

Numerical Simulation for Heat Transfer Analysis in Laminar Flow of CuO-Water Nanofluid in Tubes

Khalid B. Saleem, Emad A. Khazal

Engineering College/Basrah University

Basrah, Iraq

khalidb77@gmail.com; emad.ab74@gmail.com

Abstract - This paper presents a two-dimensional numerical analysis to study the laminar heat transfer and flow characteristics of CuO–water nanofluids through a tube at constant heat flux boundary condition at tube wall. Based on the single-phase approach, the effects of different parameters such as nanoparticle volume concentration (1% - 5%), and Reynolds number (500- 2100) for various axial locations of tube with CuO–water nanofluids as working media were discussed in detail. The finite volume method and SIMPLE algorithm are utilized to solve the governing equations numerically. The numerical results shows that with increasing Reynolds number, local Nusselt number enhanced. The variations of the local Nusselt number relative to volume concentrations are not uniform. According to the results, an equation was obtained for Nusselt number predicted data using the dimensionless numbers. The relation between local Nusselt number and Re number also compared for other previous work. There are agreement in results and found the maximum difference between results reach to be 6.3% approximately which validate the current computational model.

Keywords: Nanofluids, Heat transfer, Tubes, Laminar flow

Nomenclature

Symbols	Meaning	Units
C_p	Specific heat at constant pressure	J/kg.K
D	Diameter of the tube	m
h	Heat transfer coefficient	W/m ² .K
k	Thermal conductivity	W/m.K
Nu	Nusselt number	-
P	Pressure	Pa
Pr	Prandtl number	-
q''	Heat flux	W/m ²
Re	Reynolds number	-
T	Temperature	K
V	Velocity vector	m/s
x	Distance along x-axis	m
y	Distance along y-axis	m
Greek letters		
μ	Molecular dynamic viscosity	N.s/m ²
ϕ	Volume fraction	-
ρ	Density	kg/m ³
α	Thermal diffusivity	m ² /s

Subscripts

bf	base fluid
f	Fluid
nf	Nanofluid
s	Solid of particle

1. Introduction

Fluids heating and cooling represent the essential parts in large portions of industrial applications, for example, power stations, production processes and transportation. The majority of the strategies for heat transfer depend on the structure variety, vibration of the heated surface, and injection or suction of fluid [1, 2]. Heat Transfer inside the conventional fluids represented by water, oil and ethyleneglycol innately has low thermal conductivity in contrast with the metals and metallic oxides. Consequently, fluids with suspended solid particles are expected to have better heat transfer properties [3]. An innovative strategy for enhancing heat transfer by using ultra fine solid particles in the fluids has been utilized widely during the last decade. The term nanofluid alludes to these sorts of fluids by suspending nano- scale particles in the base fluid [4]. Alumina and copper oxide are the most regular and cheap nanoparticles which might be used inside the experimental investigations [5].

Choi and Eastman [6] utilized the particles in nanometer dimensions as a suspended solution. They demonstrated that the nanofluid thermal conductivity considerably increased. Lee et al. [7] confirmed that the suspension of 4% with 35 nm CuO particles in ethylene glycol had 20% augmentation in the thermal conductivity. Choi et al. [8] watched 60% improvement in the thermal conductivity of engine oil with 1.0% carbon nanotube. Das et al. [9] examined the temperature reliance of thermal conductivity in the nanofluids. They had been determined that a 2–4 increment in the thermal conductivity of nanofluid can occur over a temperature scope of 21–51°C.

Bai, M., et al. [10] investigated the heat-transfer character of nanofluids and applied nanofluids to engine cooling system. CFD numerical simulation technique became employed to analyze the utility value of nanofluids in engine cooling. The simulation results showed that nanofluids could improve engine heat dissipating capacity and Cu-water nanofluids had better heat-transfer ability. They likewise found that with increment of nanoparticles concentrations, more enhancement of engine dissipating capacity will happen. They concluded further to that when the concentration become 5%, the heat dissipating capacity would enhanced by 44.1%.

Namburu et al. [11] numerically examined turbulent flow and heat transfer of Al₂O₃, CuO, and SiO₂ nanoparticles suspended in ethylene glycol and water based fluids in a straight pipe. They showed that SiO₂ with nanoparticle diameter of 20 nm offers the best thermal conductivity enhancement because of better viscosity value of lower nanoparticle diameter.

