Parallel Finite Element Approach for large Thermal Problems Applied to Glass Bending Furnace

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Abstract - In this paper, iterative sub-structuring methods are applied to an FEM discretized model of an industrial glass thermoforming furnace. The FEM-based model is dubbed the component interaction network (CIN) and has been previously used in many thermal problems modelling. Applying iterative sub-structuring methods to CIN led to the creation of multi-CIN, used for large heat problems, in particular the glass thermoforming furnace problem. The substructures are then simulated simultaneously using the same direct thermal solver used for the general problem. Multiprocessing technology was exploited for parallel solving of the sub-structures, while a coupling algorithm handled interfacial coupling. In addition, a time-marching scheme was developed, including a coarse numerical problem solving for stabilization phases and adaptive time stepping for stable repetitive phases. The model was compared to the experimental measurements. The results were met with good reproduction of the thermal behavior of the components and acceptable accuracy.

Keywords: Domain decomposition, Energy optimization, Heat transfer, Numerical modelling

1. Introduction

Glass thermoforming is the process of bending glass by heating it in a large furnace, in order to create bended products like windshields. The furnace is composed of many cells, and has hundreds of heating ceramic plates that emit radiation. To obtain the desired final concavity of the windshield, experts must try and guess different power values for each ceramic heater until they get the desired shape, creating thus a "heating recipe" to be used for the production of the specific shape. However, guessing this recipe takes time, consumes energy and results in a lot of glass wasted while guessing. Therefore, it is of interest to simulate the process of thermoforming to calculate the temperature distribution on the glass, from which concavity can be deduced. With thermal simulation, the right heating recipe could be predicted without the need of wasting material and energy on actually guessing it. Previous studies led to the creation of a thermal simulation software called SGO, that was able to calculate this distribution for a glass undergoing thermoforming in a pilot furnace composed of 3 heating cells, within an accuracy of +/-15°C on temperatures in the order of 600°C and up. This software was based on the method of CIN (Component Interaction Network) that establishes the thermal network of lumped elements, interconnected with their respective heat exchanges.

However, thermoforming on a large industrial furnace operates differently than on a small pilot furnace, and thus has different output. An industrial furnace has around 6 times more heating cells and receives not only a single glass load, but a continuous injection of glass loads. Therefore, passing from small scale pilot furnace to large scale industrial furnace heavily increases the number of variables, heat exchanges, and, in consequence, the thermal matrix to resolve, making the resulting thermal network quite large. Simulation on the same model is thus complex and not practical.

Iterative sub-structuring methods were encountered in literature as convenient numerical methods for large simulations.

2. Multi-CIN Thermal Model
2.1. Iterative Sub-Structuring Methods

Iterative sub-structuring methods are a particular case of domain decomposition methods used to solve very large linear systems of algebraic equations for sub-domains that are non-overlapping. These equations are typically generated from the discretization of differential equations using finite element / volume / differences methods.

Many methods and algorithms of iterative sub-structuring methods are proposed in literature [1]–[9]. These methods and algorithms all have a common logical ladder and are similar in handling a large problem numerically.

Considering a system of linear equations resulting from the finite-element discretization of heat exchanges equations acting upon a large domain:

$$Ku = f \tag{1}$$

Where **K** represents the global thermal matrix, the temperature vector across the domain, and the heat fluxes vector.

Eq. (1) can be re-written by distinguishing the interior zone and the boundary zone as follows:

$$\begin{bmatrix} K_{II} & K_{IB} \\ K_{BI} & K_{BB} \end{bmatrix} \begin{bmatrix} u_I \\ u_B \end{bmatrix} = \begin{bmatrix} f_I \\ f_B \end{bmatrix} \tag{2}$$

