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Temperature Analysis in a Ni-Cr Hot-Wire with a Novel Thermo-Electrical Model

Kiril Petkov, Jesper Hattel
Technical University of Denmark
Nils Koppels Alle, Produktionstorvet, building 425, 2800 Kgs. Lyngby, Denmark
kipetk@mek.dtu.dk; jhat@ mek.dtu.dk

Abstract **-** A one-dimensional thermo-electrical mathematical model describing the heating and cooling of thin Ni-Cr20% wires in a free air environment is presented. The basis of the model is a one-way coupling of the heat conduction equation and the electrical diffusion equation in a 1-D control volume finite difference framework, while the effect of the natural convection is implemented in the model based on well-established correlations from literature for horizontal cylinders. The model is tested against experimental data and is found to be in a good agreement with the natural convection correlations of Hatton and Mikheyev.

Keywords**:** horizontal cylinders, natural convection, Ni-Cr wire, thermo-electrical modelling

1. Introduction

Electrically heated wires in different environmental conditions have been a topic of interest ever since the relationship between electricity and heat was discovered. Electrical wire resistance heating is widely used in a variety of engineering applications and for this reason much work on energy transfer through wires has been done. One of the most studied wire-heat-transfer applications is probably the hot-wire anemometers, and the major part of the scientific work done on the field, deals with determination of the power-fluid-velocity relation, while very few publications, as those of Sandborn et al.(1975), Gessner and Moller(1970) and Champagne et al.(1967) present a study on the temperature of the hot-wires in shear flow. Moreover, most of these works are focused on forced convection, and in general, disregard natural convection.

Hence, in the current work, we present a new numerical, thermo-electrical model for transient temperature simulations of a hot-wire due to natural convection as well as radiation. The model uses the temperature dependent voltage distribution through the wire and based on this it calculates the local wire temperature distribution as a function of time. Different correlations from literature for the heat transfer coefficient due to natural convection are implemented and the results are compared and discussed.

2. Modelling and governing equations

The model is based on the heat conduction equation and the electrical diffusion equation in a two-way coupling such that the voltage affects the Joule heating term in the former and the temperature affects the electrical conductivity in the latter, i.e.:

$$\rho c_p(T)\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k(T)\frac{\partial T}{\partial x}\right) + \dot{Q}_e + \dot{Q}_{loss} \tag{1}$$

$$\frac{\partial}{\partial x}\left(\sigma(T)\frac{\partial V}{\partial x}\right) = 0 \tag{2}$$

Where ρ, T=T(t,x,V), x, t, V=V(t,x,T), $\dot{Q}_e = \dot{Q}_e$(t,x,V) and $\dot{Q}_{loss} = \dot{Q}_{loss}$(t,x,T,est) are density, temperature of the wire, position in direction of the length of the wire, time, voltage, volumetric specific energy gained due to electric resistance heating and volumetric specific energy lost to the surroundings, respectively. The terms k(T), σ(T) and c_p(T) denote thermal conductivity, electrical conductivity and specific heat capacity, all dependent on temperature. The temperature, voltage and terms relating gain and loss of energy are coupled between the equations. The energy generated in the wire due to resistance heating,$\dot{Q}_e$, is written as:

$$\dot{Q}_e = \sigma(T)E^2 = \sigma(T)\left(\frac{\partial V}{\partial x}\right)^2 \tag{3}$$

Where E is the electric field intensity. The energy lost to the surroundings ($\dot{Q}_{loss}$) is primarily governed by the local boundary condition at the wire surface (a combination of natural convection of air and thermal radiation). The heat transfer at the mountings at each end of the wire is negligible due to the dimensions considered in the present work and hence it is modelled as an adiabatic boundary. The heat loss is given by Newton's law of cooling, where the total heat transfer coefficient h is the summation of the individual heat transfer coefficients for natural convection and thermal radiation, i.e.:

$$\dot{Q}_{loss} = hA_c(T_{air} - \mathrm{T})/V_m \tag{4}$$

$$h = h_{nc} + h_{rad} \tag{5}$$

$$h_{nc} = \frac{Nu\, k_f}{D_w}, \quad \text{where } T_f = 0.5(\mathrm{T} + T_{air}) \tag{6}$$

$$h_{nc} = \epsilon\Omega((T[K])^2 + (T_{air}[K])^2)(\,T[K] + T_{air}[K]\,) \tag{7}$$

