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# **Energy Harvesting in Vehicle's Drive**

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**Abstract** - The analysis of dynamical performance and effectiveness of developed Vibration Energy Harvesters based on Electromagnetic Tuned Mass Dampers applicable for types of Automobiles is presented. The composed Vibration Energy Harvesters consist of two main components: regular Tuned Mass Damper and Generator of Electrical Signals. The invented Vibration Energy Harvester could be fabricated in two main versions: for rectilinear vibration applications and for angular rotational applications. The considered Vibration Energy Harvesters are used successfully for equipping of driving sources: internal combustion engine, hybrid and electric motors. The presented solutions demonstrate high efficiency and would be applied for a design in all kinds of Hybrid Cars. The structural study of invented schemes shows some perspective features and directions for design. Specific particularities and control properties are provided for explanation of effectiveness of Vibration Energy Harvester is analyzed. Practical applications of Vibration Energy Harvesters for different variations of drives were tested. Test results illustrate the efficiency of proposed Vibration Energy Harvester and some new opportunities for their applications.

*Keywords*: Electromagnetic Vibration Energy Harvester.

### 1. Introduction

Energy harvesting is the modern technology allowing capture, convert and transfer small amount of wasted power [1], [2]. Energy harvesting is widely used in particular for feeding miniature, wireless autonomous electronic devices. The authors did not include in present paper the detailed analysis of a various published materials regarding considered subject trying to concentrate only on authors results. First Vibration Energy Harvesters (VEH) developed by authors in [3] - [5] were based on Controlled Tuned Mass Dampers. The implementation of VEH in vehicles was dictated by two combined reasons: necessity of a reduction of intensive vibrations and proper usage of damped vibration energy. For instance, the Hybrid Complexes are the sources of intensive vibrations [6] - [8]. The application of developed VEH (having Controlled Tuned Mass Damper scheme) in vehicle propulsion environment for reaching two goals: reduction of vibration levels and effective usage of wasted vibration is demonstrated in the current study using some examples.

## 2. Schematic Analysis of Vibration Energy Harvesters

There are several types of Energy Harvesters. There is hybrid drive is selected for current study, hence Electromagnetic Vibration Energy Harvesters (EVEHs) are planning be used, and one of the main reason for this choice is that hybrid drive operates practically with constant speed. EVEH could be presented in two general versions: "rectilinear vibration operation" and "angular – torsional operation". The core graphical model of rectilinear operating EVEV ([9] – [14]) is shown in Figure 1. Let's assume that a certain object (for instance, Hybrid Engine) is vibrating along coordinate axis X as shown in Figure 1. The mass of vibrating object is  $\mathbf{M}$ . The body 1 of EVEH is attached firmly to object. EVEH consists of two major parts: Tuned Mass Damper and Alternator. Tuned Mass Damper having mass  $\mathbf{m}$  is attached to

EVEH's body by means of spring 2 and damping element 3. Alternator part consists of pick-up coil 4 and magnet 5 (for instance, configured as a ring). Magnet 5 is attached firmly to the mass  $\mathbf{m}$  of Tuned Mass Damper. Tuned Mass Damper has to have own resonant frequency equal to Hybrid Object operational frequency. The amplitude of mass  $\mathbf{m}$  vibrations would be maximal in this case, and "productivity" of Alternator would be the maximal as well.



Fig. 1: EVEH arrangement for linear one-dimension application.

The core graphical model of torsional operating EVEH (in accordance to [15]) is shown in Figure 2. There is a hub **705** ([15]) rigidly affixed to an outer ring **750**. An inner ring **714** is co-centered with the hub **705** and connected by springs **712**. The outer ring is connected to cantilevers **752**. These elements **752** are directed radially inward toward the inner ring **714** and terminate in sheaths **770** that surround an arc of the ring **714** (like two inter-linked annuli). The ring **714** has a function of inertia mass similar to mass **m** shown in Figure 1. Where the sheaths **770** and ring **714** have an applied current, an electromagnetic bond is established between them. It is shown the detailed composition of system "sheaths **770** surrounding an arc of the ring **714**. As a result the torsional vibrations and irregularity of rotational speed of shaft would be eliminated, and subsystem "coil **4** – magnet **5**" would generate the power. So the mechanical energy of the rotational Tuned Mass Damping subsystem **712** - **714** is converted into electrical energy in electromagnetic subsystem of Alternator **4** – **5**. It is necessary to point out that reciprocal and rotational schemes are equivalent.