Where the indices I and B designate the internal and boundary nodes, respectively. The terms having the form of K_{ij} represent physically the effect that has the force from zone j on the scalar field of zone i. Moreover, the sub-matrixes in Eq. (2) are expressed as follows:

$$K_{II} = \begin{bmatrix} K_{II}^{[1]} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & K_{II}^{[N]} \end{bmatrix} \tag{3}$$

$$K_{IB} = (K_{BI})^T = \begin{bmatrix} K_{IB}^{[1]} R_B^{[1]} \\ \vdots \\ K_{IB}^{[N]} R_B^{[N]} \end{bmatrix} \tag{4}$$

$$K_{BB} = \sum_{i=1}^{N} R_B^{[i]^T} K_{BB}^{[i]} R_B^{[i]} \tag{5}$$

$$f_I = \sum_{i=1}^{N} R_I^{[i]^T} f_I^{[i]} \tag{6}$$

$$f_B = \sum_{i=1}^{N} R_B^{[i]^T} f_B^{[i]} \tag{7}$$

Where the exponents in brackets designate the sub-structure index, $R_B^{[i]}$ and $R_I^{[i]}$ are logical matrixes of Boolean type that project the global degrees of freedom into local degrees with respect to each sub – domain on the boundary interface and the sub-structure interior domain, respectively.

Iterative sub-structuring methods consist of isolating the problem on the domain interfaces and solving it apart, paving the way for the resolution of each sub-structure independently and simultaneously. Based on Cholesky factorization, Eq. (2) can be re-written in a parallelized form of:

$$\begin{bmatrix} K_{II} & K_{IB} \\ 0 & S \end{bmatrix} \begin{bmatrix} u_I \\ u_B \end{bmatrix} = \begin{bmatrix} f_I \\ g \end{bmatrix} \tag{8}$$

From Eq. (8) it is possible to solve each system separately using the same direct solver for the general domain, while an additional algorithm ensures the problem resolution at interface. The two resulting systems to solve are:

$$K_{II}^{[i]}u_I^{[i]} = f_I^{[i]} - K_{IB}^{[i]}R_B^{[i]}u_B \quad i = 1, \ldots, N \tag{9}$$

$$Su_B = g \tag{10}$$

Eq. (9) is the sub-structure internal problem and Eq. (10) is the boundary problem. The matrix S is dubbed the Schur complement matrix. The expressions of S and g are as follows:

$$S = \sum_{i=1}^{N} S^{[i]} = \sum_{i=1}^{N} R_B^{[i]^T} \left(K_{BB} - K_{BI}^{[i]} \left(K_{II}^{[i]} \right)^{-1} K_{IB}^{[i]} \right) R_B^{[i]} \tag{11}$$

$$g = \sum_{i=1}^{N} g^{[i]} = \sum_{i=1}^{N} R_B^{[i]^T} \left(f_B^{[i]} - K_{BI}^{[i]} \left(K_{II}^{[i]} \right)^{-1} f_I^{[i]} \right) R_B^{[i]} \tag{12}$$

2.2. CIN Thermal Network

CIN, which stands for component interaction network is a numerical approach that consists of decomposing the thermal analysis model into a number of components possessing physical distinctions. The CIN approach is based on FVM method of model discretization. For each component, an intrinsic model of heat transfer is defined, that can either be zero-dimensional (no temperature variation), one, two or three-dimensional. Heat and mass exchanges occur between the components at their boundary limits. The overall system of components interconnected by their respective heat and mass fluxes forms the component interaction network (CIN). The method has been used in many thermal problems[10]–[16]. An example of CIN application is shown in Fig.1
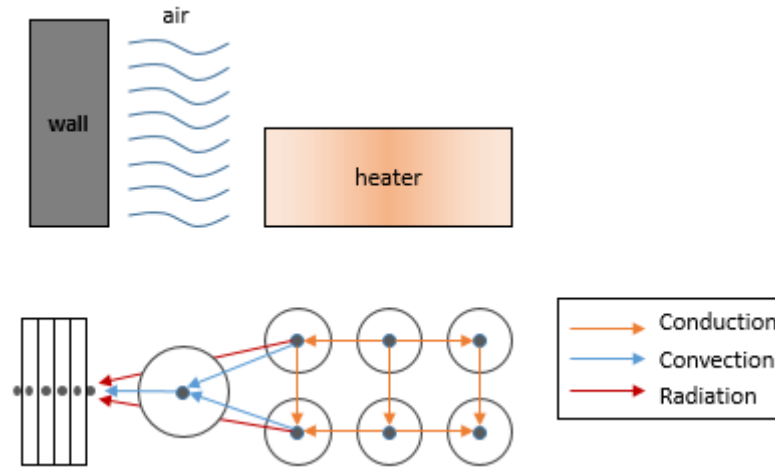

Fig. 1 Example of a small heater-wall problem discretized in CIN. Dots represent nodes of uniform temperatures, circular elements are 0-D, whose temperature are represented by the node, and rectangular ones are 1-D branches whose temperature are represented by a succession of nodes.

