

# Simulation of Wave Propagation through the Bifurcated Compliant Arterial Network

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## Extended Abstract

Propagated pressure and flow pulse waves carry information relating to the functionality of the cardiovascular system, such as cardiac output, peripheral resistance, arterial dispensability and geometry deformation. Patient specific numerical simulation can be valuable for the diagnosis and corresponding treatment of disease. The basic system of equations written in the following matrix conservative form is derived

$$\dot{Q} + F_x - H_{xx} = R \quad (1)$$

$$Q = \begin{pmatrix} A \\ u \end{pmatrix}, \quad F = \begin{pmatrix} uA \\ \frac{1}{\rho} \frac{dp_e}{d\eta} + u^2/2 \end{pmatrix}, \quad H = \begin{pmatrix} 0 \\ \frac{\Gamma}{2\rho} u \end{pmatrix}, \quad R = \begin{pmatrix} 0 \\ f \end{pmatrix}, \quad (2)$$

Here  $A=A(x,t)$ -cross sectional area of the vessel,  $u=u(x,t)$  – averaged flow velocity,  $p=p(x,t)$  – static pressure,  $f$  – the source friction term,  $\rho$  – density of incompressible fluid, Voight type visco-elastic tube law in a form  $\Gamma$  is a viscous constant of a Voight type visco-elastic wall model. The unknown vector of primitive variables should satisfy partial differential equation supplemented with initial and boundary conditions, as well as internal boundary conditions, which are the flow matching conditions in junction. We have used the Total Variation Diminishing finite volume numerical scheme second order of accuracy in space and time to approximate fluxes, and central difference approximations to approximate diffusive terms.

While pressure and flow waveforms, calculated from computationally inexpensive one dimensional analysis, reflect the real picture of the process related to the inviscid flow, local three-dimensional viscous flow analysis brings an additional insight, highlighting the area of elevated level of circulations and wall shear stress. The latter according the experimental evidence is leading to the atherosclerotic lesions and triggers unfavorable biological events in the development of the intimal hyperplasia. To validate the implementation of the code in an arterial network, analysis has been applied to the simple “fork” type bifurcation. Three-dimensional Navier – Stokes simulation has been used to optimize the graft artery junction geometry from the view point of wall shear stress level minimization. We managed to reduce recirculation zone in the bifurcated area by minimization the maximum values of a wall shear stress gradient across all vessels varying the bypass vessel orientation angle within the frame of implied geometric constraints. It was shown that the typically used for the bypass T-junction topology is not the best. The best orientations of a bypass configuration was found for a set of specified flow rates across each bifurcated vessel.

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