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Pulse Wave Velocity Prediction in Multi-Layer Thick Wall Arterial Segments

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Abstract- Pulse wave velocity (PWV) is an important index of arterial hemodynamics, which lays the foundation for continuous, noninvasive blood pressure automated monitoring. The goal of this paper is to re-examine the accuracy of PWV prediction based on a traditional homogeneous structural model for thin-walled arterial segments. In reality arteries are described as composite heterogeneous hyperelastic structures, where the thickness dimension cannot be considered small compared to the cross section size. In this paper we present a hemodynamic fluid - structure interaction model accounting for the 3D material description of multilayer arterial segments based on its histological information. The model is suitable to account for the highly nonlinear orthotropic material undergoing finite deformation for each layer. An essential ingredient is the notable dependence of results on nonlinear aspects of the model: convective fluid phenomena, hyperelastic constitutive relation for each layer, and finite deformation. The dependence of PWV on pressure for three vessels of different thicknesses is compared against a simplified thin wall model of a membrane shell interacting with an incompressible fluid. Results show an asymptotic accuracy of an order of h/r_0 is predicted. This work help lays the foundation for continuous, noninvasive blood pressure automated monitoring based on PWV.

Keywords: Pulse wave velocity, modeling, noninvasive blood pressure, convective fluid phenomena, hyperelastic, finite deformation

Nomenclature

A	Cross sectional area (m^2)
u	Axial flow velocity (m/s)
p	Transmural pressure (Pa)
ρ	Density of incompressible fluid (kg/m^3)
f	Friction term (m/s^2)
r_{i0}, r_0	Internal wall and mid-wall radii in a zero stress condition respectively (m)
η	Ratio of the wall deflection to r_0
$\lambda_r, \lambda_\theta, \lambda_x$	Stretch ratios in a radial, circumferential and axial directions respectively
σ_θ, σ_x	Circumferential and axial Cauchy stress components (Pa)
E_θ, E_r	Circumferential and axial Green-Lagrange strain components
$\mathbf{A}$	$\begin{bmatrix} A_{11} & A_{12} \\ A_{12} & A_{22} \end{bmatrix}$ Symmetric tensor of material constants
Subscripts	
(t,x)	Derivatives by time and axial coordinates
Superscripts	
T	Transposition

1. Introduction

The potential of estimating arterial blood pressure based on PWV has been investigated based on statistical regression models, or empirical representation of an incremental isotropic elastic modulus as a function of a transmural pressure [1,2]. Relating physically based characterizations verified *in vitro* [3,4] and *in vivo*, have been created by modeling arteries as fluid-filled compliant thin walled cylindrical membrane shells. The present paper describes a mathematical model predicting PWV propagation with rigorous account of nonlinearities in the fluid dynamics model, blood vessel elasticity, and finite dynamic deformation of multi-layer thick wall arterial segments. This study is a continuation of the previous work [3-4] in the context of *in-vivo* validation and application of the proposed methodology to continuous, noninvasive blood pressure measurements. In the present work, the arterial wall is considered as a heterogeneous composite hyperelastic structure. Healthy arteries are composed of three distinct layers: the tunica intima (the innermost layer), the tunica media (the middle layer) and the tunica adventitia (the outer layer), as shown in Fig. 1. We discuss a fully 3D material description of each layer, based on a material description of an artery in a passive state originally proposed by Fung [5]. A novel mathematical model predicting PWV is proposed accounting for nonlinear aspects of a convective fluid phenomena, hyperelastic constitutive relations, and finite deformation of a thick arterial wall.

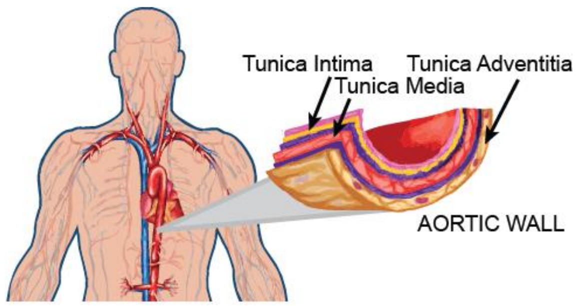

Fig. 1. The anatomy of the aortic wall.

2. Materials and Methods

2. 1. Fluid-Structure Interaction Model

One dimensional models simulating blood flow in arteries effectively describe pulsatile flow in terms of averages across the section flow parameters. Although they are not able to provide the details of flow separation, recirculation, or shear stress analysis, they should accurately represent the overall and averaged pulsatile flow characteristics, particular PWV. Derivations of one dimensional models can be found in a number of papers, see for instance [3, 4, 6], and are not repeated here.