Zamzaman et al. [12] investigate the effect of forced convective heat transfer of nanofluids of aluminum oxide and copper oxide prepared in ethylene glycol in turbulent flow. They discover vast enhancement in convective heat transfer coefficient of the nanofluids in contrast to the base fluid. Furthermore, their outcomes show that with expanding nanoparticles concentration and nanofluid temperature, the convective heat transfer coefficient of nanofluid will increases.

Naik et al. [13] analyzed turbulent convective flow of CuO nanofluids of propylene glycol–water as the base fluid and flow through a circular tube, subjected to a regular and uniform heat flux at the wall. They found that nanofluids containing extra concentrations have proven higher heat transfer coefficient. They compared their numerical results with the experimental data and affordable appropriate agreement is performed. Sheikholeslami, et al. [14] observed numerically the impact of natural convection heat transfer in a nanofluid (CuO) crammed enclosure with elliptic internal cylinder. . They observed that Nusselt number increments with an increment each of nanoparticles volume fraction, Rayleigh numbers and inclination angle. . Likewise, they found that expanding Rayleigh number prompts a lessening in heat transfer enhancement. For high Rayleigh number the minimum heat transfer enhancement ratio occurs at slanted elliptic cylinder is 90°.

Hsien-Hung Ting and Shuhn-Shyurng Hou[15] numerically research the convective heat transfer of water-based Al₂O₃ nanofluids flowing through a square cross-section duct with a fixed heat flux under laminar flow conditions. They investigated the influences of nanoparticle concentration and Peclet number on the heat transfer attributes of Al₂O₃-water nanofluids. The nanoparticle diameter is 25 nm and six particle concentrations (0.2, 0.5, 1, 1.5, 2, and 2.5 vol.%) are taken into consideration. They confirmed that the heat transfer coefficients and Nusselt number of Al₂O₃-water nanofluids increment with expansions within the Peclet number and similarly particle volume concentration. The heat transfer coefficient of nanofluids is elevated via 25.5% at a particle volume concentration of 2.5% and a Peclet quantity of 7500 as contrasted with that of the base fluid (pure water).

Bouhalleba, M. & Abbassi, H. [16], analyzed numerically heat transfer and fluid flow of natural convection in inclined cavity full of CuO-water nanofluid and partially heated. . The Prandtl number is kept constant at 7.02 corresponding to water. Aspect ratio and solid volume fraction are changed from 0.5 to 4 and from 0% to 4% respectively, and the inclination angel is varied from 0° to 90°. They found that the efficiency of heat transfer is enhanced by the increment of nanoparticles ratio into base liquid; but there is an optimum solid volume fraction which promotes the heat transfer rate. Additionally they found that

the diameter of solid particle is an imperative parameter that influences the heat transfer efficiency, its effect is more critical than the concentration itself.

Ningbo Zhao, et al [17], studied a three-dimensional numerical analysis of the laminar heat transfer and flow characteristics of Al₂O₃–water nanofluids through a flat tube at constant heat flux boundary condition. They discovered with their numerical results that with the addition of nanoparticle will enhances the heat transfer and the pressure loss of base fluid in all of the flat tubes at different Reynolds number and temperature. Both the relative average convective heat transfer coefficient and pressure drop can be enhanced by increasing nanoparticle volume concentration and reducing nanoparticle size. And the heat transfer and pressure drop enhancements of nanofluids are more evident at smaller Reynolds number and higher temperature. Also, they derived new correlation models for thermal conductivity and viscosity of Al₂O₃–water nanofluids and confirmed it.

In this research, the convective heat transfer in the developed region of the tube flow containing water and CuO nanofluid under constant heat flux was examined by the usage of Computational Fluid Dynamics (CFD) techniques. CuO nanoparticles with average diameter of 29 nm was dispersed in water. The nanofluids with three different CuO nanoparticle concentrations (1%, 3.0%, and 5% volume concentrations) were used. Effects of nanoparticle concentrations on the convective heat transfer coefficient were also investigated with different Reynolds number and find the best correlation for heat transfer coefficient. A comparison of the results predicted form the current model with experimental data from literature will made.