In the CIN method, all of the heat transfer modes are expressed in the form of:

$$Q_{ij} = G_{ij}(T_i - T_j) \tag{13}$$

Where Q is the heat flux going from the hotter node i to the colder node j, G_{ij} is the thermal conductance between i and j and T is the uniform temperature of the node. In case of radiation, conductance is function of emissivity and geometrical view-factor, and they are pre-processed using the zonal method [17], [18] on a different solver. Eq. (13) combined with the general heat equation for each node leads to the system of algebraic equations:

$$AT + B = C\frac{dT}{dt} \tag{14}$$

Where A is a square matrix having dimensions $N \times N$, dubbed the heat exchange matrix that regroups all the conductances of the heat exchanges between the nodes, N being the total number of nodes. A is expressed as:

$$A = \begin{bmatrix} \overline{G_{11}} & \cdots & \overline{G_{1N}} \\ \vdots & \ddots & \vdots \\ \overline{G_{N1}} & \cdots & \overline{G_{NN}} \end{bmatrix} \tag{15}$$

Where $\overline{G_{\iota J}}$ is the overall heat conductance between nodes i and j which is expressed as:

$$\overline{G_{\iota J}} = \sum_j \left(G_{ij,cond} + G_{ij,rad} + G_{ij,conv} \right) \tag{16}$$

And **C** is dubbed the heat capacity matrix, which is a diagonal square matrix having dimensions of and has the following form:

$$C = \begin{bmatrix} C_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & C_N \end{bmatrix} \tag{17}$$

CIN makes it easier to solve a heat problem by transforming integro-differential equations into a system of algebraic equations, having the general form of Eq. (14) – (17).

2.3. Multi - CIN

Applying the iterative sub-structuring methods on the CIN thermal model leads to the creation of a new method, the multi-CIN, which applies to large heat transfer problems. Multi-CIN consists of partitioning the large thermal CIN network into sub-networks with common boundary layers. The FEM system of algebraic equations is arranged in a way that boundary equations are written in the end, so that the boundary system can be solved in parallel separately. The resulting system of equations to resolve is thus Eq. 18 and Eq. 10, with the condition shown in Eq. 19:

$$K_{II}^{[i]}T_I^{[i]} + K_{IB}^{[i]}R_B^{[i]}T_B^{[i]} = R_I^{[i]^T}\left[C_I^{[i]}\frac{dT_I^{[i]}}{dt} - B_I^{[i]}\right] \tag{18}$$

$$g = \sum_{i=1}^{N_U} g^{[i]} = \sum_{i=1}^{N_U} R_B^{[i]^T}\left(\left(C_B^{[i]}\frac{dT_B^{[i]}}{dt} - B_B^{[i]}\right) - K_{BI}^{[i]}\left(K_{II}^{[i]}\right)^{-1}\left(C_I^{[i]}\frac{dT_I^{[i]}}{dt} - B_I^{[i]}\right)\right)R_B^{[i]} \tag{19}$$

Applying multi-CIN allows the exploitation of multi-cores CPU, each sub-structure being handled on a different CPU, while communication between solvers across the CPUs and the interfacial resolution is handled simultaneously by an additional algorithm. Fig 2. Illustrates the parallelization process using sub-structuring methods.
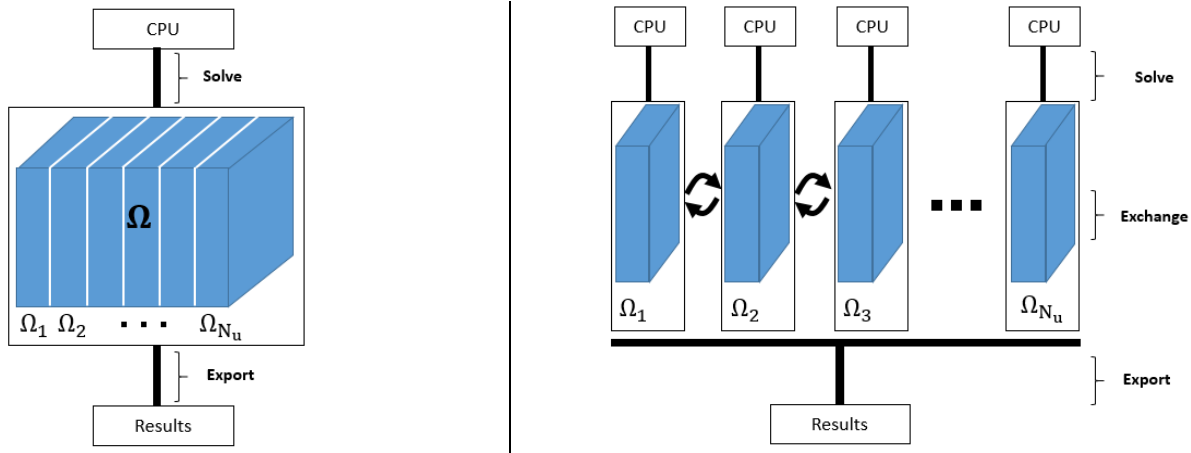

