
Amen Younes, Lyes Kadem
Department of Mechanical and Industrial Engineering, Concordia University
1455 de Maisonneuve Blvd. W, Montreal, QC, Canada, H3G 1M8
amyounes2001@gmail.com; lyes.kadem@concordia.ca

Ibrahim Hassan
Mechanical Engineering Department, Texas A&M University at Qatar
P O Box 23874, Doha, Qatar
Ibrahim.hassan@qatar.tamu.edu

Abstract- An analytical model for predicting heat transfer in annular flow regime for saturated flow boiling in a horizontal micro-tube subjected to a uniform heat flux has been developed based on one-dimensional separated flow model. Four main parameters have been considered, namely, vapor velocity, liquid film velocity, liquid film thickness, and the local pressure. Liquid film evaporation has been considered as a dominant heat transfer mechanism in the annular flow regime. Based on Taylors’ law, the initial value of the liquid film thickness at the onset of annular flow is estimated by employing the proper correlations available in literature. The main model equations obtained have been solved numerically based on explicit Runge-Kutta method. More than 420 two-phase heat transfer data points of annular flow regime for various working fluids, R134a and CO2 have been collected for the model validation. The data points were recorded for round macro and micro single horizontal channels with a range of inner diameter of $0.244 \, mm \leq D_h \leq 3.64 \, mm$, and a heated length to diameter ratio of $100 \leq (L_h/D_h) \leq 1300$. The range of liquid to vapor density ratios is $6.37 \leq (\rho_l/\rho_g) \leq 62.34$. The model was tested for equivalent Reynolds numbers within the range of $1,130 \leq Re_{eq} \leq 55,260$. In the scope of annular flow regime, the present model predicted well the collected heat transfer data with a mean absolute error (MAE) of 26.80%.

Keywords: Flow boiling, micro-channels, uniform heat flux, annular flow regime, two-phase heat transfer coefficient, separated flow model, liquid film.

Nomenclatures

<table>
<thead>
<tr>
<th>$A$</th>
<th>cross section area</th>
<th>(m$^2$)</th>
<th>Greek symbols</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D$</td>
<td>tube diameter</td>
<td>(m)</td>
<td>$\alpha$ void fraction</td>
</tr>
<tr>
<td>$C.V.$</td>
<td>Control volum</td>
<td>(–)</td>
<td>$\Gamma$ flow rate of evaporation (kg/m$^2$.s)</td>
</tr>
<tr>
<td>$f$</td>
<td>Friction factor</td>
<td>(–)</td>
<td>$\delta$ liquid film thickness (m)</td>
</tr>
<tr>
<td>$G$</td>
<td>mass flux</td>
<td>(kg/m$^2$.s)</td>
<td>$\rho$ Density (kg/m$^3$)</td>
</tr>
<tr>
<td>$h_f$</td>
<td>liquid specific enthalpy</td>
<td>(J/kg)</td>
<td>$\tau$ shear stress (N/m$^2$)</td>
</tr>
<tr>
<td>$h_g$</td>
<td>vapor specific enthalpy</td>
<td>(J/kg)</td>
<td></td>
</tr>
<tr>
<td>$h$</td>
<td>Heat transfer coefficient</td>
<td>(W/m$^2$.K)</td>
<td></td>
</tr>
<tr>
<td>$k_f$</td>
<td>Liquid Thermal conductivity</td>
<td>(W/m.K)</td>
<td></td>
</tr>
<tr>
<td>$L_h$</td>
<td>heated length</td>
<td>(m)</td>
<td></td>
</tr>
<tr>
<td>$MAE$</td>
<td>Mean absolute error</td>
<td>(–)</td>
<td></td>
</tr>
<tr>
<td>$p$</td>
<td>pressure</td>
<td>(N/m$^2$)</td>
<td></td>
</tr>
</tbody>
</table>