2. Problem Description and Governing Equations

The basic flow configuration, under study, is shown in Fig. 1. A two dimensional pipe have 1.2 m length and 4.75 mm inner diameter was spotted in the simulation.

$$q'' = 5000 \text{ W/m}^2$$

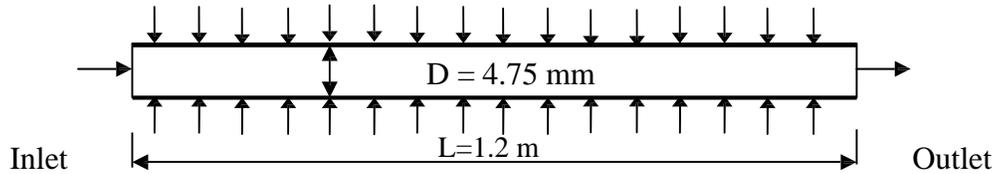


Fig. 1: Schematic diagram of the physical system.

The continuity, momentum and energy equations for a two dimensional incompressible laminar flow has been solved using appropriate boundary conditions by mean computational fluid dynamics technique. Following assumptions have been made: two-dimensional problem, there is no viscous dissipation, no gravity acts, the fluid properties are constant and radiation heat exchange was assumed negligible. At steady state conditions using above assumption, the governing equations as given below [18]:

Continuity equation:

$$\nabla \cdot (\rho_{nf} \mathbf{V}) = 0 \quad (1)$$

Momentum equation:

$$\nabla \cdot (\rho_{nf} \mathbf{V} \mathbf{V}) = -\nabla P + \nabla \cdot (\mu_{nf} \nabla \mathbf{V}) \quad (2)$$

Energy equation:

$$\nabla \cdot (\rho_{nf} C V T) = \nabla \cdot (k_{nf} \nabla T) \quad (3)$$

The effective physical properties of the nanofluids in the above equations are:

The viscosity of the nanofluids can be approximated as viscosity of a base fluid μ_f is given by [19, 18]:

$$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}} \quad (4)$$

The density of the nanofluid is given as:

$$\rho_{nf} = (1-\phi)\rho_f + \phi\rho_s \quad (5)$$

The thermal diffusivity of the nanofluid is given as:

$$\alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}} \quad (6)$$

The heat capacity of the nanofluid is expressed as [20, 21]:

$$(\rho C_p)_{nf} = (1-\phi)(\rho C_p)_f + \phi(\rho C_p)_s \quad (7)$$

In this research, the single-phase method was implemented. Solid particles with less than 100 nm diameter were spotted inside the single-section approach. Furthermore, the thermal conductivity of nanofluid is approximated as given in references [20, 21]:

$$k_{nf} = k_f \frac{(k_s + 2k_f - 2\phi(k_f - k_s))}{(k_s + 2k_f + \phi(k_f - k_s))} \quad (8)$$

Thermophysical properties of the nanofluid are given in Table (1). In current formulation, thermophysical properties of the nanofluid are assumed to be constant.

Table 1: Thermophysical Properties.

Property	Fluid phase (water)	Nanoparticle (CuO)
C_p (J/kg K)	4182	540
ρ (kg/m ³)	997.1	6500
k (W/m K)	0.613	18
α (m ² /s)	1.47×10^7	57.45×10^7

The boundary conditions are prescribed as follows:

- At the tube inlet: $u(y) = U_i$ and $T = T_i = 298$ °K
- At the tube outlet: pressure outlet boundary $P = 0$
- At the wall : no-slip condition, $q'' = 5000$ W/m².

The above boundary conditions are shown in Fig. 1. The total length of the computational domain is taken as ($L = 1.2$ m) to ensure fully developed outlet boundary condition, and the constant heat flux of 5000 (W/m²) as a boundary condition at the pipe wall was applied.

3. Numerical Procedure

The CFD (Computational Fluid Dynamic) is used to solve the two-dimensional flow of CuO-water nanofluid into a tube with uniform heat flux. The governing equations were discretized with control volume scheme. For the convective and diffusive terms, a first order upwind method was utilized while SIMPLE method was established for solving iteratively the velocity-pressure coupling algebraic equations. The physical boundary conditions for the geometry are defined and applied as inlet, pressure outlet and wall of the tube as shown in Fig. 1. For single phase approach, solid particles with diameter less than 100 nm have been noticed.

