

# **Investigation of Complex Fluid Behaviour Indices on the Performance of Electrokinetic Flow in Nano-channel**

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**Abstract-** In this paper, an electrokinetic flow of an electrolyte in a nano-channel will be studied. This study will be with the existence of the Electric Double Layer (EDL) and fully analytical. Working fluid assumed to be non-Newtonian which follows power law. Governing equation for the EDL is Poisson-Boltzmann. In addition, Navier-Stokes equations will be considered. With existence of the small zeta potential, some simplifications are made on the governing equations and consequently, solution can be analytical. The focus of this investigation is on consideration of the effects of fluid behavior indices of both pseudoplastic and dilatant non-Newtonian fluids on some physical properties especially viscosity and electrokinetic mobility. Results show that, the less the fluid behavior index, the closer the velocity profile to the plug flow pattern and full development of the velocity is obtained faster. In addition, results show that, for larger Debye-Huckel parameters, faster flows are obtained. This finding is significantly more clear in the case pseudoplastic fluids, when dramatically nonlinear nature of dilatant fluids are compared with the patterns obtained by pseudoplastic ones. In the case of viscosity, as predicted, pseudoplastic and dilatant fluids reflect completely different response to viscosity, while in pseudoplastics, the farther the distance to the channel wall, the less viscous the fluid, and vice versa in dilatants. In this case, results show that, responses of pseudoplastic fluids to viscosity variations based on fluid behavior indices are more dramatic. In the case of electrokinetic mobility, nonlinearity mentioned for viscosity is seen again, but in the opposite direction, as expected. Furthermore, results show that, Electrokinetic mobility response to zeta potential is very critical. In addition, Debye-Huckel dimensionless length effects on non-Newtonian fluids are investigated.

**Keywords:** Electric Double Layer, Complex Fluid, Electrokinetic Phenomena, Viscosity, Debye-Huckel Parameter.

## **1. Introduction**

In recent decades, after introducing micro- and nano-fabrication technologies, several possibilities in the case of micro- and nano-fluidic devices have been invented. Decreasing in size and hydraulic diameter some of the physical parameters such as surface tension will be more significant while they are negligible in normal sizes. After Helmholtz who discovered electric double layer (EDL), many researchers develop this idea (Kandlikar et al., 2006; Burgreenand and Nakache, 1964; Lu and Chan, 1994). In most of these cases, based on applications, working fluid assumed to be Newtonian. However, in some important cases, non-Newtonian fluids are in existence such as polymers, colloids and suspensions. The two more important applications of the complex fluids for electrokinetic phenomena are fuel cells and biomedical issues. In this area, some researches have been done such as (Berli and Olivares, 2008; Zimmermann et al., 2006; Zhao and Yang, 2012; Chen et al. 2014).

## **2. Mathematical Modelling**

In this paper, an electrokinetic flow of an electrolyte in a nano-channel will be studied. This study will be with the existence of the Electric Double Layer (EDL) and fully analytical. Working fluid assumed to be non-Newtonian which follows power law. Governing equation for the EDL is Poisson-Boltzmann. In addition, Navier-Stokes equations will be considered. With existence of the small zeta

potential, some simplifications are made on the governing equations and consequently, solution can be analytical.

$$\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \varphi}{\partial r} \right) = \frac{\varphi}{\varepsilon^2} \quad (1)$$

In this equation,  $\varphi$  is electric potential,  $\varepsilon$  is Debye-Huckel dimensionless length and  $r$  is Nanochannel diameter. According to simplifications mentioned before, Generalized Bessel Functions are utilized in order to model the Electrokinetic flows in nano-channel with small zeta potential.

In this paper, complex fluids with power law have been considered as working fluid. In this case, viscosity of the fluids follows power law instead of Newtonian formula as below:

$$\mu = \mu_0 \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u}{\partial r} \right) \right]^{m-1} \quad (2)$$

In this formula,  $u$  is velocity,  $\mu$  is viscosity and  $\mu_0$  is similar Newtonian fluid viscosity. In this case, if  $m = 1$ , fluid is considered as Newtonian,  $m > 1$  as dilatant and  $m < 1$  as pseudoplastic. In this case, Eq. 1 will be as follows:

$$\left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u}{\partial r} \right) \right]^m = \frac{\varphi \varepsilon_e E_0 R T}{\varepsilon^2 F U_0 \mu_0} \quad (3)$$

In the above formula,  $\varepsilon_e$  is electrolyte diffusion dielectric coefficient,  $E_0$  is electrical field,  $R$  is universal gas constant,  $T$  is temperature,  $F$  is Faraday constant and  $U_0$  is reference velocity. The above PDE can be solved analytically by Generalized Bessel Equation concept.

### 3. Results and Discussion

The focus of this investigation is on consideration of the effects of fluid behavior indices of both pseudoplastic and dilatant non-Newtonian fluids on some physical properties especially viscosity and electrokinetic mobility.

Velocity field has been solved and it can be shown in the Fig. 1 for several behavioural index,  $m$ .

