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Actuation of Flexoelectric Membranes in Viscoelastic Fluids with Application to Outer Hair Cells

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Abstract- Liquid crystal flexoelectric actuation uses an imposed electric field to create membrane bending and it is used by the Outer Hair Cells (OHC) located in the inner ear, whose role is to amplify sound through generation of mechanical power. Oscillations in the OHC membranes create periodic viscoelastic flows in the contacting fluid media. A key objective of this work on flexoelectric actuation relevant to OHC is to find the relations and impact of the electro-mechanical properties of the membrane, the rheological properties of the viscoelastic media, and the frequency response of the generated mechanical power output. The model developed and used in this work is based on the integration of: (i) the flexoelectric membrane shape equation applied to a circular membrane attached to the inner surface of a circular capillary, and (ii) the coupled capillary flow of contacting viscoelastic phases, such that the membrane flexoelectric oscillations drive periodic viscoelastic capillary flows, as in OHCs. By applying the Fourier transform formalism to the governing equation an analytical expression for the transfer function, associated to the curvature and electrical field, power dissipation elastic storage were found. The integrated flexoelectric/viscoelastic model and the novel findings contribute to the ongoing quest for a fundamental understanding of the functioning of outer hair cells (OHC), especially on the role of membrane deformation in delivering mechanical power through electromotility and its frequency-dependent power conversion efficiency.

***Keywords*:** Rheology, Transport-Phenomena, Flexoelectric membrane actuation, Flexoelectric-driven viscoelastic capillary flow, rheological transfer function, Outer hair cells.

1. Introduction

In Nature and physiology, biological liquid crystals play significant roles as multifunctional materials (Abou-Dakka et al. 2012). This paper presents theory and simulation of a physiological actuator device whose functioning hinges on unique electro-mechanical properties of mesophases and that provides an example of responsive self-organized materials. The functioning of Outer Hair Cells (OHC) in the inner ear involves electric-field driven periodic curvature oscillations of liquid crystal (LC) elastic membranes that impart momentum and flow to the contacting viscoelastic fluids; the electric field actuation of the liquid crystal membrane is known as flexoelectricity (Abou-Dakka et al. 2012; Herrera-Valencia, Rey 2014). The key role of OHC is sound amplification in the presence of viscous dissipation and elastic storage (Abou-Dakka et al. 2012; Herrera-Valencia, Rey 2014). Hence, the full description and understanding of OHC functioning has to include the frequency response of flexoelectric membranes embedded in viscous and viscoelastic media due to an oscillating **E** field (Abou-Dakka et al. 2012; Herrera-Valencia, Rey 2014). The field of flexoelectric membranes was pioneered and developed by Petrov and co-workers (Abou-Dakka et al. 2012; Herrera-Valencia, Rey 2014). The generic and key features of the electrical to mechanical energy conversion system in OHCs). The input oscillating **E** field, through the electromechanical flexoelectric effect, produces curvature oscillations in the elastic membrane that forms the OHC (Abou-Dakka et al. 2012; Herrera-Valencia, Rey 2014) and that is surrounded by viscoelastic media. In turn, the oscillating elastic membrane displaces the contacting viscoelastic liquids through the mechanical viscoelasto-elasticity effect (Abou-Dakka et al. 2012; Herrera-Valencia, Rey 2014). The combined effect that allows the electro-mechanical energy conversion is based on the integration of flexoelectric effect (**E** field imposed on flexoelectric membrane) and the mechanical effect (membrane elasticity plus viscoelastic bulk fluid flow) (Abou-Dakka et al. 2012; Herrera-Valencia, Rey A.D. 2014). The two key issues in this energy conversion device are:

(i) How much power P is eventually delivered to the contacting viscoelastic fluids from the imposed oscillating electric field **E** and how much stored membrane elastic energy E_m is required to deliver that power, and

(ii) Under which material conditions, a well-localized resonant power peak is found (in the spectrum of P), as physiologically required.