Fig. 2: EVEH arrangement for torsional application.

#### 3. Modelling of EVEH

The simplified modeling of EVEH could be done using the scheme shown in Figure 1. Let's suppose that the Hybrid Power Object **M** is mounted on vehicle by means of mounts, supports, etc. (modeling by spring having stiffness **ko** and viscosity factor **bo**). It could vibrate in one direction only. Its movement coordinate is **X**(**t**), where **t** is time. The outer casing of EVEH is firmly attached to the Hybrid Object, and the mass of casing is included in **M**. Tuned Mass Damper **m** is connected to casing by means of springs having the stiffness **k1** and viscosity factor **b1**. Mass **m** is vibrating in the same direction as mass **M**. The coordinate of vibrations of **m** is **x**(**t**). The electromagnetic subsystem is inducing current **i**, and **L** is inductance, **R** is electrical resistance, **R**= **Rc**+**Rd**, where **Rc** is coil resistance and **Rd** is load resistance. The EVEH dynamics is described by system of ordinary differential equations (see for instance [1], [2], [5], [17]),

$$MX^{``} + \mathbf{ko} X + \mathbf{bo} X^{`} + \mathbf{k1} (X - x) + \mathbf{b1} (X^{`} - x^{`}) = H \sin (\omega t),$$
(1)  
$$mx^{``} - k_{1} (X - x) - b_{1} (X^{`} - x^{`}) + \Phi i = 0,$$
(2)

$$\mathrm{Li}^{*} + \mathrm{Ri} - \Phi \mathrm{i} = 0. \tag{3}$$

where **H** is the magnitude of external force,  $\boldsymbol{\omega}$  is the frequency of external force,  $\boldsymbol{\Phi}$  is the linkage factor. The system of equations (1) – (3) is written purposely focusing on specific link between vibrations of Tuned Mass Damping subsystem (equation (2)) and dynamic behaviour of current generating subsystem (equation (3)). Furthermore it is possible to pay attention on a specific of the dynamic properties of Tuned Mass Damper coupled to Alternator. The selected sample of object for analysis is Free-Piston Linear Internal Combustion Engine without crankshaft combined with Electrical Generator completed by auxiliary drive [6], [7]. It was established that the vibrations of an object body are along one axis mainly. Assuming that at considered case: M=75 kg, bo=14000 kg/s, ko=7\*10<sup>6</sup> kg/s<sup>2</sup>, H=641 kg m/s<sup>2</sup>, it is possible to build the amplitude - frequency characteristics as shown in Figure 3a by red solid curve. The abscissa horizontal axis on following similar Figures is for excitation frequency and ordinate vertical axis is for amplitude.



Fig. 3: a. Amplitude-Frequency Characteristics for Hybrid Complex when  $\Phi=1$ ; b.EVEH average Current and Power when  $\Phi=1$ .

The peak of this curve is at frequency equals to 44 Hz, or the resonance vibration of Hybrid Object could happen at 44 Hz. The prescribed operation frequency of Hybrid Object is 50 Hz and the amplitude of vibration is 0.145 mm for 50 Hz. It means that the Hybrid Complex will be operating at 50 Hz having amplitude of 0.145 mm if there is no Tuned Mass Damper. The parameter's selection of Tuned Mass Damper in first approximation could be m=4.553 kg, b1=100 kg/s, k1=4.249\*10<sup>6</sup> kg/s<sup>2</sup>. Let's assume for simplicity that the initial parameters of Alternator for trial are dimensionless and all of them are equal to unit, or L=1, R=1,  $\Phi = 1$ . The amplitude - frequency characteristic of Tuned Mass Damper could be reflected by brown dashed curve as shown in Figure 3a. The resonant amplitude of Tuned Mass Damper is equal to 0.807 mm at 48.5 Hz, and in operation case of 50 Hz the amplitude would be 0.758 mm. The blue dotted curve in Figure 3a stands for damped vibrations of main system (Tuned Mass Damper installed). EVEH induced current would be presented by green dash – dotted curve as shown in Figure 3b, and EVEH peak of useful power could be plotted by solid magenta curve in Figure 3b. EVEH useful power is calculating by formula  $P(\omega) = 0.5 Rd i^2$ , where Rd = 0.5 R. One can see that the shape and location of green dash – dotted curve (induced current) in Figure 3b is coinciding with the shape and location of magenta solid curve (peak of useful power) in Figure 3b is coinciding with the shape and location of green dash – dotted curve (runed Mass Damper) in Figure 3a. The shape and location of magenta solid curve (peak of useful power) in Figure 3b is coinciding with the shape and location of green dash – dotted curve (induced current) in Figure 3b. EVEH useful power) in Figure 3a. The shape and location of magenta solid curve (peak of useful power) in Figure 3b is coinciding with the shape and location of green dash – dotted curve (induced current) in Figure 3b. EVEH useful power) in Figure 3a. The shape and locatio

