval were extracted. This extraction process facilitates the tracking of real, physical frequency variations over time, offering insights into the dynamic characteristics of the signal. This process also suppresses noise via the use of eigenvectors designating particular physical frequencies as dominant for a particular time interval. Finally, Markovian modeling was applied to the estimated maximum frequency or eigenfrequency time series. This modeling entails constructing a transition matrix characterizing the probabilistic transitions between different eigenfrequency states over time [14,15,16]. Frequency state intervals for the transition matrix were found from the extracted maximum frequency value time series by first selecting the maximum and minimum from the set of maximum frequencies. The array of maximum frequencies was divided into 17 equal intervals and the maximum likelihood estimate of the transition matrix was calculated via counting the number of times different frequency intervals were visited in the eigenfrequency time series [17]. This was done for each of the four signals. The transition matrix provides the transition probabilities which reflect the likelihood of moving from one frequency state to another and captures the average dynamic behavior of a signal's time dependent spectral matrix.

Transition matrices were also estimated from spectral data matrices using optimal spectral dictionaries generated from simplex-based endmember extraction. Spectral simplex endmember extraction is a technique for extracting a finite number of spectral data vectors which span a spectral data matrix set and is used in hyperspectral imagery processing [18]. Unlike the NMF technique used in generating the eigenvector-based encyclopedia, a generated endmember matrix is a subset of real spectra extracted from the original spectral data matrix which represent the volumetric span of all spectral data. Eigenfrequency time series, from which transition matrices were generated for each of the four signals, were derived using a similar method as outlined above. The simplex endmember extraction algorithm was applied to each of the four signals where 18 endmembers were generated for each signal. The following algorithm was then applied to each of the four signals where each possessed an endmember spectral matrix derived from a spectral data matrix.

The LASSO algorithm was applied to the time dependent spectral data matrix using the extracted 18 optimal endmember set. Application of this algorithm entailed taking each data spectrum within the spectral data matrix and assigning a specific optimal endmember. The maximum value for each optimal endmember associated with each time interval was then found allowing for the creation of a maximum frequency time series. This endmember-based eigenfrequency time series was then transformed into a Markovian transition matrix in the same way outlined above, quantifying the probability of transitions from one frequency state to another.

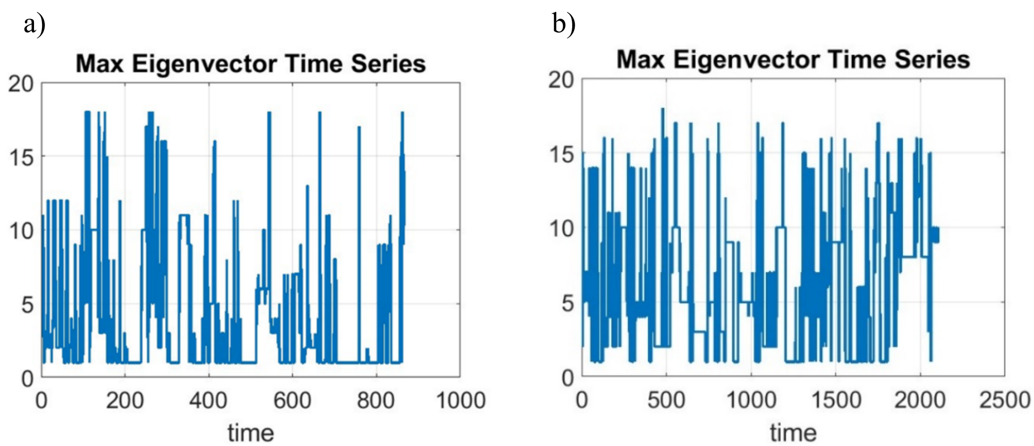


Figure 3: Maximum eigenvector time series for the uncorrupted and corrupted music snippet 1 which have different lengths. a) first music snippet and b) its corruption. X-axis designates the number of temporal spectral time intervals associated with time window spectral data estimates having a resolution of 0.22 s. Y-axis is the eigenvector label designation for the selected optimal eigenvector within the dictionary. The span of the eigenvector dictionary is 18.