As expected, the issues (i) and (ii) identified above depend on (a) the **E**-frequency ω as well as on (b) the material properties of the bio-device components discussed below (Abou-Dakka et al. 2012; Herrera-Valencia, Rey 2014).

1.1. Frequency response

The intensity of the linear momentum transfer from the oscillating membrane to the contacting viscoelastic fluids depends on the imposed frequency (Abou-Dakka et al. 2012; Herrera-Valencia, Rey A.D. 2014). Hence frequency-dependent viscoelasticity is an essential ingredient of this important biological LC electro-mechanical oscillator (Abou-Dakka et al. 2012; Herrera-Valencia, Rey A.D. 2014). Viscoelasticity is an important frequency-dependent property of synthetic and biological materials and processes (Abou-Dakka et al. 2012; Herrera-Valencia, Rey 2014). Biological systems respond differently to inputs of different frequencies (Abou-Dakka et al. 2012; Herrera-Valencia, Rey A.D. 2014). Some systems may amplify components of certain frequencies, and attenuate components of other frequencies (Abou-Dakka et al. 2012; Herrera-Valencia, Rey 2014) , and this property is crucial to understand the processes that control the functioning of OHCs (Abou-Dakka et al. 2012; Herrera-Valencia, Rey A.D. 2014) and hearing processes (Abou-Dakka et al. 2012; Herrera-Valencia, Rey 2014). The frequency response (Abou-Dakka et al. 2012; Herrera-Valencia, Rey 2014) is the relationship between the system input and output in the Fourier domain.

1. 2. Materials

Nematic liquid crystals (NLC) are multifunctional self-organizing viscoelastic anisotropic materials whose orientational order responds to external flow, electromagnetic, chemical, optical and surface fields (Petrov 2014; Rey 2005, 2006a-c, 2008a,b) the orientational order is defined by the director **n** and the elastic distortions by director gradients $\nabla\mathbf{n}$ (Petrov 2014; Rey 2005, 2006a-c, 2008a,b). A distinguishing and novel property of nematics is flexoelectricity (Petrov 2014; Rey 2005, 2006a-c, 2008a, b) which describes the coupling between orientational gradients and electric polarization, such that an applied electric field creates orientational distortions and distortions create macroscopic polarization (Petrov 2014; Rey 2005, 2006a-c, 2008a,b). Thin layers and biological membranes behave like liquid crystals, membranes should also exhibit flexoelectricity or couplings between polarization and bending

(Abou-Dakka et al. 2012; Herrera-Valencia, Rey 2014, Rey 2005, 2006a-c, 2008a, b). Figure 1 shows a schematic of flexoelectric polarization in rod-like and banana-like molecules and the corresponding membrane flexoelectric polarization; as noted above the physics and modeling is affected by identifying the director field n with the membrane normal **k**.

As a partial result, both the direct and converse membrane flexoelectric effects are sensor-actuator properties when membrane curvature and polarization are coupled as in nematic liquid crystals. Membrane flexoelectricity due to its inherent sensor-actuator capabilities is an area of current interest in soft matter materials (Abou-Dakka et al. 2012; Herrera-Valencia, Rey 2014). Over the years, much literature has dealt with the problem of measuring flexoelectric coefficients in various liquid crystals

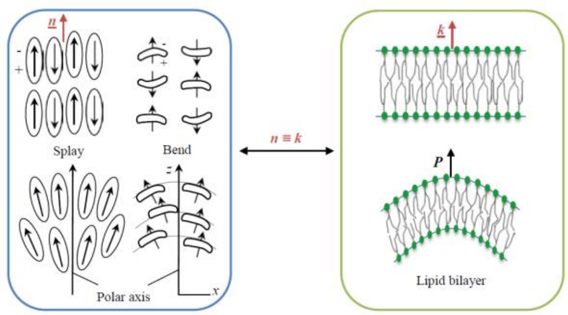
(Abou-Dakka et al. 2012; Herrera-Valencia, Rey 2014). For typical LC membranes, these coefficients range from 3-20 pC/m, but recent experiments have reported flexoelectricity coefficients of up to 35 nC/m in bent-core liquid crystals (Abou-Dakka et al. 2012; Herrera-Valencia, Rey 2014). Such large bend coefficients make bent-core liquid crystals practical materials for mechano-electric transduction (Abou-Dakka et al. 2012; Herrera-Valencia, Rey 2014).