Conservation of mass and momentum results in the following system of one dimensional equations

$$A_t + (uA)_x = 0 \tag{1}$$

$$u_t + \left(\frac{u^2}{2} + \frac{p}{\rho}\right)_x = f \tag{2}$$

For an impermeable thick wall vessel the pressure – strain relationship is maintained by equilibrium condition as a function p=p(η), based on relevant constitutive relations. Noting that $A = \pi r_0^2 (1+\eta)^2$, and assuming that transmural pressure is a smooth function of a wall normal deflection (derivative $p_\eta = \partial p / \partial \eta$ exists at any point), the total system of equations can be presented in the following non-conservative form

$$U_t + H(U)U_x = F \tag{3}$$

Where

$$U = \begin{bmatrix} \eta \\ u \end{bmatrix}; \; H = \begin{bmatrix} u & \dfrac{1+\eta}{2} \\ \dfrac{p_\eta}{\rho} & u \end{bmatrix}; \; F = \begin{bmatrix} 0 \\ f \end{bmatrix} \tag{4}$$

The characteristics analysis shows that the system (3) is strictly hyperbolic, with real and distinct eigenvalues. PWV is associated with the forward running wave velocity, hence it is identified as

$$PWV = u + \sqrt{\frac{1+\eta}{2\rho} p_\eta} \tag{4}$$

2. 2. Hyperelasticity of the Vessel Wall

Numerous formulations of constitutive models for arteries have been proposed in the literature. In a comparison paper [7] it is concluded that the exponential descriptor of the passive behavior of arteries, due to Zhou-Fung, is “the best available”.

According to Zhou-Fung [5] the strain energy density function for the pseudo elastic constitutive relation may be presented in the form

$$W = \frac{1}{2} c(e^Q - 1), \qquad Q = \mathbf{E}^T \mathbf{A} \mathbf{E}; \tag{5}$$

Where c is the material coefficient, Q is the function of the Green-Lagrange strain $\mathbf{E}$ [5] and material parameters $\mathbf{A}$ tensors. Applying power series expansion, strain energy can be presented in a form

$$W = \frac{1}{2} c(\mathbf{E}^T \mathbf{A}\mathbf{E} + \frac{(\mathbf{E}^T \mathbf{A}\mathbf{E})^2}{2!} + ...) \tag{6}$$

Where the first term corresponds to the classical linear theory, and $c\mathbf{A}$ has a connotation of a linear symmetric elastic anisotropic stiffness tensor. The Cauchy – Green stress components are defined as the following [5].

$$\begin{aligned} \sigma_r &= \lambda_r^2 \frac{\partial W}{\partial E_r} = \lambda_r^2 (A_{11} E_r + A_{12} E_\theta) e^Q \\ \sigma_\theta &= \lambda_\theta^2 \frac{\partial W}{\partial E_\theta} = \lambda_\theta^2 (A_{12} E_r + A_{22} E_\theta) e^Q \end{aligned} \tag{7}$$

Neglecting inertia forces, the problem of an artery subjected by transmural pressure is described by solving equation of equilibrium

$$\frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \tag{8}$$

Supplemented by constituent equations (8), relations for the principal stretch ratios [5]

$$\lambda_r = \frac{dr}{dR}, \quad \lambda_\theta = \frac{r}{R}, \quad E_i = \frac{1}{2}(\lambda_i^2 - 1), \quad (i = r, \theta) \tag{9}$$

And boundary conditions at un-deformed inside and outside radii of the tube.

$$\sigma_r(R_i) = -p, \quad \sigma_r(R_o) = 0 \tag{10}$$

3. Results and Discussion

To solve the strongly nonlinear boundary value problem (8) – (11) the method of differentiation by a load parameter was used [9]. The input data relating to the Fung's model material constants correspond to the experimental number 71 presented in [7] (here we avail typically used multicomponent notation for the material constants, so that $b_1=A_{22}$, $b_3=A_{11}$, $b_6=A_{12}$): c=26.95kPa; A_{11}=0.0089; A_{12}=0.0193; A_{22}=0.9925 for the tunica media. Following [7] we restrict attention to a two layer model incorporating the tunica media and tunica adventitia only, since the tunica intima contributes negligible mechanical strength to the arterial wall. Experimental tests, according to [7] indicate that the tunica media is about an order of magnitude stiffer than the tunica adventitia (which was based on a several porcine aortas). The Symmetric tensor material $\boldsymbol{A}$ is scaled accordingly in the model for the adventitia layer.

We are now able to compare the performance of a Fung's single layer model with a two-layer structural model and a linear elastic model using Fung's anisotropic material constants for a human aorta with an inner radius of 10.5mm and an outer radius of 14.5mm. Figure 2 depicts the effect of internal pressure on deflection, taking into account the location in the arterial wall. The nonlinear single layer model of tunica media only is 4mm thick (marked as "nl1"), nonlinear two layer model of tunica media and tunica adventitia that is 3mm and 1mm thick respectively (marked as "nl2"), and a linear single layer model which is 4mm thick (marked as "l1") [7]. The linear model is described (7), where the first term corresponds to the classical linear theory, and $c\mathbf{A}$ has a connotation of a linear symmetric elastic anisotropic stiffness tensor. We verify the accuracy of the method of differentiation by a load parameter by comparison with the exact solution (marked by diamonds). Figure 2 shows that due to material nonlinearity, arterial compliance associated with the deflection of an inner surface, is not practically changed when we account for the adventitia layer. The latter means that the Fung's model, based on a homogeneous single layer structure for the aortic wall, is sufficient to predict PWV. It follows also from the Figure 2 that the linear model reduces dramatically the quality of a mechanical response modeling of an arterial wall.