4. Nonnegative Matrix Factorization-Markovian Algorithmic Results

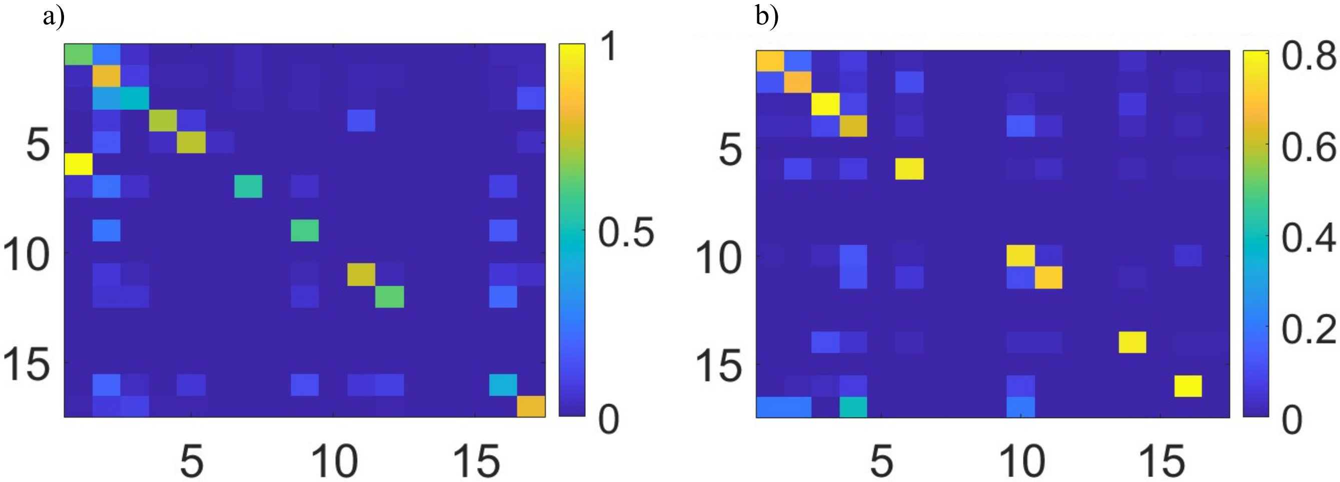
The important aspect of this analysis is the demonstration that clear and significantly different spectral frequency transition structure exists when corrupted and uncorrupted signals are compared. The playing of the notes in the score shown in Fig. 1a)-b) produces large amounts of harmonics. Therefore, specific transition frequency states can and do appear in the Markovian modeling of the uncorrupted and corrupted signals which are not captured in the musical score. However, the filtering process utilized in this analysis seeks to capture the dominant frequency changes which promotes and solidifies the idea that visible differences in the transition structure are associated with real signal corruptions and are not pure harmonic artifacts. The transition matrix for the uncorrupted signal shown in Fig. 1a) is displayed in Fig. 4a). The transition matrix displays a strong diagonal structure across the full spectral bandwidth occupying intervals 1-17 which stretches from 245 to 2383 Hz. Very little off-diagonal probabilistic values exist with the only noteworthy off-diagonal probabilistic values occurring at the frequency transition interval of 5 to 1 which is the high to low frequency state transition of [873 998] Hz into [245 370] Hz. This is approximately the musical note transition of [A5 B5] to [B3 F#4]. A major spectral region of low frequency state transition probability occurs over the region spanning the intervals of 13-15 which is the frequency range of [1753-2130] Hz (musical note transition of A6 to C7). This is consistent with the score structure where notes at these high frequencies are about 1 octave above the highest frequency note in the score where no frequency state transitions occur.

The transition matrix for the corrupted version of the signal shown in Fig. 1a), which is the signal shown in Fig. 1b), is displayed in Fig. 4b). Just as in the case of the uncorrupted signal, very little off-diagonal probabilistic values exist. A strong diagonal structure stretching across approximately the same full spectral bandwidth of [158.4 2253.3] Hz is displayed. Noticeable regions of low transition probability appear at intervals spanning 7-9 and 12-13 which correspond to the frequency transition ranges of [898 1021] Hz (A5 to C6) and [1514 1637] Hz (F#6 to G#6) respectively. The major point is that the transition matrices for the uncorrupted and corrupted spectral signals are structurally different, demonstrating that the combination of the NMF decomposition and Markovian theory is successful in distilling statistically different and distinct temporal-spectral fingerprints.

The transition matrix for the uncorrupted signal shown in Fig. 1c) is displayed in Fig. 4c). It is structurally different from the temporal-spectral fingerprint shown in Fig. 4a) with a spectral bandwidth spanning the frequency interval of [202 4105] Hz. Probabilistic frequency transition values are clearly and distinctly bandlimited, with non-zero probabilistic values appearing along the diagonal for the frequency transition intervals 1- 9 or [202 2267] Hz. A region with extensive near zero probability values appears in the spectral bandwidth range of [2267 – 4105] Hz which is the frequency transition interval of 9 to 16.