The specific objectives of this paper are:

(1) To derive a high order dynamic linear model for a flexoelectric membrane attached to a capillary tube that contains viscoelastic liquids and is subjected to a fluctuating small amplitude electric fields of arbitrary frequency;

(2) To compute the frequency response of the electromechanical device, taking into account the viscoelastic nature of the contacting fluids;

(3) To use the modelling results to characterize the role of membrane flexoelectricity and contacting fluid viscoelasticity on the transfer function of the device;



(a) **(b)**

Fig. 1. (a) Flexoelectricity in rod-like and banana-shaped nematic liquid crystals due to slay and bend deformations of the director **n**. (b) Flexoelectricity in biological membranes due to bending curvature described by surface gradients of the unit normal **k**. The correspondence between nematic flexoelectricity and membrane flexoelectricity is obtained when the director **n** is identified with the membrane unit normal **k**. Adapted from Abou-Dakka et al. (2012).

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The specific objectives of this paper are:

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(5) To compute the frequency response of the electromechanical device, taking into account the viscoelastic nature of the contacting fluids;

(6) To use the modelling results to characterize the role of membrane flexoelectricity and contacting fluid viscoelasticity on the transfer function of the device;

2. Physical Problem

In this work, we model the system in the frequency domain, include momentum inertia, and develop a generic approach that can be used in the future with any viscoelastic constitutive equation, as required by experimental results (Abou-Dakka et al.2012; Herrera-Valencia & Rey 2014). The physical set-up and geometry of the flexoelectric membrane tethered to a capillary tube containing two viscoelastic fluids is depicted in Figure 2.

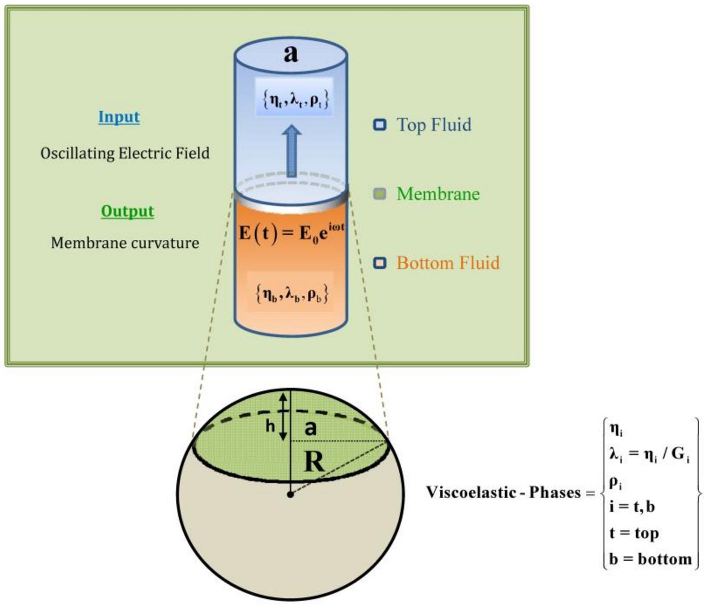

Fig. 2. Schematic of the geometry and operation of flexoelectric mechanics defined in Figure 2, in a capillary geometry of radius a and axial length L.