Where h=h(x,t) is the total heat transfer coefficient, V_m the wire volume, h_{nc} the natural convective heat transfer coefficient, h_{rad} the radiation heat transfer coefficient, A_c the area of wire circumference, T_{air} the temperature of air in [C], Nu the Nusselt number, D_w the diameter of wire, k_f the thermal conductivity of air at film temperature T_f, ϵ= ϵ (x) the emissivity of wire and Ω the Stefan-Boltzmann constant. Temperatures denoted by [K] are given in Kelvin temperature scale.

Since there is no single well established method for predicting the Nusselt number for natural convection in air, different well recognized approaches from literature are used for its determination and later implemented in the model.

Table 1: Table listing the natural convection correlations for horizontal cylinders applied in the model and their working range. The correlation of Hesse and Sparrow (1974) is not listed, because it is described by a table.

Correlation	Expression	Range of Ra
Mikheyev(1966)	$Nu = 1.18Ra^{1/8}$	$10^{-4} - 10^{3}$
Van der Hegger Zijnen(1956)	$Nu = 0.35 + 0.25Ra^{1/8} + 0.45Ra^{1/4}$	$10^{-7} - 10^{9}$
Tsubouchi and Masuda(1966)	$Nu = 0.36 + 0.52Ra^{1/4}$	$10^{-6} - 10^{1}$
Hatton1(1970) computed	$Nu\left[T_f/T_{inf}\right]^{-0.154} = 0.525 + 0.422Ra^{0.315}$	$10^{-3} - 10^{1}$
Hatton2(1970) suggested fit	$Nu\left[T_f/T_{inf}\right]^{-0.154} = 0.384 + 0.59Ra^{0.154}$	$10^{-3} - 10^{1}$

Churchil and Chu (1974)	$Nu = \left(0.6 + 0.378\left(\frac{Ra}{((0.559/Pr)^{9/16})^{6/9}}\right)^{1/6}\right)^2$	$10^{-5} - 10^{12}$

The boundary conditions applied for the differential equations (1) and (2) are as follows:

$$\left.\frac{\partial T}{\partial x}\right|_{x=0} = \left.\frac{\partial T}{\partial x}\right|_{x=L} = 0 \quad (8)$$

$$V_{x=0} = V1 \,\&\, V_{x=L} = 0 \quad (9)$$

Where V1 expresses the actual applied voltage at the left boundary ($x = 0$) and L the length of the wire.

Table 2: Parameters of the experimental set up.

Parameter	ρ [kg/m^3]	L [m]	V1 [V]	Dw [mm]	Tair [C]	Tinitial [C]	ϵ [-]
Value	8400	0.226	2.563	0.5	26	70	0.94

3. Numerical Methods

In order to solve the system of equations given by eq.(1)- (2), control volume finite difference method with a formulation similar to the one given in Hattel (2005) is applied. The thermal boundary conditions are implemented by 2 ghost points, one point at each side of the geometry. The electrical diffusion equation (eq.2) is discretized in a similar manner, however the voltage is evaluated at the boundaries of each cell, due to the nature of the physics applied. This corresponds to the well-known staggered grid approach originally suggested for fluid flow by Patankar(1980) and further developed for solid mechanics in Hattel and Hansen(1995). The temperature dependent data for c_p(T) and σ (T) are taken from Petrovic et al.(1993). However a small discrepancy in the electrical conductivity between the local Ni-Cr supplier's data and Petrovic's data was found for the data values at room temperature, and in order to account for that Petrovic's electrical conductivity data was decreased with a constant value of 0.062[µOhm.m]. The temperature dependent conductivity k(T) is assumed to follow the Smith-Palmer formulation as written in Endo et al.(2010).