The transition matrix for the corrupted version of this signal, shown in Fig. 1d), is displayed in Fig. 4d). A similarly strong diagonal structure over a spectral bandwidth of [287 3115] Hz is displayed, which is slightly larger than the [202 2267] Hz range containing non zero frequency transition probability for the uncorrupted music snippet version. What is clearly distinctive is a region of very low probability for the frequency transition spectral bandwidth of [1119 – 1784] Hz, or frequency transition intervals 6 to 9, except for an exceptionally high frequency transition probability value occurring in the 7th frequency transition interval of [1285 1452] Hz. It is conjectured that the stretching of the uncorrupted musical snippet from 8 s to 22 s represents enough of a distortion of the original signal, causing the appearance of frequency transition intervals with extremely low probability. The noticeable regions of low frequency transition probability found for the corrupted version of music snippet 2 in the middle of the full frequency transition range is a characteristic shared with corrupted version of music snippet 1. Both sets of corrupted and uncorrupted signals behave similarly in this respect, demonstrating at the least how the transition matrix can quantify significant temporal-spectral frequency changes in signals. The transition matrix, in other words, for the uncorrupted and corrupted spectral signals are structurally different enough and the NMF decomposition and Markovian theory-based algorithm sensitive enough to successfully estimate statistical, temporal-spectral signal fingerprints which are visibly different.

a) b)



c) d)

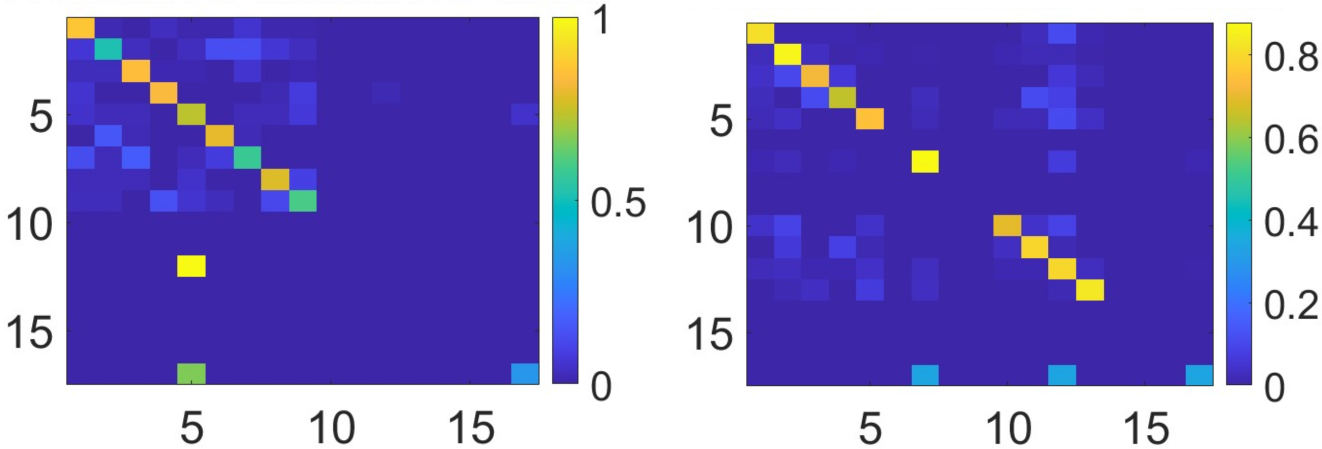


Figure 4: Transition matrices obtained using the NMF-based optimization algorithm. Signal fingerprint for a) musical snippet 1. 17 equally space intervals spanning 244.5 - 2382.5 Hz. Signal fingerprint for b) corrupted version of music snippet 1. 17 equally space intervals spanning 158.4 – 2253.3 Hz. Signal fingerprint for c) music snippet 2. 17 equally space intervals spanning 201.5– 4105.2 Hz. Signal fingerprint for d) corrupted version of music snippet 2. 17 equally space intervals spanning 287.6– 3114.6 Hz.

The simplex-based endmember estimation algorithm provides transition matrix estimates for all cases. However, the transition probability structure in these matrices did not seem to be significantly different from each other. Fig. 5a)-b) shows the endmember-based transition matrices for the first music signal and its corrupted version. The full spectral bandwidth for each signal respectively is [245 2383] Hz and [245 1866] Hz. The bandwidths of both signals are not drastically different and both signals share the same middle spectral frequency transition state region where low transition probabilities occur. The similarity of the transition matrices suggests that the simplex-based endmember extraction method, which is based on the extraction of real spectral signals from the spectral data matrix, volumetrically captures similar higher dimensional temporal-spectral structure in both uncorrupted and corrupted signals. This characteristic is most likely due to the uncorrupted and corrupted signals emanating from a similar source or musical key producing similar endmembers responsible for the production of similar transition matrices. Spectral simplex endmembers are not the result of the matrix factorization process performed above, which actually filters a spectral data matrix, but are physical spectra which volumetrically bind the spectral data. With the lack of generation of a kernel for the spectra data matrix via the matrix factorization process, the separation of noise from the spectral data signals is not instituted which is thought to be important in distinguishing between corrupted and uncorrupted signals.