The input $\mathbf{E}$ field distorts the initially flat circular membrane into a spherical cap of radius R and height h. The flexoelectric actuation creates a capillary viscoelastic flow in the contacting top (t) and bottom (b) fluids of viscosities $\{\eta_t, \eta_b\}$, relaxation times $\{\lambda_t, \lambda_b\}$ and fluid densities $\{\rho_t, \rho_b\}$ respectively. Adapted from Abou-Dakka et al. (2012)

A capillary tube of radius “a” contains an edge-fixed flexoelectric membrane located at z = 0. Above and below the membrane there are two viscoelastic fluids with column heights z = L, viscosities $\{\eta_b, \eta_t\}$, relaxation times $\{\lambda_b, \lambda_t\}$ and densities $\{\rho_b, \rho_t\}$ respectively. The pressure at the top of the upper layer and at the bottom of the lower layers are equal to a constant, i.e. $p_t(\xi = 2L, t) = p_b(\xi = 0, t) = p_0$. By imposing a fluctuating electrical field $\mathbf{E}$ (t) the membrane oscillates and displaces the upper and lower incompressible viscoelastic fluids; we emphasize that the Poiseuille flow is only generated by the flexoelectric effect of the membrane caused by the imposed $\mathbf{E}(t)$ field (Rey 2005, 2006a-c, 2008a,b; Abou-Dakka et al. 2012; Herrera-Valencia & Rey 2014). The membrane deformation is described by a spherical dome of height h and radius R (Rey 2008a-c).

3. Electro-rheological Model

Here, we show that in the inertia-less regime the model can be mapped into a standard mechanical spring-dashpot model (Herrera-Valencia & Rey 2014). By neglecting the momentum inertia, and characterizing the viscoelastic media with two Maxwell fluids, the following second order linear differential equation was obtained by Abou-Dakka et al. (2012) and generalized by Herrera-Valencia & Rey (2014).

$$\left\{b_2^*\left(k,\ \bar{\lambda}_t\bar{\lambda}_b\right)\frac{d^2}{dt^2}+b_1^*\left(k,\Sigma_{\bar{\eta}}\right)\frac{d}{dt}+1\right\}\bar{H}\left(\bar{t}\right)=a_0^*\frac{1\text{-}k}{k}\left\{kb_2^*\left(k,\ \bar{\lambda}_t\bar{\lambda}_b\right)\frac{d^2}{dt^2}+\frac{d}{dt}+1\right\}\bar{E}\left(\bar{t}\right) \qquad (1)$$

Where $1\text{-}k/k$ is the inverse of the dimensionless effective membrane tension, i.e. $1\text{-}k/k = 1/\bar{M}$. The curvature viscous $b_1^*\left(k,\Sigma_{\bar{\eta}}\right)$ and curvature inertial $b_2^*\left(k,\ \bar{\lambda}_t\bar{\lambda}_b\right)$ material functions are defined by:

$$b_1^*\left(k,\Sigma_{\bar{\eta}}\right)=1+\left(\frac{1-k}{k}\right)\Sigma_{\bar{\eta}} \quad , \quad b_2^*\left(k,\ \bar{\lambda}_t\bar{\lambda}_b\right)=\frac{\bar{\lambda}_t\bar{\lambda}_b}{k} \qquad (2a, b)$$