$Exp$ experimental

subscripts

$Exp$ experimental

$\rho_l$ liquid or liquid film

$g$ gas or vapor phase

$h$ heated
1. Introduction

Micro-channels heat sinks are very important in industries and aerospace applications, since they provide high thermal effectiveness, low weight, and small size. Consequently, in recent years, there has been an increasing interest in studying and investigating flow boiling in micro-channels. Additionally, predicting heat transfer coefficient for flow boiling in micro-channels is one of the most important steps for design micro-channel heat sinks. Several experimental investigations have been done recently for flow boiling in micro-channels. Some of them were performed for horizontal micro-channels at adiabatic flow conditions such as those presented by Cavallini et al. (2009), Zhang et al. (2004), Yun et al. (2005), Basu et al. (2011), Oh et al. (2011), Ali et al. (2012), Mahmoud and Karayiannis (2013), and Ducoulombier et al. (2011), while few others were performed at diabatic flow conditions, as those presented by Tibiriçă et al. (2011), Yan and Lin (1998), and Wu et al. (2011). On the other hand, only a few analytical models of flow boiling heat transfer in micro-channels have been presented in the literature (e.g. Moriyama and Inoue (1992), LaClair and Mudawar (2009), Na and Chung (2011), Das et al. (2006), Jacobi and Thome (2002), Thome et al. (2004)).

Heat transfer mechanism is influenced by flow pattern regime and it is of importance to know the dominant heat transfer mechanism for developing a more realistic flow boiling model. Many researchers, (e.g. Kew and Cornwell, (1997), Bao et al. (2000), Yen et al. (2003), Kandlikar (2004), and Pamitran et al. (2011)), have revealed that dominant heat transfer mechanism for boiling in small-and micro-channels is a flow pattern dependent. Furthermore it is influenced by the channel size as has been studied by Park and Hrnjak (2007), who found that the nucleate boiling heat transfer is dominant in conventional tubes, whereas the convective boiling heat transfer becomes dominant as the tube diameter decreases. According to some other investigators (e.g. Lee and Lee (2001), Qu and Mudawar (2004), Ong and Thome (2009), and Thome and Consolini (2010)), it has been proven that at intermediate and high vapor quality, where the annular flow pattern is usually observed (e.g. Tran et al. (2000)), the evaporation of liquid film is the dominant heat transfer mechanism.

Identifying the flow pattern and its boundaries is an essential key for choosing the suitable prediction method. Consequently, several researches for developing a flow pattern map for flow boiling in micro-channels have been carried out recently (e.g. Kandlikar (2002), Garimella et al. (2003), Wu and Cheng (2003), Revellin and Thome (2007a), Saisorn and Wongwises (2008), Karayiannis et al. (2010), Harirchian and Garimella (2009), and Thome and Consolini (2010)). However, most of these flow pattern maps were established based on adiabatic flow database. Thome and El Hajal (2003) have developed a flow pattern map proposed by Zürcher et al. (2002), which was able to predict fairly the two-phase flow regimes of seven various refrigerants. Thus, it has been used in this work for identifying the boundaries of annular flow pattern. In summary, most of the investigations conducted for predicting two-phase heat transfer for flow boiling in micro-channels are experimental and applicable for specific operating conditions while, only few analytical models have been developed. Thus, developing more analytical models which can be applicable for wider range of operating conditions and fluids is in need. The aim of this paper is to develop an analytical model for predicting two phase heat transfer coefficient of flow boiling in a horizontal micro-channel subjected to a uniform heat flux.

2. Analytical Modelling of Annular Flow Pattern

It has been verified in many previous studies (e.g. Jiang et al. (2001), Celata et al. (2001), Thome (2004), Revellin and Thome (2007b), Cioncolini and Thome (2012)), that annular flow is promised to be
one of the main dominant flow patterns for flow boiling in micro-channels. A physical description of a two-phase annular flow model is shown in Fig. 1. As can be seen, the annular flow consists of two continuous phases, the vapor core phase which flows with a velocity, namely, , and the liquid film, whose thickness is , which moves with a velocity, namely, , in the axil flow direction, . The micro-tube with inner diameter, , is subjected to a uniform heat flux, .

