

## Optimal Control of a Stirling Engine

**Karsten Schwalbe, Abdellah Khodja, Mathias Scheunert, Karl Heinz Hoffmann**

Technische Universität Chemnitz

Str. der Nationen 62, 09111 Chemnitz, Germany

karsten.schwalbe@physik.tu-chemnitz.de; abdellah.khodja@physik.tu-chemnitz.de

mathias.scheunert@physik.tu-chemnitz.de; hoffmann@physik.tu-chemnitz.de

### Extended Abstract

Many industrial engines, like machine tools for example, produce a lot of waste heat during operation. To reduce energy costs, this heat can be recuperated using an appropriate heat engine. The Stirling engine is such a heat engine as it can make use of the external heat source and produce usable mechanical power. Although the Stirling engine can theoretically reach the Carnot efficiency in the reversible limit, in reality effects like irreversible heat and mass transfer as well as friction losses lower its performance significantly. Therefore, thermodynamic models are needed to estimate the performance of real Stirling engines.

In this study an endoreversible model of the Stirling engine is presented. In Endoreversible Thermodynamics [1] all irreversibilities are restricted to interactions between reversibly working subsystems. The model of the Stirling engine consists of two gas chambers with pistons and a regenerator in between. The gas chambers are in contact with an external heat source and a heat sink for cooling, respectively. The pistons can be moved freely and independent from each other with the help of linear motors. The investigated system can be described using, among others, the ideal gas assumption, the Newtonian heat transport law, a simplified mass transfer law, and friction losses proportional to the squared rotation speed. The needed transport coefficients are derived using experimental data.

On the basis of this model the optimal trajectory of the engine's pistons is calculated using Pontryagin's maximum principle. This leads to an ordinary differential equation system with mixed start and final conditions for which a special solution method has to be used to ensure convergence. It is found that the optimized paths differ significantly from the conventional harmonic paths. While the latter show a sinusoidal behaviour, the former exhibit a relatively long isochoric phase. As a result, the power output is much larger than in case of a conventional piston movement. For the considered engine parameters the power output nearly doubles.

Additionally, the influence of the time period of the cyclic piston motion on the performance of the engine is investigated. We find that there is a local maximum of the power output for a certain time period depending on the engine parameters. Consequently, there exists an optimal time period for the Stirling engine that should be used for operation.

Investigating the influence of the friction coefficient on the power output, it turns out that the power output decreases monotonously with an increasing friction coefficient. Furthermore it follows from the numerical data that the decrease of the power is relatively large for small friction coefficients, so that it is very crucial to lower the friction losses of a high performing Stirling engine as much as possible.

Finally, some possible modifications for the Stirling engine model are discussed. Here, Endoreversible Thermodynamics shows one of its greatest benefits: A high degree of adaptability while the model equations remain manageable.

### References

- [1] K. H. Hoffmann, J. M. Burzler, and S. Schubert, "Endoreversible thermodynamics," *J. Non-Equilib. Thermodyn.*, vol. 22, no. 4, pp. 311-355, 1997.