Let's examine the impact of each electromagnetic parameter on Tuned Mass Damper amplitude and useful power. Suppose that each of parameter variation is in interval (1...4), or 1 < L < 4; 1 < R < 4;  $1 < \Phi < 4$ ; and  $\omega = 314$  1/s = 50 Hz.

The relationship between amplitude of Tuned Mass Damper and an inductance L is plotted by blue curve in Figure 4a; and on the same figure there is a relationship between the peak of useful power P and an inductance L plotted by red curve. Other parameters are  $R = \Phi = 1$ . The horizontal axis (dimensionless) is standing for inductance, and the vertical axis is standing for amplitude of Tuned Mass Damper (in mm), and useful power (in Watt). The analysis of this chart shows that the amplitudes of Tuned Mass Damper are growing slightly with increased numbers of inductance, and values of useful power are falling down with increased numbers of inductance. It could be concluded that for power efficiency it is better to select the inductance with minimal values.

The relationship between amplitude of Tuned Mass Damper and a resistance R is plotted by the similar blue curve in Figure 4b; and a relationship between the peak of useful power P and a resistance R is plotted by red curve. Other parameters are  $\Phi = L = 1$ . The horizontal axis (dimensionless) is standing for resistance, and the vertical axis is the same as per Figure 4a. The analysis of this graph shows that the amplitudes of Tuned Mass Damper are reducing slightly down to some minimum at R=1.65 and raising slightly after that, and the values of useful power at this time are growing up slightly to some maximum at R=1.65 and reducing slightly after that. It could be stated that for power efficiency it is better to select the resistance values in vicinity of R = 1.65 for that particular combination of parameters.



Fig. 4: a. Amplitude of Tuned Mass Damper and peak of useful power vs. Inductance; b. Amplitude of Tuned Mass Damper and peak of useful power vs. Resistance; c. Amplitude of Tuned Mass Damper and peak of useful power vs. linkage factor.

The relationship between amplitude of Tuned Mass Damper and a linkage factor  $\Phi$  is plotted by the similar blue curve in Figure 4c; and a relationship between useful power P and a linkage factor  $\Phi$  is plotted by red curve. Other parameters are R = L = 1. The horizontal axis (dimensionless) is standing for linkage factor, and the vertical axis is the same as in Figure 4a. The analysis of this graph shows that the amplitudes of Tuned Mass Damper are reducing down. The observer would see the inflection point at  $\Phi$  = 2.5. The values of useful power P are growing significantly up to point  $\Phi$  = 2.5 and reducing after that. It could be stated that for power efficiency it is better to select the resistance values in vicinity of  $\Phi$  = 2.5 in this case. This study illustrates that the implementation of Alternator subsystem into Tuned Mass Damper design lead to changes of magnitudes of its amplitudes, and increasing values of useful power means the reduction of Tuned Mass Damper's amplitudes numbers. The analysis of graphs presented in Figures 4a – 4c tells that the linkage factor provides the most significant influence on amplitudes of Tuned Mass Damper.

#### 4. Modelling in Case of Rectilinear Vibrating Complex

The Free-Piston Linear Internal Combustion Engine without crankshaft combined with Electrical Generator completed by auxiliary drive [6], [7] is an example of device vibrating in one line direction (rectilinear) or object having single degree of freedom. Its model can be described by an equation (1). An attachment of Tuned Mass Damper completed with Alternator to Hybrid Complex is providing two opportunities to engineer: a reduction of level of body vibrations and an effective conversion of damped vibrations into useful power. Actually these opportunities could involve Tuned Mass Damper completed with Alternator in conflict situation. Trying to reach the best result for reduction of Complex vibrations at 50 Hz, the system was readjusted in the following way: the viscosity coefficient b1 in initial conditions was selected as b1=70 kg/s, an inductance was changed to L=4 in the same time, and the electrical resistance was changed to R=1.5.



Fig. 5: Dynamics of EVEH focused on tuning damping of Hybrid Complex vibrations.