Fig. 1. Sketch of an annular flow model in horizontal circular channel

2. 1. Assumptions For Derivation Of Model Equations:
Two-phase annular flow boiling is a complex phenomenon and has many challenges to model its real features. Thus, for model simplification, the following assumptions have been taken into account:
1. The flow boiling is considered as a one dimensional steady state and fully developed flow.
2. The dominant heat transfer mechanism in annular flow regime is the evaporation of the liquid thin film.
3. The micro-tube is subjected to a circumferential and axial uniform heat flux, and the working fluid enters the micro-tube at saturation conditions, and the liquid and vapor phases are considered to be at thermodynamic equilibrium.
4. As the channel size decreases, the gravity force effect decreases. Thus, in the present model the influence of gravity force is assumed to be neglected.
5. No entrained liquid droplets exist in the vapor core and no bubbles in the liquid film trapped in the annular flow regime for flow boiling in micro-channels.
6. The conduction heat transfer in axial flow direction is neglected.
7. Taking the above assumptions into account, the conservation equations are derived as in the following sections.

2. 2. Conservation Equations:
The basic equations of the model have been derived by applying the mass, momentum, and energy balances for each phase in the chosen control volume, as depicted in Fig. 2., considering the above listed assumptions.

Equations of Mass Conservation: The mass balance has been applied for each phase, liquid film and vapor core phases, in the chosen control volume as shown in Fig. 2a. Consequently, the mass conservation equation of the liquid film phase and vapor phase for one dimensional and steady flow can be deduced and expressed by Eqs. (1)–(2) respectively.
Equations of Momentum Conservation: The forces acting on the chosen control volume are shown in 2-b. The body and virtual mass forces were neglected here. For flow boiling in micro-channels, the effect of gravitational body force is small compared to the viscous and inertial forces as it has been investigated in several previous studies, (e.g. Kandlikar (2004), Serizawa et al. (2002)). So it was neglected. The virtual mass force was neglected for model simplifications. Newton’s Law is applied to each phase in the control volume for deriving the phasic momentum equations which are expressed by Eqs. (5 and 6) as follows:

\[
\frac{d}{dz}\left[\rho_f u_f^2 \delta(D - \delta)\right] = -\frac{d}{dz}\left[p\delta(D - \delta)\right] - \tau_w D - \Gamma\left(\frac{u_i}{2}\right) - \tau_i(D - 2\delta) \\
\frac{d}{dz}\left[\rho_g u_g^2 (D - 2\delta)^2\right] = -\frac{d}{dz}\left[p(D - 2\delta)^2\right] + 4\Gamma\left(\frac{u_i}{2}\right) - 4\tau_i(D - 2\delta)
\]

Where, \(\tau_w\) and \(\tau_i\), are the wall and interfacial shear stresses respectively, and \(u_i\), is the interfacial velocity. The wall shear stress is defined as follows:
\[ \tau_w = \frac{1}{2} \rho_f f_r u_f^2 \]  

(7)

For wide range of flow conditions which cover laminar and turbulent flow conditions, the liquid film friction factor \( f_r \) is calculated in terms of liquid film Reynolds number for better results as presented in Kim and Mudawar (2012) as follows:

\[
f_r = \begin{cases} 
\left( \frac{64}{Re_f} \right) & \text{for } Re_f \leq 2000 \\
0.079Re_f^{-0.25} & \text{for } 2000 < Re_f \leq 20,000 \\
0.046Re_f^{-0.20} & \text{for } 20,000 < Re_f 
\end{cases}
\]  

(8)

Since the liquid film velocity is considered lower than the vapor core velocity and the tangential components of both velocities at the interface are considered to be the same. Thus, it would be reasonable to assume \( u_i = (0.5u_g) \) for simplifying the analysis of the present model.