Consequently, single phase approach was selected for nanofluid modeling [22]. The fluid was entered the pipe with a constant velocity in every run. The constant heat flux of 5000 (W/m²) as a boundary condition at the pipe wall was applied.

4. Effect of the Mesh Refinement

It is important to have a good mesh to get an accurate solution. There are some general guidelines to create a good mesh. A good mesh should be fine enough with high quality cells and good distribution of these cells is essential. Moreover the mesh should not have more cells than the available computer resources can handle. The grid independence was checked. The cases considered are as following:

For two dimensional pipe the hexahedral structured elements mesh was used. Four mesh sizes were used and the governing equations were solved based on these four meshes respectively. The results obtained from these meshes at Re = 1700 and volume fraction 3% are summarized in table (2).

From these results (table (2)) it can be seen that the solution becomes independent of grid size and increasing the size of mesh more than the mesh No. 3 do not have a significant effect on the results just increasing the run time and memory requirements. Therefore for more accuracy the mesh No.3 (3600 cells) will be considered in all calculations.

Table 2: Mesh independent case.

Mesh No.	Mesh size (Number of cells)	Heat Transfer coefficient, h (W/m ² .K)
No. 1	12000	748.41
No. 2	24000	729.12
No. 3	36000	727.305
No.4	48000	727.031

5. Results & Discussion

Results were obtained for single-phase flow using relations mentioned at article 2 for different values of Re, ϕ and axial distance (x/D) with a fixed heat flux at tube wall (*i.e.* $q=5000$ w/m² K).

The local heat transfer coefficient and local Nusselt number were evaluated using the following equations:

$$Nu(x) = \frac{h(x)D}{k} \quad (9)$$

$$h(x) = \frac{q''}{(T_w(x) - T_f(x))} \quad (10)$$

Where, D, q'' , k, T_w and T_f are pipe diameter, heat flux, and thermal conductivity of the fluid, tube wall temperature and nanofluid bulk temperature, respectively at an axial position.

Fig. 2a shows the variation of heat transfer coefficient with Re at an axial location (x/D=150) with 29 nm particles diameter. The results obtained for both pure water and CuO- water nanofluid with three different ϕ values (*i.e.* $\phi=1\%, 3\%$ and 5%). The results obtained shows that the heat transfer coefficient was increases with increasing Re and this due to the increment of fluid velocity which makes the fluid less heated and this cause decrease in temperature difference between the wall temperature and fluid bulk temperature, also it seen that the heat transfer coefficient for pure

water is less than it in nanofluid because the particles ratio makes the fluid more heated. Fig. 2b shows the heat transfer coefficient in tube versus axial location for pure water and CuO-water nanofluid with three different concentrations ($\phi=1\%, 3\%$ and 5%). As seen from the figure the heat transfer coefficient decreases with increasing the axial distance because there is increase at the temperature difference between the tube wall and fluid bulk temperatures, also the heat transfer coefficient for nanofluid is higher than it in pure water because the CuO particles make the fluid more heated then decreases the temperature difference between wall temperature and bulk temperature then h will increase.