3. 1. Dimensionless numbers

The governing Equation (4) contains five dimensionless numbers $\left\{\bar{\lambda}_t\bar{\lambda}_b,\Sigma_{\bar{\eta}},k,De,a_0^*\right\}$, which are associated with the following mechanisms: (i) Memory $\left(\bar{\lambda}_t\bar{\lambda}_b\right)$: product of the viscoelastic dimensionless times $\bar{\lambda}_t$, and $\bar{\lambda}_b$, it obeys, $\bar{\lambda}_t+\bar{\lambda}_b=1$ and defines the elastic asymmetry of the fluids. When $\bar{\lambda}_t\bar{\lambda}_b<<1$ (highly asymmetric case) one of the fluids is nearly inelastic, and when $\bar{\lambda}_t\bar{\lambda}_b=1/4$ (highly symmetric case) both fluids are equally elastic; (ii) Bulk Viscosity $\left(\Sigma_{\bar{\eta}}=\bar{\eta}_t+\bar{\eta}_b=\bar{G}_t\bar{\lambda}_t+\bar{G}_b\bar{\lambda}_b\right)$: total viscosity in the system, where the elastic dimensionless moduli satisfy $\bar{G}_t+\bar{G}_b=1$. The numerical value of this number is controlled by the product between the two dimensionless Maxwell time numbers $\bar{\lambda}_t\bar{\lambda}_b$, $\Sigma_{\bar{\eta}}=\Sigma_{\bar{\eta}}\left(\bar{\lambda}_t\bar{\lambda}_b\right)$; (iii) Elastic ratio (k): dimensionless ratio between the membrane and the total system elasticity : $0<k=\left(1+1/\bar{M}\right)^{-1}<1$. A floppy (soft) and stiff (rigid) membrane corresponds to k<<1 and $k\cong 1$ respectively. The elastic ratio, $k=k\left(\bar{M}\right)$ is determined by the dimensionless elastic membrane modulus; (iv-v). The Deborah De and flexoelectric a_0^* numbers given by:

$$De=\frac{t_i}{t_{ve}}=\frac{a\sqrt{\left(\rho_t+\rho_b\right)/\left(G_t+G_b\right)}}{\lambda_t+\lambda_b}; \quad a_0^*=\frac{c_f\Im E_0 a/4L}{M} \qquad (3a, b)$$

a_0^* is the dimensionless conversion of electric to elastic energy or equivalently the static transfer function at zero frequency (Abou-Dakka et al. 2012; Herrera-Valencia & Rey 2014).

3. 2. Response mode classification

To satisfy Equations (4-6), besides the restrictions noted above, the maxima and minima values of the total dimensionless bulk-viscosity number $\left(\boldsymbol{\Sigma}_{\bar{\boldsymbol{\eta}}\min},\boldsymbol{\Sigma}_{\bar{\boldsymbol{\eta}}\max}\right)$ must be bounded by the values of the Maxwell relaxation times in the bottom and the top fluids. Under perfect symmetry (identical elasticity in top and bottom fluids) $\bar{\lambda}_t\bar{\lambda}_b=1/4$, and the total viscosity is fixed at $\boldsymbol{\Sigma}_{\bar{\boldsymbol{\eta}}\max}=\boldsymbol{\Sigma}_{\bar{\boldsymbol{\eta}}\min}=0.5$, while under nearly total asymmetry $\bar{\lambda}_t \bar{\lambda}_b \approx \varepsilon << 1$, the total viscosity can vary between $\boldsymbol{\Sigma}_{\bar{\eta}\max} = 1, \boldsymbol{\Sigma}_{\bar{\eta}\min} = \varepsilon << 1$.

According to the magnitudes of the three dimensionless numbers $\left\{\bar{\lambda}_t \bar{\lambda}_b, \boldsymbol{\Sigma}_{\bar{\eta}}, k\right\}$, the system (Equations 4-6) displays six distinct modes, summarized in Table I. These six modes arise since the memory symmetry can be high (HS) or low (LS), the total viscosity high (HV), medium (MV) or low (LV), and the membrane can be floppy (FM) or stiff (SM). For example, in Table I the third row mode {LS, LV, FM} corresponds to low symmetry, low viscosity and floppy membrane. This effective mode classification narrows down the parametric envelope of biological significance.

Table 1. Device response modes.

