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Reynolds-constrained Subgrid-scale Model for Large-eddy Simulation of Turbulent Wall Flows

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Abstract- A novel subgrid-scale (SGS) modelling approach is introduced for large-eddy simulation (LES) of both incompressible and compressible turbulent wall flows. The proposed new model is composed of two parts depending on the distance to the nearest wall. In the near-wall region, the mean SGS stress tensor is constrained by an external Reynolds stress tensor to ensure the total target stress, while the fluctuating stress tensor is closed in a traditional fashion using a residual model parameterization. In the far-wall region, the conventional SGS model can be directly employed with necessary smoothing operation in the neighbourhood of the interface. For compressible case, the SGS heat flux vector, as an addition to the SGS stress tensor, shall be parameterized in a similar way. The validity and fidelity of the developed models are tested via large-eddy simulations of both attached and detached flow configurations, including turbulent channel flows, flows around a circular cylinder, flow over periodic hills, flow around two tandem cylinders, etc. The obtained results are well compared with those from numerical simulations using various approaches and experimental measurements. The constrained large-eddy simulation (CLES) method can capture fruitful multiscale flow structures, which are largely lacking in the flow field given by hybrid RANS/LES methods. Moreover, the CLES method proves to be much less sensitive to the grid resolution than traditional LES method, and make pure LES of flows of engineering interest feasible with moderate grids.

Keywords**:** SGS modelling, SGS stress tensor, SGS heat flux vector, CLES, Turbulent wall flows

1. Introduction

The computational fluid dynamics (CFD) techniques are playing increasingly important roles in both fundamental research and industrial engineering, in which turbulent wall flows are the most frequently encountered flow configuration. In nearly all the commercial or in-house CFD software, the Reynolds Averaged Navier-Stokes (RANS) simulations are commonly employed due to the low computational cost. The well-known Reynolds stress closures include the Spalart-Allmaras (SA) one-equation model (Spalart and Allmaras, 1994), the $k-\omega$ two-equation model, the Menter's shear stress transport (SST) model (Menter, 1994), etc. However, most of the RANS models suffer from the lack of robustness and universality when used to solve unsteady non-equilibrium separated flows. Large-eddy simulation (LES) method possesses the capability to resolve the three-dimensional (3D) unsteady and separated flows much more accurately when the required mesh resolution is fully satisfied because the subgrid-scale (SGS) models are believed to be more "universal" than the Reynolds stress models as needed in RANS. However, the application of pure LES technique to turbulent wall flows of engineering interest is still far from feasible due to its fairly fine grids requirement (Piomelli and Balaras, 2002).

The invention of hybrid RANS-LES method has infused new life and energy into the numerical simulation of turbulent wall flows, especially with massive separations (Fröhlich and von Terzi, 2008). In hybrid RANS/LES method, the RANS equations are solved in the near-wall region, while the LES equations are integrated in the rest region. A representative member in this family is referred to as detached-eddy simulation (DES) method (Spalart, 2009). Although the DES-type approaches have received favourable attention from the CFD community, they may suffer from the inherent drawbacks of unphysical transition from the smooth RANS filed to the fluctuating LES field. The log-layer mismatch (LLM) phenomenon observed in DES may cast doubts on the effectiveness and fidelity of DES approach for simulation of wall-bounded flows (Nikitin et al., 2000; Piomelli and Balaras, 2002; Spalart, 2009).

Mostly, the celebrated SGS models for LES of turbulent flows perform less well than expected due to the lack of sufficient grids and physically meaningful constraints. Chen et al. (2012) propose that an external Reynolds stress constraint should be imposed on the SGS modelling in the near-wall region for LES of incompressible turbulent wall flows (see also Xiao et al., 2014). For the SGS modelling in compressible case, however, the calculated near-wall Reynolds heat flux is suggested to be constrained by modelled Reynolds heat flux in addition to the Reynolds stress (Jiang et al., 2013; Chen et al., 2013; Xia et al., 2013; Hong et al. 2013). The aim of this paper is at providing a review of the constrained large-eddy simulation (CLES) technique for turbulent wall flows and evaluating its applicability for simulation of turbulent wall flows of engineering interest.