**Equations of Energy Conservation:** Considering the rate of evaporation of the liquid film \( \Gamma \), as represented by Eq. (3), and applying the first law of thermodynamic for the liquid film and vapor core phases, the equations of energy conservation of liquid and vapor phases respectively are written as follows:

\[
\left[ \rho_f u_f \left( h_f + \frac{u_f^2}{2} \right) - \rho_g u_g h_f \right] \frac{d\delta}{dz} + \rho_g h_f \frac{(D-2\delta)}{4} \frac{du_g}{dz} + \rho_f \left( h_f + \frac{3}{2} u_f^2 \right) \frac{\delta(D-\delta)}{D-2\delta} \frac{du_f}{dz} = q_w D + \tau_i u_i 
\]

(9)

\[
(2\rho_g u_g^3) \frac{d\delta}{dz} + \frac{3}{2} \rho_g u_g^2 (D-2\delta) \frac{du_g}{dz} = -4\tau_i u_i 
\]

(10)

Where, \( q_w \), is the uniform heat flux applied at the external surface of the micro-channel, \( h_f \), and \( h_g \) are the enthalpy of the liquid film and the vapor core respectively. The interfacial shear stress is one of the most important parameters in the annular flow regime, and has a significant effect on the liquid film thickness. Taking into account the energy equation of the vapor phase, Eq. (10), and the assumption that \( u_i = (0.5u_g) \), the interfacial shear stress can be expressed as follows:

\[
\tau_i = \rho_g u_g^2 \frac{\Delta \delta}{dz} - 3\rho_g u_g \frac{(D-2\delta)}{4} \frac{du_g}{dz} 
\]

(11)

Substituting by the interfacial shear stress as defined by Eq. (11) in Eqs. (5 and 6), the momentum equations of liquid and vapor phases can be recast as follows:

\[
\left[ \rho_f u_f^2 - \frac{3}{2} \rho_g u_g^2 + p \right] \frac{d\delta}{dz} + \frac{7}{8} \rho_g u_g (D-2\delta) \frac{du_g}{dz} + 2\rho_f u_f \frac{\delta(D-\delta)}{D-2\delta} \frac{du_f}{dz} + \frac{\delta(D-\delta)}{D-2\delta} \frac{dp}{dz} = \tau_w \frac{\delta(D-\delta)}{D-2\delta} 
\]

(12)

\[
\frac{1}{2} \rho_g u_g^2 + p \frac{d\delta}{dz} - \left[ \frac{3}{8} \rho_g u_g (D-2\delta) \right] \frac{du_g}{dz} + \frac{1}{4} (D-2\delta) \frac{dp}{dz} = 0
\]

(13)

The liquid energy equation, Eq. (9), can be recast as follows:

\[
\left[ \rho_f u_f \left( h_f + \frac{u_f^2}{2} \right) - \rho_g u_g \left( h_f + \frac{u_g^2}{2} \right) \right] \frac{d\delta}{dz} + \left[ \rho_g \frac{(D-2\delta)}{4} \left( h_f + \frac{3}{2} u_g^2 \right) \right] \frac{du_g}{dz} + \rho_f \left( h_f + \frac{3}{2} u_f^2 \right) \frac{\delta(D-\delta)}{D-2\delta} \frac{du_f}{dz} = q_w^* \frac{D}{D-2\delta} 
\]

(14)
3. Solution Procedure

The final form of the present model is represented by four main equations which are the mass conservation equation of the whole mixture, Eq. (4), the momentum conservation equation of the liquid film, Eq. (12), and the momentum conservation equation of the vapor phase, Eq. (13), the energy conservation equation of the liquid film, Eq. (14), which have been solved numerically. In the present model, the initial liquid film thickness is predicted using the correlations developed by Han and Shikazono (2009, 2010), as expressed by Eq. (15) as follow:

\[
\left(\frac{\delta_o}{D}\right) = \min \left[\left(\frac{\delta_o}{D}\right)_{\text{steady}}, \left(\frac{\delta_o}{D}\right)_{\text{accel}}\right]
\] (15)

Where \( (\delta_o/D)_{\text{steady}} \) is the ratio of the initial values of the liquid film thickness and tube diameter for steady case, while the term \( (\delta_o/D)_{\text{accel}} \) is the ratio of the initial values of the liquid film thickness and tube diameter considering the influence of bubble acceleration. The initial values of \( u_{go} \), and \( u_{fo} \) are calculated in terms of initial liquid film thickness using Eq. (16) and Eq. (17) respectively.