Fig. 2: Visual comparison between monolithic method and sub-structuring method in multi-CPU utilization.

3. Experimental Validation and Algorithm

The method was applied to an already existing thermal model for an R&D glass thermoforming furnace, in order to be able to extend the model into a large industrial furnace. This model is called SGO and is based on the CIN network method.

The schematic of the furnace is seen in Fig. 3. The furnace is composed of two zones: fixed zone, and a circulating zone. The circulating zone consists of the metallic support and the glass, while the fixed zone consists of the ceramic heaters, lateral walls and other furnace elements.

On the metallic support, a vertical head board set on the front of the moving parts acts as a radiation shield, isolating radiation and other heat transfer phenomena in each chamber. This headboard is composed of an insulating clothing.
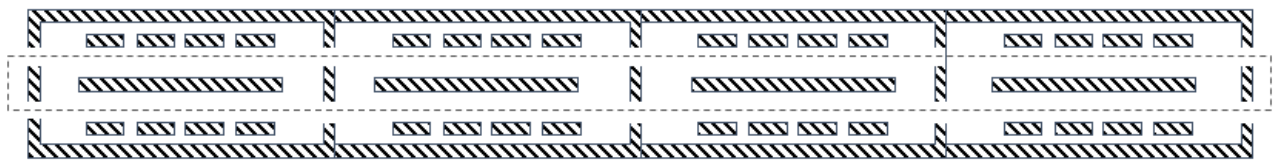

Fig. 3: schematic representation of the industrial furnace with only 4 thermoforming cells showing out of 18.

Using optimum domain partitioning [19], [20] methods, the partitioning was chosen along this headboard. That is, each sub-structure constitutes a heating chamber. At the interface, and due to the headboards acting the role of isolators, only advection is present. Advection has the characteristic of being a unidirectional flux. In other words, the heat flux gain from mass flow of air node I to air node j is:

$$Q_{ij} = \dot{m}_{ij} C_{p,A}(T) T_i \tag{20}$$

Where $C_{p,A}(T)$ is the heat capacity of air at temperature T and $\dot{m}_{ij}$ is the mass flux from i to j.

Therefore, multi-CIN is applied with the following conditions: For N_A air volumes interacting at the interface, $\left[A_1^{[i]}, A_2^{[i]}, \ldots, A_{N_A}^{[i]}\right]$ being the air volumes at interface of cell i and $\left[A_1^{[j]}, A_2^{[j]}, \ldots, A_{N_A}^{[j]}\right]$ being the air volumes at interfaces touching cell i, that is $j = i + 1$ or $j = i - 1$, the sub-matrices reviewed in paragraph 2.1 are as follows:

$$
K_{IB}^{[i]} = \begin{bmatrix}
\dot{m}_{A_1^{[j]} \to A_1^{[i]}} C_{P,air} & \cdots & 0 \\
0 & \ddots & \vdots \\
0 & \cdots & \dot{m}_{A_{N_A}^{[j]} \to A_{N_A}^{[i]}} C_{P,air} \\
0 & \cdots & 0 \\
\vdots & \vdots & \vdots \\
0 & 0 & 0
\end{bmatrix}
\tag{21}
$$