System's Modes	$\bar{\lambda}_t \bar{\lambda}_b$	$\boldsymbol{\Sigma}_{\bar{\eta}} = \boldsymbol{\Sigma}_{\bar{\eta}}\left(\bar{\lambda}_t \bar{\lambda}_b\right)$	**k**
I Low Symmetry, Low Viscosity, Floppy membrane **{LS, LV, FM}**	$\boldsymbol{\varepsilon}$	$\boldsymbol{\varepsilon}$	$\mathbf{k} << \mathbf{1}$ $\mathbf{k} \cong \boldsymbol{\varepsilon}$
II Low Symmetry, Low Viscosity, Stiff Membrane **{LS, LV, SM}**	$\boldsymbol{\varepsilon}$	$\boldsymbol{\varepsilon}$	$\mathbf{1}$
III Low Symmetry, High Viscosity, Floppy Membrane **{LS, HV, FM}**	$\boldsymbol{\varepsilon}$	$1-\boldsymbol{\varepsilon}$	$\mathbf{k} << \mathbf{1}$ $\mathbf{k} \cong \boldsymbol{\varepsilon}$
IV Low Symmetry, High Viscosity, Stiff Membrane **{LS, HV, SM}**	$\boldsymbol{\varepsilon}$	$1-\boldsymbol{\varepsilon}$	$\mathbf{1}$
V High Symmetry, Intermediate Viscosity, Floppy Membrane **{HS, IV, FM}**	$1/4$	$1/2$	$\mathbf{k} << \mathbf{1}$
VI High Symmetry, Intermediate Viscosity, Stiff Membrane **{HS, IV, SM}**	$1/4$	$1/2$	$\mathbf{1}$

$\bar{\lambda}_t \bar{\lambda}_b$: memory, $\Sigma_{\bar{\eta}}$: viscosity, k: elasticity ratio, $\boldsymbol{\varepsilon} \approx O\left(10^{-4}\right)$

The specific numerical values in Table I are selected as to be characteristic of each mode. The six modes in Table I, can be represented by the vertices of a prismatic 3D material space shown in Figure 3, spanned by fluid memory $\left\{\bar{\lambda}_t \bar{\lambda}_b\right\}$, membrane elasticity $\{k\}$, and total fluid viscosity $\left\{\boldsymbol{\Sigma}_{\bar{\eta}}\right\}$. The front edge of the prism, defined by the line $\bar{\lambda}_t \bar{\lambda}_b = 10^{-4}$, $\boldsymbol{\Sigma}_{\bar{\eta}} = 1$, 0<k<1

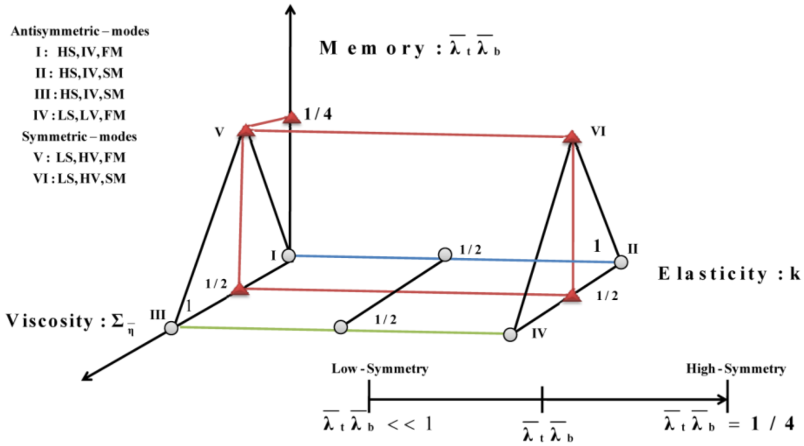

Fig. 3. Prismatic material space for the six possible modes of Equations (1-3), shown in Table I. The vertical axis is the memory of the fluids, the horizontal is the elasticity ratio k, and the axis into the page is the total viscosity of the fluids. The six vertices correspond to the six modes in Table I.