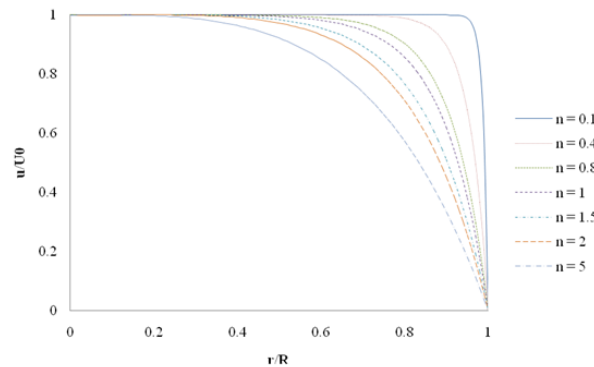


Fig. 1. Velocity profile for several sample complex fluids

Results show that, the less the fluid behavior index, the closer the velocity profile to the plug flow pattern and full development of the velocity is obtained faster. . In addition, results show that, for larger Debye-Huckel parameters, faster flows are obtained (Figs. 2 and 3).

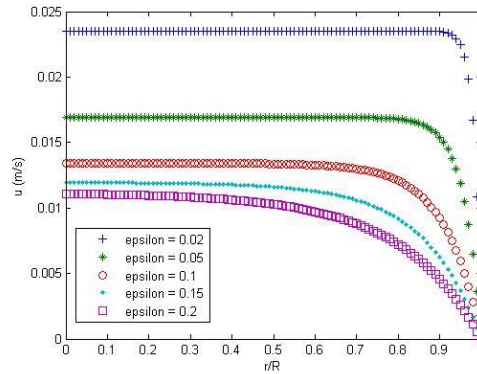


Fig. 2. Velocity distribution profiles for a sample complex pseudoplastic fluid ( $m = 0.8$ ) with different Debye-Huckel parameter

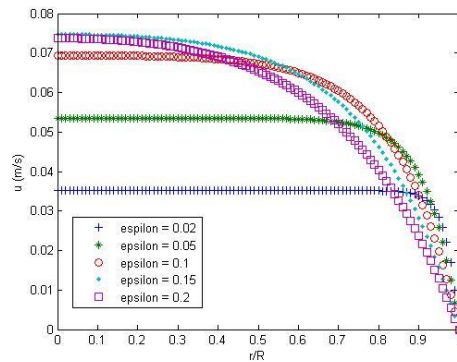


Fig. 3. Velocity distribution profiles for a sample complex dilatant fluid ( $m = 1.5$ ) with different Debye-Huckel parameter

This finding is significantly clearer in the case pseudoplastic fluids, when dramatically nonlinear nature of dilatant fluids are compared with the patterns obtained by pseudoplastic ones. In the case of viscosity, as predicted, pseudoplastic and dilatant fluids reflect completely different response to viscosity, while in pseudoplastics, the farther the distance to the channel wall, the less viscous the fluid, and vice versa in dilatants (Fig. 4). In this case, results show that, responses of pseudoplastic fluids to viscosity variations based on fluid behavior indices are more dramatic.

In the case of electrokinetic mobility, nonlinearity mentioned for viscosity is seen again, but in the opposite direction, as expected (Fig. 5)

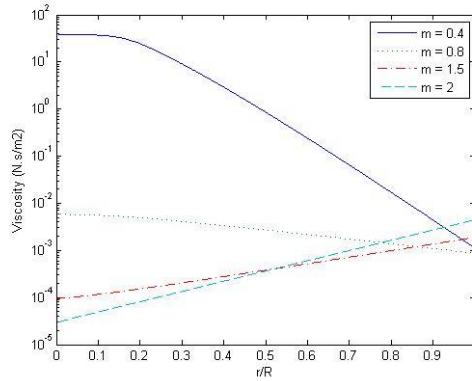


Fig. 4. Viscosity distribution profiles for several complex fluids

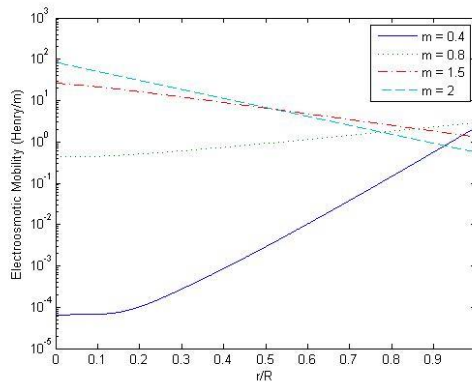


Fig. 5. Electrokinetic mobility distribution profiles for several complex fluids

Furthermore, results show that, Electrokinetic mobility response to zeta potential is very critical, that is, by increasing zeta potential from 2.5 mV to 25 mV, Electrokinetic mobility increased about 7 times for a case studied pseudoplastic fluid ( $n=0.8$ ) in the center of the channel. however, in the case of dilatants, in the center of the channel, Electrokinetic mobility is almost constant in different zeta potential (Figs. 6 and 7).