2. Reynolds-constrained SGS models

In the CLES approach for turbulent wall flows, the low-pass filtered Navier-Stokes equations are solved in the entire domain with the SGS models constructed in different forms within the near-wall and far-wall regions. In the far-wall region, traditional SGS models are employed, whereas in the near-wall region, the mean SGS models are constrained by given external Reynolds quantities.

2. 1. Incompressible Case

In incompressible case, our tusk is to close the SGS stress $\tau_{ij}^{LES} = \overline{u_i u_j} - \overline{u}_i \overline{u}_j$ based on the resolved velocity field $\overline{u}_i$. Comparing the RANS and the LES equations yields

$$\langle \tau_{ij}^{LES} \rangle = \tau_{ij}^{RANS} - (\langle \overline{u}_i \overline{u}_j \rangle - \langle \overline{u}_i \rangle \langle \overline{u}_j \rangle). \tag{1}$$

Here, the angle brackets denote ensemble average, and τ_{ij}^{RANS} is the modelled external Reynolds stress tensor, such as the SA one-equation model (Spalart and Allmaras, 1994), and the Menter's SST model (Menter, 1994) parameterized in terms of the resolved physical quantities. Therefore, the SGS stress model in the near-wall region can be decomposed into a mean part and a fluctuating part:

$$\tau_{ij}^{LES} = \langle \tau_{ij}^{LES} \rangle + (\tau_{ij}^{LES})'. \tag{2}$$

In Eq. (2), the mean SGS stress is determined by Eq. (1), and the fluctuating SGS stress is parameterized as

$$(\tau_{ij}^{LES})' = C_S'(\Delta^2 |\overline{S}| \overline{S}_{ij} - \langle \Delta^2 |\overline{S}| \overline{S}_{ij} \rangle). \tag{3}$$

Here, the Smagorinsky-type parameterization is employed, Δ is the filter width, $\overline{S}_{ij} = (\partial \overline{u}_i / \partial x_j + \partial \overline{u}_j / \partial x_i)/2$ is the filtered strain rate tensor, and $|\overline{S}| = (2\overline{S}_{ij}\overline{S}_{ij})^{1/2}$. The model coefficient C_S' can be identified using the dynamic procedure as proposed by Lilly (1992) based on the Germano identity (Germano, 1992). In summary, the SGS stress model in the near-wall region takes the following form

$$\tau_{ij}^{LES} = \tau_{ij}^{RANS} - (\langle \overline{u}_i \overline{u}_j \rangle - \langle \overline{u}_i \rangle \langle \overline{u}_j \rangle) - C_S'(\Delta^2 |\overline{S}| \overline{S}_{ij} - \langle \Delta^2 |\overline{S}| \overline{S}_{ij} \rangle). \tag{4}$$

In the far-wall region, the conventional dynamic Smagorinsky model (DSM) is used as usual (Lilly, 1992). More details on the derivation can be found in Chen et al. (2012).

2. 2. Compressible Case

In compressible case, both the SGS stress tensor $\tau_{ij}^{LES} = \bar{\rho}(\widetilde{u_i u_j} - \widetilde{u}_i \widetilde{u}_j)$ and the SGS heat flux $q_i^{LES} = \bar{\rho} C_p (\widetilde{T u_i} - \widetilde{T} \widetilde{u}_i)$ need to be modelled based on the resolved density, velocity and temperature fields, i.e., $\overline{\rho}$, $\widetilde{u}_i$ and $\widetilde{T}$, respectively. Here, an *overbar* denotes spatial filtering, and a *tilde* denotes Favre (or density-weighted) filtering. In analogy to the incompressible case, the SGS stress and heat flux are split into a mean part and a fluctuating part. The mean SGS stress and heat flux are constrained by prescribed external Reynolds stress and heat flux, τ_{ij}^{RANS} and q_i^{RANS}, respectively, and the fluctuating SGS effects can be closed according to the traditional compressible Smagorinsky formulations (see Martín et al., 2000). Formally, the SGS stress and heat flux in the near-wall region are given by:

$$\begin{aligned}\tau_{ij}^{LES} &= \tau_{ij}^{RANS} - \langle \overline{\rho} \rangle (\{\tilde{u}_i \tilde{u}_j\} - \{\tilde{u}_i\}\{\tilde{u}_j\}) \\ &\quad -2C'_S \left[\bar{\rho}\Delta^2 |\widetilde{S}| \left(\widetilde{S}_{ij} - \frac{1}{3}\widetilde{S}_{kk}\delta_{ij} \right) - \left\langle \bar{\rho}\Delta^2 |\widetilde{S}| \left(\widetilde{S}_{ij} - \frac{1}{3}\widetilde{S}_{kk}\delta_{ij} \right) \right\rangle \right] \\ &\quad +\frac{2}{3} C'_I \left[\bar{\rho}\Delta^2 |\widetilde{S}|^2 - \langle \bar{\rho}\Delta^2 |\widetilde{S}|^2 \rangle \right] \delta_{ij}, \end{aligned} \tag{5}$$

$$\begin{aligned} q_i^{LES} &= q_i^{RANS} - \langle \bar{\rho} \rangle C_p (\{\tilde{u}_i \tilde{T}\} - \{\tilde{u}_i\}\{\tilde{T}\}) \\ &\quad -\frac{C'_S}{Pr_T} \left(\Delta^2 \bar{\rho} C_p |\widetilde{S}| \frac{\partial \widetilde{T}}{\partial x_i} - \left\langle \Delta^2 \bar{\rho} C_p |\widetilde{S}| \frac{\partial \widetilde{T}}{\partial x_i} \right\rangle \right). \end{aligned} \tag{6}$$

Here, $\{\cdot\}$ represents the Favre (or density-weighted) average, C_p is the specific heat at constant pressure, C'_S and C'_I are the Smagorinsky constants, and Pr_T is the SGS Prandtl number, which can be prescribed empirically or calculated instantaneously through a dynamic procedure.

In the far-wall region, traditional Smagorinsky models for the SGS stress and heat flux are used as in Martín et al. (2000). Readers are referred to the paper by Jiang et al. (2013) for more details.

3. Results

The LES techniques using the Reynolds-constrained SGS models for both incompressible and compressible turbulent wall flows have been applied to simulations of various flow configurations, including incompressible turbulent channel flow (Chen et al., 2012), compressible turbulent flows in a plane channel (Jiang et al., 2013), past a circular cylinder (Hong et al., 2014), and around a commercial aircraft (Chen et al., 2013), etc. The CLES method has also been validated in simulation of the flow interaction around two tandem cylinders as studied experimentally by Jenkins et al. (2005) in the NASA-Langley Basic Aerodynamics Research Tunnel (BART). The Reynolds number based on the cylinder diameter is 1.66×10^5, and the cylinder spacing-to-diameter ratio is 3.70. Figs. 1 (a) and (b) show the color contours of the mean streamwise and vertical velocity components, which compare very well with the experimental results (the lower half of each figure). The improved delayed DES (IDDES) method can capture similar flow patterns and performs much better than other methods, e.g., DES (not shown here).

Shown in Fig. 1 (c) are the pressure distributions ($C_P = (p_\infty^{static} - p)/p_\infty^{dynamic}$) on the rear cylinder. It can be seen that the curves predicted by CLES and IDDES agree with the experimental data more than other methods (i.e., DES and DDES) do on nearly the whole cylinder surface but the vicinity of the forward stagnation point. Similar conclusions can be drawn for the upstream cylinder (not shown).

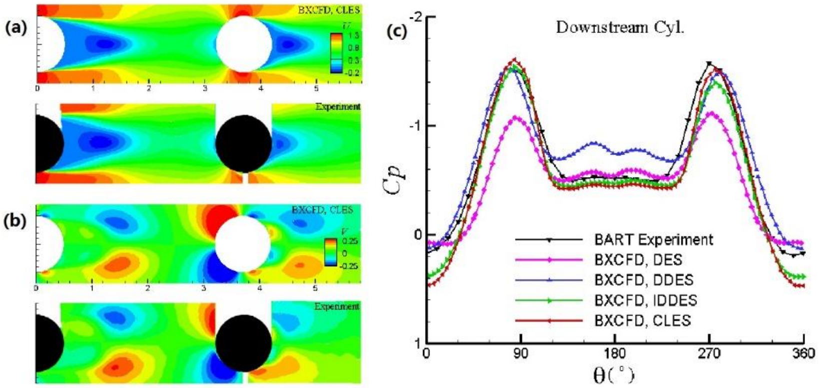

Fig. 1. (a) Mean streamwise velocity, (b) mean vertical velocity, and (c) pressure distribution on downstream cylinder. The experimental results are from Jenkins et al. (2005).