4. Fluid Power Dissipation

The average power delivered to the viscoelastic fluids $\bar{P}(\bar{\omega})$ by the oscillating membrane is the period average of the product of the input force $\bar{E}(\bar{t},\bar{\omega}) = \mathrm{Exp}(i\bar{\omega}\bar{t})$ times the flow rate $\bar{\Im}(\bar{t},\bar{\omega}) = -2^{-1} d\bar{H}(\bar{t},\bar{\omega}) / d\bar{t}$ and is proportional to imaginary part of the transfer function of Equation (7), i.e., $\mathrm{Im}[F_D(\bar{\omega})]$:

$$\bar{P}(\bar{\omega}) = \left\langle \mathrm{Re}\left[\bar{E}(\bar{t},\bar{\omega})\right] \cdot \mathrm{Re}\left[\bar{\Im}(\bar{t},\bar{\omega})\right]\right\rangle = \frac{1}{2}\bar{\omega}\left|\mathrm{Im}\left[F_D(\bar{\omega})\right]\right| \tag{4}$$

Notice that, in this particular case, It is used an analytical complex function (exponential function) to describe the input force (electrical field), but it can be generalized with a stochastic function trough a Fourier series (Herrera et al. 2009, 2010).

5. Numerical Results

Figure 4 shows the power dissipation as a function of the dimensionless frequency $\bar{\omega}$ for the modes {I, III, V}, without inertia (a) and the power dissipation as a function of the elastic ratio in the mode III. Inertialess conditions generate a broader power peak only in mode III (large viscosity) since dissipative modes persist with higher frequencies. These facts follows from the fact that the power is proportional to the imaginary part of the transfer function (see Equation 10) and according to the asymptotic results of Appendix C, only under finite inertia $\mathrm{Im}\left[F_D(\bar{\omega})\right]$ converges at large frequency to its static value. Hence except for mode III, inertia-less conditions do not generate power pulses. The material properties used in the simulation correspond to mode III {LS, HV, FM}. It is clear that the elastic ratio k plays an important role in the amplitude, and affects the symmetry and frequency bandwidth of the resonance. As expected more floppy membranes will result in higher dissipation as they store more energy.

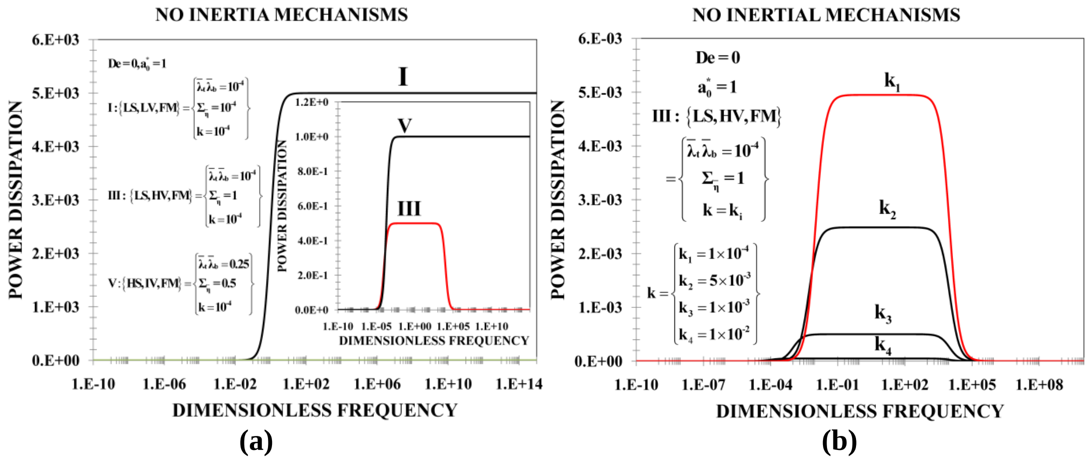

Fig. 4. (a) Power dissipation $\overline{P}\left(\overline{\omega}\right)$ as a function of the dimensionless frequency $\overline{\omega}$ for the mode {I, III, V} in the cases where the inertial mechanisms are absent. (b) Power dissipation as a function of the elastic ratio k . The material parameters used in the simulation correspond to mode III. Softer membrane generates more power dissipation.