4. Results

The results from the computer simulations and the experiments are presented in fig.1. Seven different correlations for the Nu-number and hence the heat transfer coefficient(HTC) at the surface of the wire were evaluated and compared with the experimental measurements. Figure 1 is arranged such that the sub-figure "A" (left) shows the comparison between the experiment and the HTC correlations suggested by Hatton 1 and 2, van der Hegge Zijnen (abbreviated v.d.H.Z) and Hesse and Sparrow (all of them implemented in the thermo-electrical model as shown in eq.(6)), whereas the sub-figure "B" (right) shows the comparison between the experiment and the correlations suggested by Mikheyev, Tsubouchi and Masuda, and Churchill and Chu implemented in the numerical model.

From Fig.1 it can be seen that the best agreement between the experimental and simulated results is achieved with the Mikheyev HTC correlation as the absolute error at t=37s is -2.1^{o}C, while for correlation of Hatton1, the error is -2.7^{o}C(absolute error=T_{sim}-T_{exp}). The simulations applying the correlations of Churchill and Chu, as well as Tsubouchi and Masuda predict high temperatures, with absolute errors over 30^o C. During the cooling phase(t>37s), there is a certain disagreement between the experimental and simulated results as the wire in the experiment set up is cooling slightly faster than the predicted cooling of the simulations. The general behavior of cooling is well re-presented by the simulations, i.e. the wire is cooling in an expected exponential fashion. However the temperature error at the end of the cooling is 11.1^{o} C for the correlation of Mikheyev.

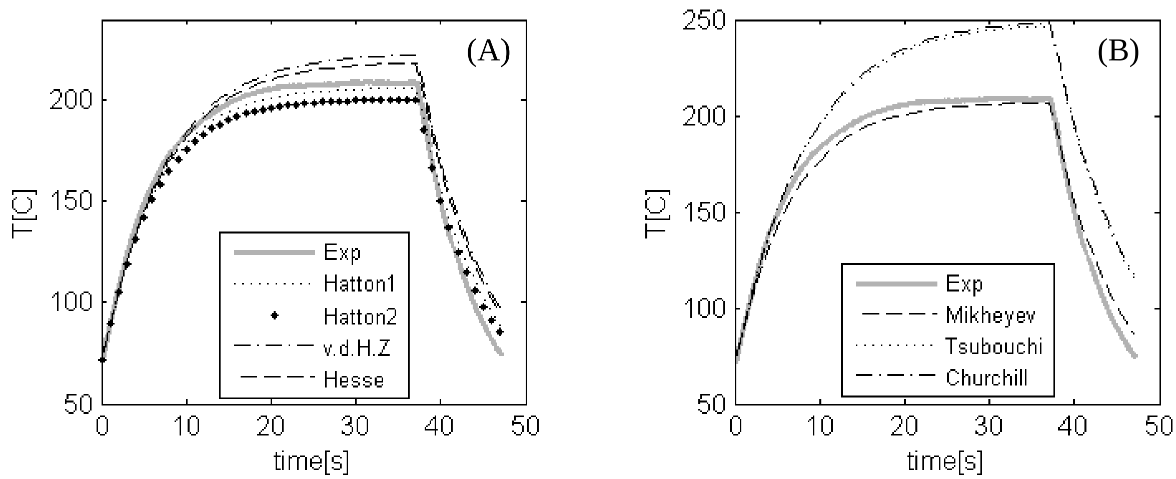

Fig. 1. Figures showing experimental and simulation results for a wire of diameter 25swg(0.5mm) in free air. The figures show the temperature with respect to time during heating and cooling of a Ni-Cr(80/20) resistance wire. Different correlations applied in the model are shown and compared.

5. Conclusions

A new thermo-electrical mathematical model is presented capable of capturing the time dependent heating of thin Ni-Cr20 wires of thickness 0.5mm subjected to natural convection flow of air. Comparing the modelling results with corresponding experimental data shows that by implementing the correlations of Mikheyev (1966) we can decrease the absolute error in the predicted temperature down to around 2^{o} C.

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