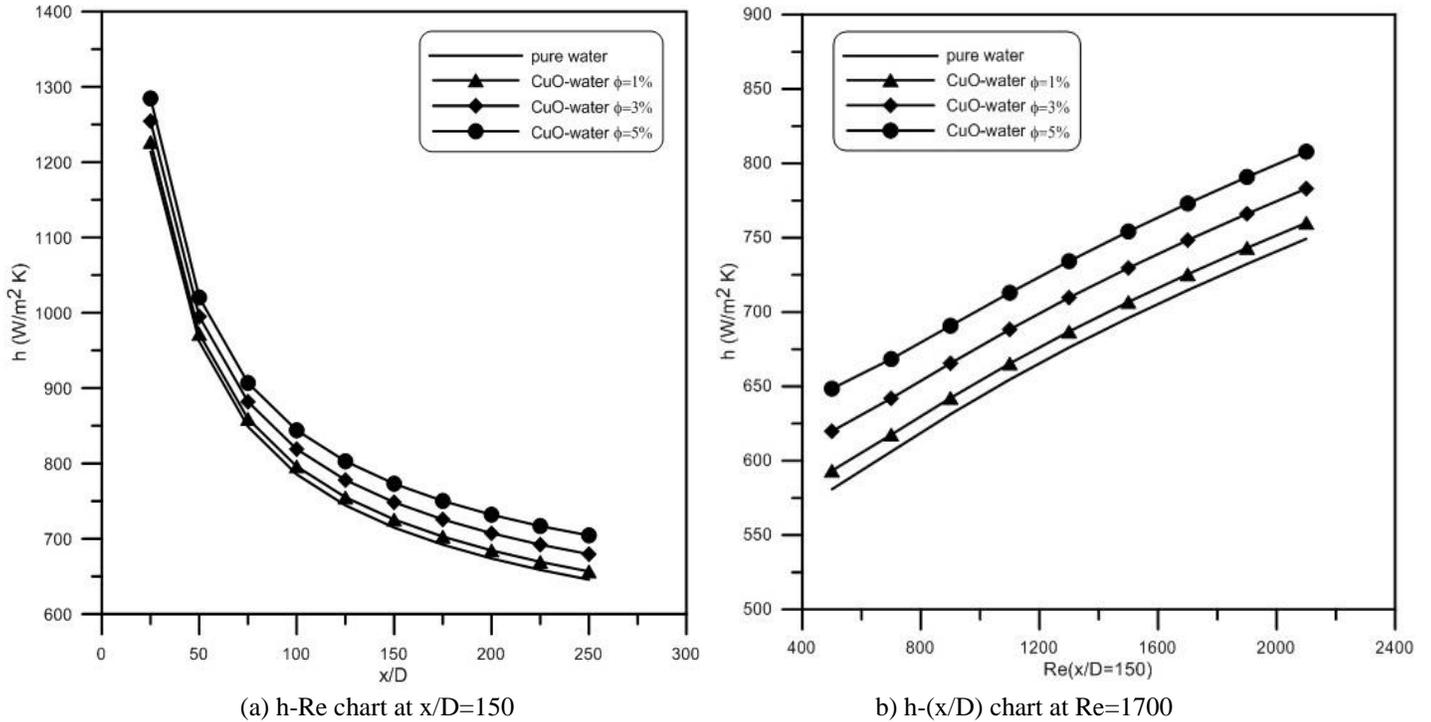


Fig. 2: Heat transfer coefficient variation with Re number and axial distance.

Fig. 3a shows Nusselt number versus Re at axial distance ($x/D=150$) for pure water and CuO-water nanofluid with three particle concentrations (*i.e.* $\phi=1\%, 3\%$ and 5%) with nanofluid diameter of 29 nm. Usually Nu will increase with increasing the axial distance due to increasing h as describe by Fig.3a but Nu for pure water is higher than it in CuO-water nanofluid and this due to the increment in thermal conductivity of nanofluid than thermal conductivity of pure water and the thermal conductivity of nanofluid is increase when ϕ increases which make reduction in Nu as illustrated in the figure. Fig. 3b shows Nusselt number versus axial distance for pure water and CuO-water nanofluid with three particle concentrations (*i.e.* $\phi=1\%, 3\%$ and 5%) with nanofluid diameter of 29 nm. The figure shows that Nu will decrease along the axial distance due to h decrement and it decreases too with increasing ϕ due to increasing of thermal conductivity of nanofluid as illustrated previously.

Figs. 4a and 4b shows dimensionless temperature profile along vertical distance at different axial locations with $Re=500$ for pure water and CuO-Water with $\phi=5\%$ respectively. It is important to note that such a decrease of the fluid temperature at the tube wall exists along the tube length and seems to be more interested toward the tube end. These results have clearly shown the helpful impact because of the nanoparticles, an impact that may be clarified by the way that with the presence of these particles, the thermal properties of the resulting blend have been largely enhanced.