\[
u_{go} = \left(\frac{G x_e/\rho_g}{D^2/(D - 2\delta_o)^2}\right)
\] (16)

\[
u_{fo} = \left(\frac{G(1 - x_e)/\rho_f}{D^2/(4\delta_o(D - \delta_o))}\right)
\] (17)

Where, \( x_e \) is the vapor quality defined by Eq. (18) in terms of the heated length-to-diameter ratio \( (L_h/D_h) \), and Boiling number, \( Bl = (q_w/Gh_{fg}) \).

\[
x_e = 4Bl(L_h/D_h)
\] (18)

4. Results And Discussion

By predicting the variation of liquid film thickness \( \delta(z) \) in terms of heated length, and assuming that the heat transferred by conduction through the channel wall equals to that transferred by convection through the liquid film. Thus, the local two-phase heat transfer coefficient \( h_{tp}(z) \), under uniform heat flux conditions may be represented by Eq. (19) as follows:

\[
h_{tp}(z) = k_f/\delta(z)
\] (19)

Where \( k_f \), is the thermal conductivity of the liquid phase, which is always much larger than that of the vapor phase, of the working fluid estimated at the inlet saturation conditions. By obtaining the local two-phase heat transfer coefficient along the heated length of the micro-channel, the average two-phase heat transfer coefficient is calculated by the following expression, Eq. (20).

\[
\overline{h_{tp}} = \frac{1}{N} \int_0^{L_h} \frac{h_{tp}(z)}{dz}
\] (20)

Where \( N \) is the number of segments divided along the micro-tube, and \( L_h \) is the heated length. Two-phase heat transfer coefficient has been calculated by the present model and compared with various experimental heat transfer data for wide range of operating conditions for flow boiling in single mini/micro-channels. For instance, Fig. 3. shows the variation of two-phase heat transfer coefficient of saturated flow boiling of the refrigerant CO2 in terms of vapor quality as predicted by the present model for flowing through a mini-circular tube has an inner diameter, 1.5 mm, at an inlet saturation temperature \( T_{sat} = 10 \, ^\circ C \). It is apparent that by increasing the mass flux and the vapor quality, the heat transfer coefficient increases. Compare to the heat transfer data measured by Choi et al (2007), a good agreement with the tested data at an intermediate and a high vapor quality range, \( 0.15 \leq x_{exit} \leq 0.85 \), can be seen.
clearly. For the tested range of equivalent Reynolds numbers, $8,700 \leq \text{Re}_{\text{eq}} \leq 12,060$, the model predicted the heat transfer data of the CO2 with MAE = 7.19%. Overall, the present model has been compared with 423 two-phase heat transfer data points which have been collected from different resources for annular flow regime for two working fluids (R134a, and CO2), and various operating conditions. Fig. 1 shows that the present model predicted well the tested experimental heat transfer data and gave a good agreement with a MAE of 26.80%.

![Graph](image)

Fig. 3. Variation of heat transfer coefficient for flow boiling of the refrigerant CO2 with exit vapor quality, and comparison with the experimental data of Choi et al. (2007).

### 5. Conclusions

An analytical model to predict the two-phase heat transfer in annular flow regime for flow boiling in a horizontal circular micro-channel subjected to a uniform heat flux, has been developed. The present model was developed based on the separated flow model for two-phase flow. The influences of working fluid, channel size, and operating conditions on the two-phase heat transfer have been taken into account. It has been noted that, in the annular flow regime, the two-phase heat transfer coefficient increase as the mass flux and exit vapor quality increase. The model has been compared with 423 two-phase heat transfer data points collected from the literature for flow boiling conditions that cover a range of equivalent Reynolds number $1,127 \leq \text{Re}_{\text{eq}} \leq 33,184$. The tested heat transfer data points have been predicted fairly by the present model with MAE of 26.80%. The main application of the present model can be addressed here is predicting the two-phase heat transfer coefficient in the annular flow regime for flow boiling in horizontal micro-channels, and consequently estimating the cooling load of the micro-channel heat sink based on the chosen operating condition.
Fig. 1. Heat transfer coefficients predicted by the present model against those measured experimentally (423 data points) for various working fluids and operating conditions.

References