4. Conclusion

In this paper, the fundamental concepts and mathematical formulations of the Reynolds-constrained SGS modelling are briefly introduced. The performance of the CLES method has been evaluated, in comparison with other well-known methods, when used to simulate a variety of turbulent wall flows. The CLES method proves to possess in a certain degree superiority over traditional LES method and the widely used DES family in predicting typical mean and statistical quantities. It is suggested that the proposed CLES method be promising numerical tool for turbulent wall flows in the sense of both scientific research and engineering application.

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References

Chen S., Chen Y., Xia Z., Qu K., Shi Y., Xiao Z., Liu Q., Cai Q., Liu F., Lee C., Zhang R., Cai J. (2013). Constrained Large-Eddy Simulation And Detached Eddy Simulation Of Flow Past A Commercial Aircraft At 14 Degrees Angle Of Attack, *Sci. China-Phys. Mech. Astron.*, 56 (2), 270-276.

Chen S., Xia Z., Pei S., Wang J., Yang Y., Xiao Z., Shi Y. (2012). Reynolds-Stress-Constrained Large-Eddy Simulation of Wall-Bounded Turbulent Flows, *J. Fluid Mech.*, 703, 1-28.

Fröhlich J., von Terzi D. (2008). Hybrid LES/RANS Methods for the Simulation of Turbulent Flows, *Prog. Aerosp. Sci.*, 44, 349-377.

Germano M. (1992). Turbulence: The Filtering Approach, *J. Fluid Mech.*, 238, 325-336.

Hong R., Xia Z., Shi Y., Xiao Z., Chen S. (2014). Constrained Large-Eddy Simulation of Compressible Flow Past a Circular Cylinder, *Commun. Comput. Phys.*, 15(2), 388-421.

Jenkins L.N., Khorrami M. R., Choudhari M.M., McGinley C.B. (2005). Characterization of Unsteady Flow Structures around Tandem Cylinders for Component Interaction Studies in Airframe Noise, *AIAA Paper*, 2005-2812.

Jiang Z., Xiao Z., Shi Y., Chen S. (2013). Constrained Large-Eddy Simulation Of Wall-Bounded Compressible Turbulent Flows, *Phys. Fluids*, 25, 106102.

Lilly D.K. (1992). A Proposed Modification of Germano Subgrid-Scale Closure Method, *Phys. Fluids A*, 4, 633-635.

Martín M.P., Piomelli U., Candler G.V. (2000). Subgrid-Scale Models For Compressible Large-Eddy Simulations, *Theor. Comput. Fluid Dyna*, 13, 361-376.

Menter F.R. (1994) Two-Equation Eddy-Viscosity Turbulence Models For Engineering Applications, *AIAA Journal*, 32 (8), 1598-1605.

Nikitin N.V., Nicoud F., Wasistho B., Squires K.D., Spalart P.R. (2000). An Approach to Wall Modeling In Large-Eddy Simulations, *Phys. Fluids*, 12, 1629-1632.

Piomelli U., Balaras E. (2002) Wall-Layer Models For Large-Eddy Simulations, *Annu. Rev. Fluid Mech.*, 34, 349-374.

Spalart P.R. (2009). Detached-Eddy Simulation, *Annu. Rev. Fluid Mech.*, 41, 181-202.

Spalart P.R., Allmaras S.R. (1994). A One-Equation Turbulence Model for Aerodynamic Flows, *Recherche Aerospatiale*, 1, 5-21.

Xia Z., Shi Y., Hong R., Xiao Z., Chen S. (2013). Constrained Large-Eddy Simulation of Separated Flow in a Channel with Streamwise-Periodic Constrictions, *J. Turbul.*, 14 (1), 1-21.

Xiao Z., Shi Y., Xia Z., Chen S. (2014). Comment on "A Hybrid Subgrid-Scale Model Constrained By Reynolds Stress," *Phys. Fluids*, 26 (5), 2013.