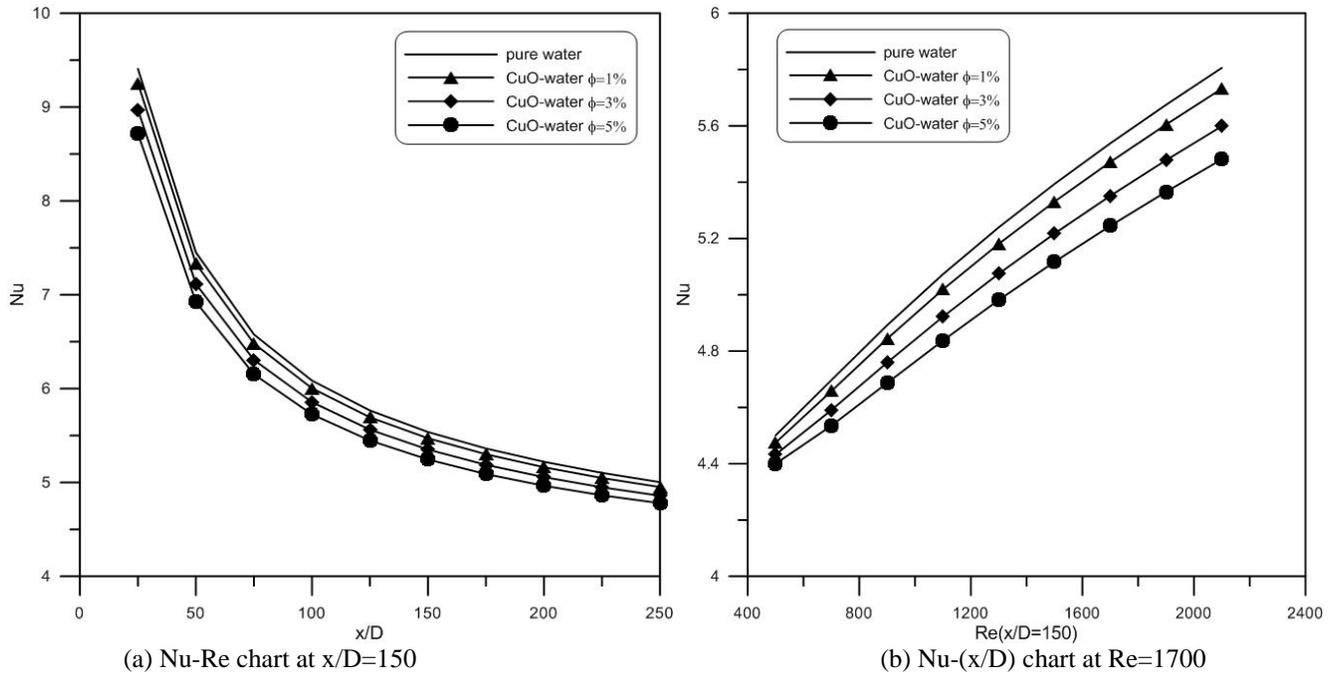


Fig. 3: Nusselt number variation with Re number and axial distance.