The amplitude - frequency characteristic of Hybrid Complex equipped by Tuned Mass Damper could be reflected by blue dotted curve as shown in Figure 5. The abscissa axis stands for frequency (in Hz) of Engine force in Figure 5, and ordinate axis stands for **a.** amplitudes of Complex vibrations in mm (red solid curve reflects the initial dynamical behaviour of Complex without Tuned Mass Damper), **b**. peak value of useful power in W (magenta dashed curve). The analysis of obtained results tells that the amplitude of Complex vibrations is reduced down to 0.051 mm at 50 Hz, or reduction of level is in 2.84 times in comparison to levels of Complex vibrations without installed Tuned Mass Damper. The peak value of useful power is 0.064 W, and average value of useful power is 0.032 W. Having the assigned parameters it could be stated that the vibration reduction results are best. However the result for using of wasted energy is far from optimal. The presented case illustrates the EVEH focused on tuning damping of Hybrid Complex vibrations.

Let's make some changes in selected electromagnetic parameters – go back to L=1. The amplitude - frequency characteristic of Hybrid Complex equipped by Tuned Mass Damper and the peak value of useful power will be plotted in Figure 6 having the same coordinates like for Figure 5. The analysis of graphs in Figure 6 shows that the amplitude of Complex vibrations is reduced down to 0.059 mm at 50 Hz, or reduction of level is in 2.46 times in comparison to levels of Complex vibrations without installed Tuned Mass Damper, and the peak value of useful power is 0.47 W, and average value of useful power is 0.235 W. The effectiveness of using wasted power increased now in 7.34 times. There is an attractive result in vibration reduction combined to satisfactory efforts of using the wasted power in latest case. This case is illustrating the EVEH focused on reasonable balance between tuning damping of Hybrid Complex vibrations and effectiveness of using wasted power.



Fig. 6: Dynamics of EVEH focused on balance between tuning damping and using wasted power.

The further examine of considered system shows that greater numbers of useful power could be achieved at the efficiency of vibration reduction. There is an illustration of that statement presented in Figure 7. The coordinates in Figure 7 are the same as in Figure 5, and the names of the plotted curves are the same as well. The new change in initial conditions is  $\Phi$ = 1.55 for this case. As a result the amplitude of Complex vibrations is reduced down to 0.073 mm at 50 Hz, or reduction of level is in 2 times in comparison to levels of Complex vibrations without installed Tuned Mass Damper, and the peak value of useful power is 0.852 W, and average value of useful power is 0.436 W, or effectiveness of using wasted power is radically increased. This is the example of the EVEH focused on effectiveness of using wasted power.



Fig. 7: Dynamics of EVEH focused on using wasted power.

The prototype of EVEH focused on tuning damping of Hybrid Complex vibrations was designed and fabricated. The design parameters were composed specifically for mounting on 18 kW Free-Piston Linear Internal Combustion Engine linked to Electrical Generator equipped by Additional Auxiliary Drive [6],[7], and main its characteristics were based on previously presented modelling. EVEH was horizontally installed, and it was capable to annihilate rectilinear vibrations. The EVEN photo is shown in Figure 8. The authors developed several configurations of EVEH applicable for vibration damping in various rectilinear directions. The prototype of EVEH focused on tuning damping of Hybrid Complex vibrations was assembled on 18 kW Free-Piston Linear Internal Combustion Engine linked to Electrical Generator equipped by Additional Auxiliary Drive [6],[7] and tested.



Fig. 8: Photo of rectilinear EVEN.

The operational frequency of Hybrid Complex was fixed and equal to 50 Hz. The time interval of one stroke was 0.01 sec. The measured average amplitude of Complex vibrations before installation of EVEH was 0.15 mm. The measured average amplitude of Complex vibrations with assembled EVEH was 0.06 mm. The comparison of magnitudes measured in an experiment to data obtained in previous theoretical study tells that the deviations of obtained data from theoretically predicted ones are 3.45% for amplitude of Complex vibrations before installation of EVEH, 17.65% for amplitude of Complex vibrations with assembled EVEH, 12.1% for vibration level reduction rate. The obtained results illustrate a satisfactory compliance of real fabricated device to theoretically developed modelling. The recorded useful power was visualized specifically for convenience purposes as a graph in coordinates "useful power in W" vs. "time in seconds" (see Figure 9). The analysis of presented graph brings us to conclusion that the measured average useful power is 0.03 Watt. The comparison of values metered during test to data received theoretically shows that the deviation of obtained data from theoretically predicted ones are 3.45% and one can say that this result confirms that it is a satisfactory compliance of real manufactured EVEH to theoretically predicted model.