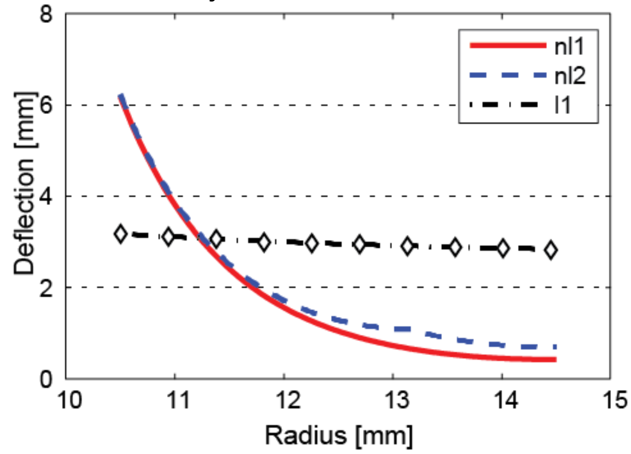

Fig. 2. Aortic wall deflection is shown as a function of the location in the arterial wall. Non-linear single layer, non-linear two layers and linear single layer are shown by nl1, nl2, l1 respectively. The total thicknesses for all 3 cases are the same, with the properties of nl2 changing by an order of magnitude at the tunica adventitia.

Figure 3 depicts the dependence of PWV on pressure for the systolic phase (marked as "SBP") and a diastole phase (marked as "DBP") for three vessels of different thicknesses. The properties and the external radius are the same as for the vessel described in the Figure 2. Following [10] we assume here that the flow velocity u=0 for the diastole phase, and is equal to the 20% of PWV for the systole phase. All results have been compared with the simplified thin walled model of a membrane shell interacting with an incompressible fluid [4]. According to theory [11] an asymptotic accuracy of a thin walled shell model is of an order of h/r_0 which correlates with the data presented in Figure 3.

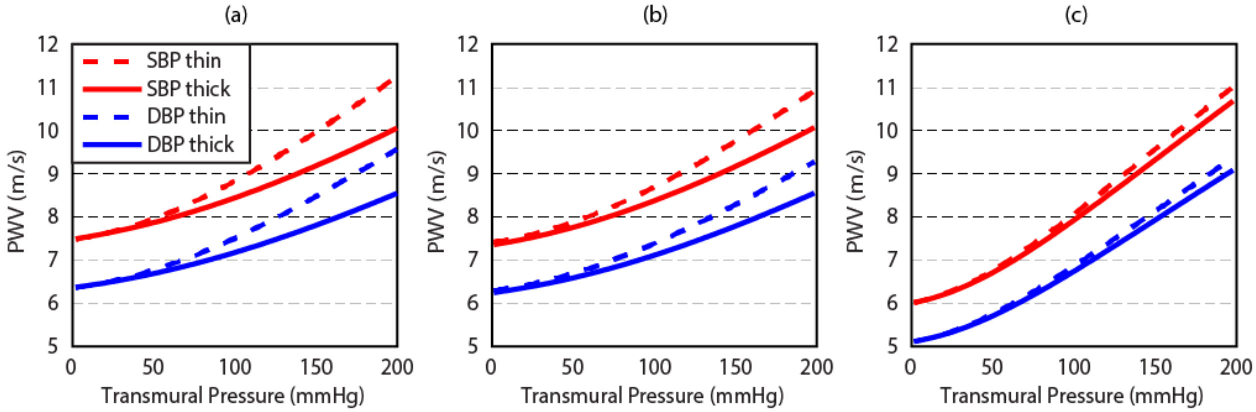

Fig. 3. PWV curves for systolic blood pressure (SBP) and diastolic blood pressure (DBP) for the non-linear single layer thin and thick aortic wall. Figure 3a corresponds to the vessel thickness of h=4mm; Figure 3b –h=2.5mm; Figure 3c – h=1mm.

4. Conclusion

A novel mathematical model predicting PWV propagation with rigorous account of nonlinearities in the fluid dynamics model, blood vessel elasticity, and finite dynamic deformation of multi-layer thick wall arterial segments was studied. It was found that the account for the multilayer model affects distribution of local parameters in the proximity of the external layer (adventitia), and does not affect stiffness related to the internal layer. The latter means that the single layer model is sufficient to predict PWV of an arterial segment. Within physiological range of blood pressure the three dimensional effects provide the difference for PWV between thick and a thin wall models that account for approximately a 4% difference as seen in Figure 3c, where the relative wall thickness ratio (h/r_0) is 0.07.

Acknowledgements

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