6. Conclusion

Membrane flexoelectricity is a novel electromechanical coupling effect that occurs in polarizable media under geometric curvature (Petrov 2006). The sensor effect is performed by bending induced electric polarization, whereas the converse actuation effect is performed by the membrane curvature induced by an imposed electric field (Rey 2005, 2006a-c; 2008a,b). Membrane flexoelectricity is relevant to the biological functioning of the Outer Hair Cells (OHC) which act as amplifiers to counteract viscous dissipation through mechanic transduction and thus allowing hearing (Brownell 2007; Abou-Dakka et al. 2012; Herrera-Valencia & Rey 2014). The key challenge is to understand the coupling of oscillatory flexoelectric actuation and the viscoelastic phenomena of the fluids that are in contact with the oscillating membrane.An efficient method to describe membrane flexoelectricity is to use the liquid crystal analogy that follows by identifying the director field of a nematic with the unit normal to the membrane (Abou-Dakka et al 2012; Herrera-Valencia 2014). A key parameter is the flexoelectric coefficient which for biological membranes is of the order of 3-20 pC/m (Abou-Dakka et al 2012; Herrera-Valencia 2014).

In this paper we explored the dynamics of the actuation flexoelectric mode. An integrated dynamical model for the average curvature of flexoelectric membranes oscillating in viscoelastic fluid media under capillary confinement was formulated using a previously presented shape equation based on the liquid crystal approach (de Gennes & Prost 1994; Petrov 2006; Rey 2005; 2006a-c; Rey 2008a,b). The membrane curvature dynamics is given by a balance between the viscoelastic stress jump from the contacting bulk liquids, the restoring membrane effective tension, and the driving flexoelectric force (Abou-Dakka et al. 2012; Herrera-Valencia & Rey 2014). Using the flexoelectric shape equation in conjunction with a viscoelastic capillary flow model for the contacting phases we obtained a new average curvature dynamic equation (Abou-Dakka et al. 2012; Herrera Valencia & Rey 2014). Applying the Fourier transform to the governing partial linear differential equation and using the relation between the speed of the average curvature and volumetric flow, a relation between the average curvature and applied electrical field was found (Abou-Dakka et al. 2012; Herrera Valencia & Rey 2014). The corresponding dynamic, is a function of the asymmetry of the viscoelastic phases, total bulk viscosity and membrane elasticity, through characteristic dimensionless numbers associated to each mechanisms (Abou-Dakka et al. 2012; Herrera Valencia & Rey 2014). A thorough parametric study was performed to identify the conditions that lead to the emergence of a power pulse (Abou-Dakka et al. 2012; Herrera Valencia & Rey 2014). It was found that, the inertial mechanisms play an important role in the resonance curves associated to the power dissipation in the relevant modes {I, III, V}, which corresponds to the cases of a low and high symmetry of the viscoelastic phases, low and sufficiently large total bulk viscosity and small elastic ratio indicating that less elasticity is stored in the membrane (Abou-Dakka et al. 2012; Herrera Valencia & Rey 2014).

An evaluation of the present model predictions based on power profile, indicates that the Helfrich-Flexoelectric-Maxwell fluid model possess the necessary physics to qualitatively capture electro-mechanical power conversion (Brownell 1985; Petrov 2006; Rabbits et al. 2009)

The linear model presented here is only valid for electric fields of sufficiently small amplitude, high dimensionless frequencies and small deformations. The present theory, model, and computations contribute to the evolving fundamental understanding of biological shape actuation through electromechanical couplings (Rey 2005; Rey 2006a-c; Rey 2008a-b; Abou-Dakka et al. 2012; Herrera-Valencia & Rey 2014).

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