Fig. 9: The recorded useful power.

### 5. EVEH Modeling in Case of Torsional Vibration Damping

The description and model of Torsional Vibration Damper containing electromagnetic subsystem generating additional power was represented in [16]. Analysis of equations for mechanical subsystem [16] allows establish the equivalence between torsional vibrations and linear vibrations examined herein earlier, see equations (1) - (2). For converting the rectilinear system into rotational one it is necessary to substitute masses by a mass moments of inertia and linear coordinates by angular ones. It gives an opportunity to apply for torsional vibration damping study all theoretically

above-obtained results. For experimental purposes the Diesel Engine 1.9L installed in Hybrid Complex was selected. The fixed operation speed was 4700 rpm. There was no Torsional Vibration Damper during preliminary tests (see Figure 10a). The 6th order torsional vibrations @ 4700 rpm were recorded and a value of peak - to - peak angle was equal to 0.63 degrees (see Figure 10a). In the Figure 10 on abscissa axis the frequencies in rpm are shown, and on ordinate axis the of torsional peak-to-peak angle vibration Damper. The Damper was designed following patented schemes in [15] and developed recommendations. The view of manufactured prototype is shown in Figure 11. This Damper was tuned to 470 Hz [16], and it included EVEH focused on reasonable balance between tuning damping of Hybrid Complex vibrations and effectiveness of using wasted power. The Figure 10b illustrates the tests results of considered Hybrid Complex with installed Damper.



Fig. 10: Test recorded values of peak - to - peak angle; a. no Torsional Vibration Damper; b. with Torsional Vibration Damper installed.



Fig. 11: Torsional Vibration Damper with EVEH.

One can see that installation of Torsional EVEH makes the changing in dynamics. Now the 6th order torsional vibrations has shifted peak to lower frequency, actually now the resonance peak is equal to 0.22 degrees @ 3615 rpm. However taking into an account that the operation is at 4700 rpm, the observer could notice that the value of peak - to - peak angle is equal to 0.09 degrees, or the level reduction rate is 7 times, in other words the value of peak - to - peak angle is reduced in 7 times in operating regime. The measured average useful power was 1.17 Watt. Such results illustrate the optimistic perspectives for usage of EVEH in torsional vibration suppression configuration.

The authors need to highlight one of the perspective ways of EVEH usage which was practically missed in R&D practice. It is the rerouting of produced power to Tuned Mass Damper subsystem for reducing its viscosity factor **b1** [10],[11]. Such EVEH usage significantly improves the Tuned Mass Damper efficiency. The engineer could find that a rerouting of the useful power signal for viscous factor compensation gives the magnificent opportunity for further suppression of negative Hybrid Complex vibrations. However the designer must take into account that this phenomenon is caused by increasing amplitude of Tuned Mass Damper, which could cause (in its turn) the further raising of a magnitudes of useful power. It is important to keep that process of Damper's displacements increasing combined to power raising in stable dynamic zone. It should be done using, for instance, the control schemes described in [15].

It is necessary to mention that the presented design solutions are not restricted, and they could be applicable in various situations in all types of vehicles.

### 6. Conclusion

Elaborated Vibration Energy Harvesters based on Electromagnetic Tuned Mass Dampers has several improved features. The universality of application of proposed scheme for "rectilinear vibration operation" and "angular – torsional operation" is one of the most important issues among others. The main purpose of design and implementation of proposed Electromagnetic Vibration Energy Harvester is to serve dual goals: **a**. reduce the vibration levels of Hybrid Complex by Controlled Tuned Mass Dampers, **b**. generating additional power which could be used in Hybrid Vehicle.

The implementation of suggested Electromagnetic Vibration Energy Harvesters provides flexibility in applications: it gives a possibility to use them in such formats as a. focused on tuning damping of Hybrid Complex vibrations, b. focused on reasonable balance between tuning damping of Hybrid Complex vibrations and effectiveness of using wasted power, c. focused on effectiveness of using wasted power.

Usage of proposed design solutions allows redirect the useful power for compensation of Tuned Mass Damper's viscous factor. Due to composed control system it is possible to build Electromagnetic Vibration Energy Harvester operating in stable zones.

The developed Electromagnetic Vibration Energy Harvesters could be installed and used successfully in all types of vehicles having vibrating modules, units and parts.

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