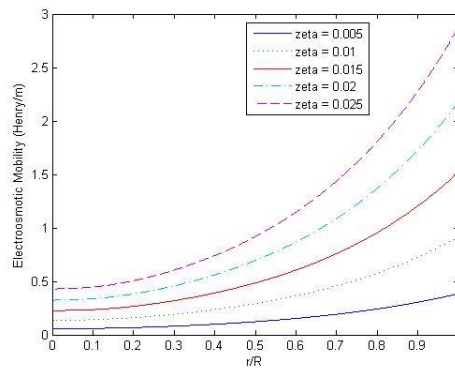


Fig. 6. Electrokinetic mobility distribution profiles for a sample complex pseudoplastic fluid ( $m = 0.8$ ) with different zeta potentials

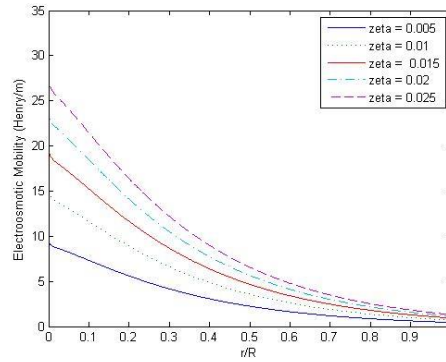


Fig. 7. Electrokinetic mobility distribution profiles for a sample complex dilatant fluid ( $m = 1.5$ ) with different zeta potentials

In addition, Debye-Huckel dimensionless length effects on non-Newtonian fluids are investigated. In this case, for 25 mV zeta potential, results have been shown in Figs. 8 and 9. As it can be seen in these figures, for pseudoplastic and dilatant complex fluids, behavior is contradictory. In the other words, for pseudoplastic fluids, electrokinetic mobility in the near wall area, will be smooth and the more the Debye-Huckel parameter, the more achievable electrokinetic mobility occurs. However, in dilatant fluids, results show some inconsistencies in the near wall area.

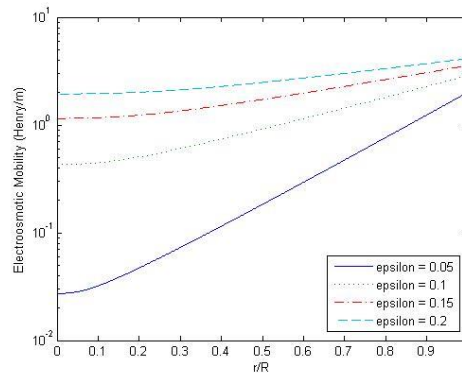


Fig. 8. Electrokinetic mobility distribution profiles for a sample complex pseudoplastic fluid ( $m = 0.8$ ) with different Debye-Huckel Parameters

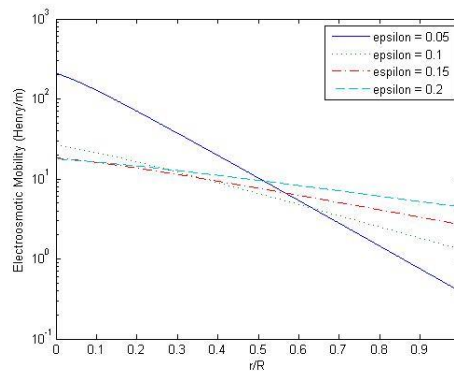


Fig. 9. Electrokinetic mobility distribution profiles for a sample complex dilatant fluid ( $m = 1.5$ ) with different Debye-Huckel Parameters

#### 4. Conclusion

In this paper, an electrokinetic flow of an electrolyte in a nano-channel was studied. With existence of the small zeta potential, some simplifications were made on the governing equations and consequently, solution was analytical. The focus of this investigation was on consideration of the effects of fluid behavior indices of both pseudoplastic and dilatant non-Newtonian fluids on some physical properties especially viscosity and electrokinetic mobility. Results show that, the less the fluid behavior index, the closer the velocity profile to the plug flow pattern and full development of the velocity is obtained faster. In addition, results show that, for larger Debye-Huckel parameters, faster flows are obtained. This finding is significantly more clear in the case pseudoplastic fluids, when dramatically nonlinear nature of dilatant fluids are compared with the patterns obtained by pseudoplastic ones. In the case of viscosity, as predicted, pseudoplastic and dilatant fluids reflect completely different response to viscosity, while in pseudoplastics, the farther the distance to the channel wall, the less viscous the fluid, and vice versa in dilatants. In this case, results show that, responses of pseudoplastic fluids to viscosity variations based on fluid behavior indices are more dramatic. In the case of electrokinetic mobility, nonlinearity mentioned for viscosity is seen again, but in the opposite direction, as expected. Furthermore, results show that, Electrokinetic mobility response to zeta potential is very critical, that is, by increasing zeta potential from 2.5 mV to 25 mV, Electrokinetic mobility increased about 7 times for a case studied pseudoplastic fluid ( $n=0.8$ ) in the center of the channel. however, in the case of dilatants, in the center of the channel, Electrokinetic mobility is almost constant in different zeta potential. In addition, Debye-Huckel dimensionless length effects on non-Newtonian fluids are investigated.

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