a) b)

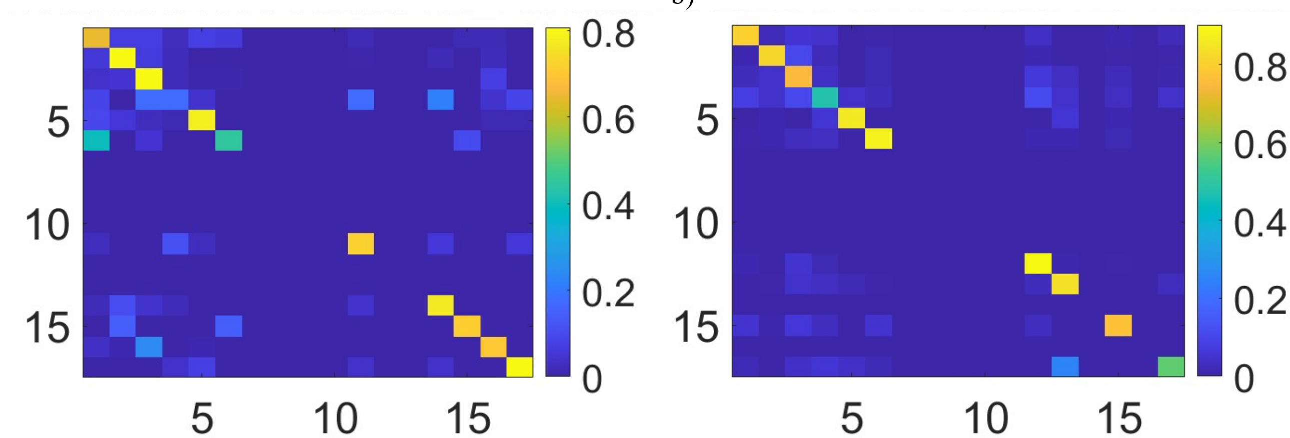


Figure 5: Transition matrices using the simplex endmember-based optimization algorithm for a) uncorrupted musical snippet 1 possessing 17 equally space intervals spanning 244.5 - 2382.5 Hz and the b) corrupted music snippet 2 possessing 17 equally space intervals spanning 244.5 – 1865.7 Hz.

5. Conclusions

The efficacy of NMF-Markovian modeling for spectral signal fingerprinting is demonstrated driven by the need for characterization and differentiation of spectral-temporal signals. While the concept of robust spectral signal fingerprinting, particularly for electromagnetic, high-frequency signals, is not novel, the approach shown here introduces some innovations. Firstly, the L1 norm-based LASSO method is employed for optimal eigenvector selection, which enhances the subsequent selection of an optimal dictionary from an overcomplete encyclopedia comprised of four individual NMF spectral eigenvector dictionaries. Secondly, by leveraging music-acoustic signals where the signal state truth and perturbations from this truth are known, a robust method for exploring the differentiation between uncorrupted and corrupted high-frequency signals is explored. The results suggest that NMF-Markovian transition matrices, although relatively simple, quickly and effectively characterize signals within the same frequency span (which for signals here is equivalent to musical key). Notably, the differentiation between uncorrupted and corrupted signals is achievable even when limited to the maximum frequencies extracted from the dominant eigenvector for each time windowed Fourier transformed interval. The investigation into simplex-endmember estimation combined with Markov chain methods did not yield significantly distinct transition matrices between uncorrupted and corrupted signal data. This may be attributed to the absence of actual spectral signal filtering performed via the expansion of time dependent spectral data in NMF eigenvector space. The NMF decomposition is conjectured to be an essential element of the technique responsible for suppressing noise by carefully selecting optimal eigenvectors and maximum frequencies.

While this analysis here focuses on the processing of music-acoustic signals, which extend into the kHz range, taken as proxies for radio-frequency (RF) signals in the GHz range, the relevance of the technique remains strong. It serves as a preliminary method for evaluation of high-frequency signals emitted from unknown sources where there is significant modulation in temporal-frequency structure. For instance, the NMF-Markovian model could serve as a preprocessor to neural network algorithms which provide more intensive structural extraction of latent information. Overall, the study allows for further exploration in the realm of spectral signal processing, emphasizing the potential of the NMF-Markovian model as a practical tool for quick understanding of complex signal behavior in high-frequency environments related to geo-intelligence.

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