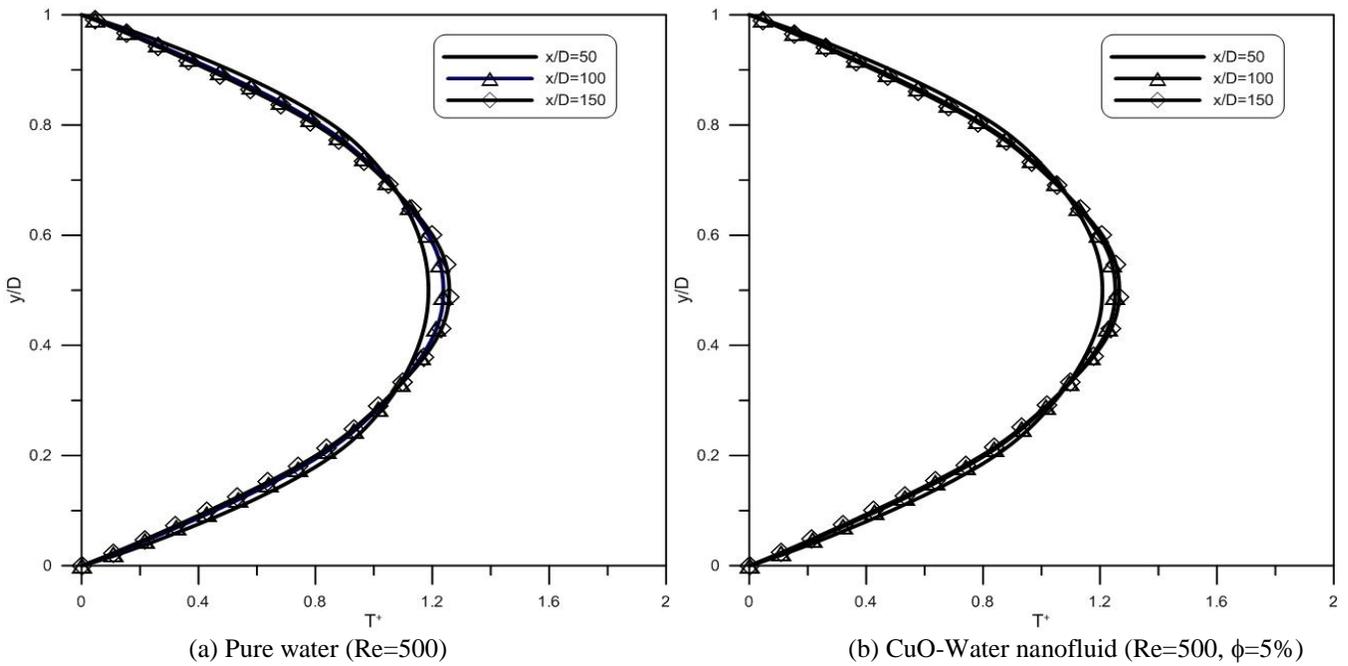


Fig. 4: Dimensionless temperature profile at different axial locations.

Fig. 5 Shows profiles of wall temperature along tube axial distance for $Re=500$ and $q=5000$ (w/m^2) at vertical level $y=4$ mm for both pure water and CuO-Water nanofluid with different concentrations ($\phi=1\%$ and 5%). The figure shows the increment in fluid temperature with the axial distance because the fluid is more heated towards the tube end its temperature increases. The concentration of the particles increases the temperature of CuO-Water more than the pure water temperature and that clearly observed at the larger concentration of particles (*i.e.* $\phi=5\%$).

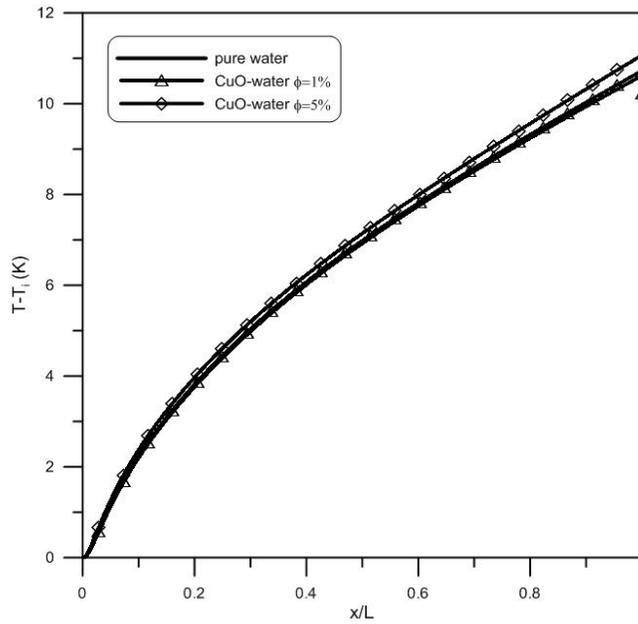


Fig. 5: Temperature difference variation with axial distance ($Re=500$, $y=4$ mm).

Fig. 6 represent a comparison of numerical and experimental results for Al_2O_3 nanofluid with particle diameter 45 nm at axial distance ($x/D=147$) [23].

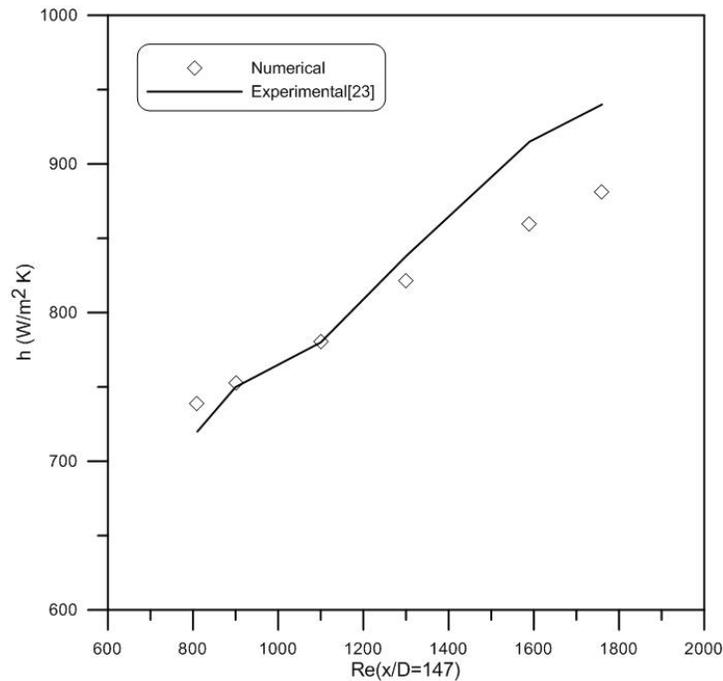


Fig. 6: Comparison of experimental and numerical results.

The figure shows for the first four heat transfer coefficient values there were a reasonable agreement between the numerical and experimental data and there is little difference between the results and experimental data with a maximum error determined to be around 6.3% so the determined results from the current model are acceptable.

6. Correlation

The convective heat transfer of the nanofluid relies upon various factors such as heat capacity, viscosity, particles volumetric concentration and axial location. Based on the results of numerical computations of new correlations are developed for Nusselt number as a function of Reynolds number range (500 -2100), particles volume fraction range (1% - 5%) with pure water and axial location (x/D).

$$Nu = 5.96Re^{0.18}(1 + \phi)^{-1.17}(x / D)^{-0.28} \quad (11)$$

This relation of correlate Nusselt number data for the nanofluid as shown in Fig.7. The correlated Nu data were in good agreement with the Nu simulated with 2% maximum error.

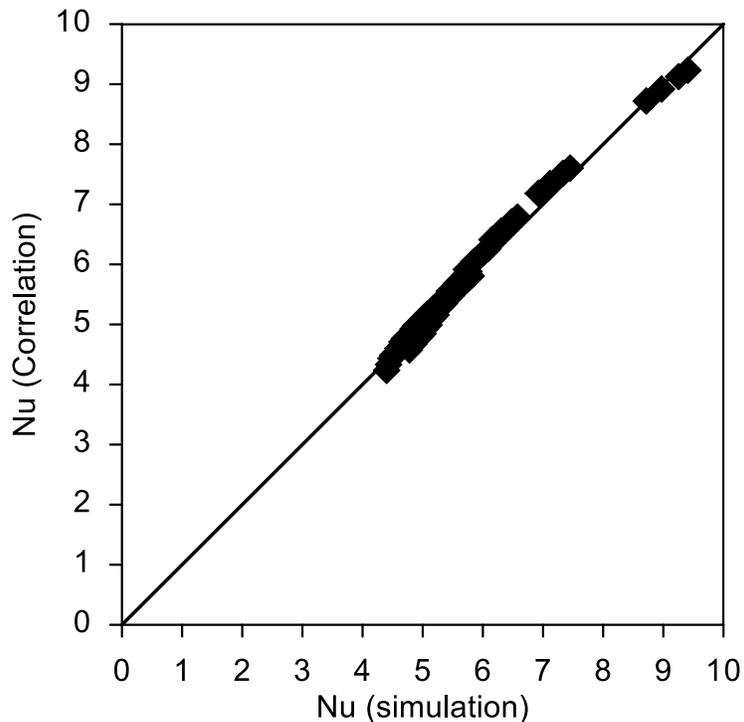


Fig.7: Parity plot comparing the prediction values and simulation results.

7. Conclusions

In the present study numerical simulation of CuO-water nanofluid flowing in a tube with uniform heating at the wall was examined with different flow parameters, the calculations were performed using finite volume method with SIMPLE algorithm at the developed region of the tube. The results shows that the heat transfer coefficient enhanced with increasing the concentration of the nanoparticles and Re number. Also both of heat transfer coefficient and Nu number decreases with the axial distance and Nu number is more decreases with increasing the particle concentration. A new correlation derived for the predicted Nu Number from the simulation based on Re, axial distance (x/D) and particle volumetric concentration (ϕ) which shows good agreement with the Nu simulated results. The current numerical model was validated with experimental data from the literature and it gives a quite agreement in the lower Re numbers.

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