$$
K_{BB}^{[i]} = 0, \; K_{BI}^{[i]} = 0
\tag{22}
$$

Where $K_{IB}^{[i]}$ has the dimensions of $N_i \times N_A$, with N_i being the number of total nodes in sub-matrix i. $K_{BB}^{[i]}$ is null due to the fact that no boundary nodes interact with each other, and $K_{BI}^{[i]}$ is null since advection is unidirectional. This implies that the Schur complement matrix is null, and reduces the problem into the resolution of the internal nodes only by resolving Eq. 18. This is achieved by the pseudo – algorithm in Fig. 4.

```
Read ModelFromSGO
Execute WriteSubStructuresModel()
Initialize T_B
For i:1 → 18: (parallel solving)
    While (t < t_total):
        Execute SGOSOLVER(i) and Solve for T_I^[i]:
        K_II^[i] T_I^[i] + K_IB^[i] R_B^[i] T_B^[i] = R_I^[i]^T [C_I^[i] (dT_I^[i]/dt) - B_I^[i]]
        Wait iteration Results
        Update SGOSOLVER(i) with T_B^[j] from SGOSOLVER(i+1)
        Update SGOSOLVER(i+1) with T_B^[i] from SGOSOLVER(i)
```

Fig. 4: Pseudo-code of multi-CIN implementation.

In the algorithm in Fig. 4, he function WriteSubStructuresModel() writes the input model for each sub-structure from the main input file. SGOSOLVER(i) is the direct solver already used. The pseudo – algorithm is dubbed SGIO, and it is an extension to the original code. It parallelizes different instances of the direct solver, handles the communication, interface initialization and data handling of the final results.

The algorithm was used to simulate the thermoforming of many windshields. In fact, when applying a heating recipe, a lot of windshields are inserted at the start to stabilize the thermal state of the furnace. This stability was observed after the insertion of 90 windshields. For safer results, windshield temperature results were chosen and obtained for the 150th windshield. This windshield's temperature distribution was compared against a test windshield that consists of two superposed sheets of steel. The choice of steel is so that thermocouples can be easily welded into the inner parts of the surfaces. The thermocouples were installed on both upper and lower sheets of the windshields as seen in Fig. 5. The inner ply is the top sheet (PV) and the outer ply is the bottom sheet (GV).
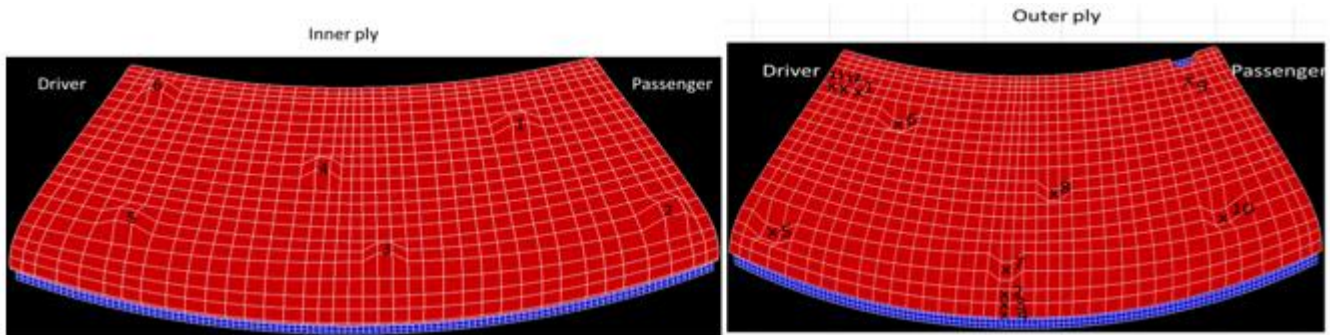
Fig. 5: The distribution of thermocouples on both inner ply(PV) and outer ply (GV).

The industrial furnace that was used for testing is composed of 18 thermoforming cells. Each cell has approximately a similar structure. 182 ceramic heaters exist in each cell. The residence time is of 21 s for the windshield in each cell. An industrial heating recipe was applied that was already used on the industrial furnace.

The results showed good agreement between the multi-CIN model and the experiment. The model reproduced correctly the thermal behaviour of the steel windshield inside the furnace, while showing good accuracy. The deviation in temperature measurements are due to a lot of factor, notably the thermocouple's precision, the data acquisition box over-heating due to radiation, the thermal model's precision and other contributing factors. In the results in Fig.6 - 7, each dotted line represents a heating cell. Each cell was simulated in parallel. The dashed grey curve represents the difference in temperature at each instant.
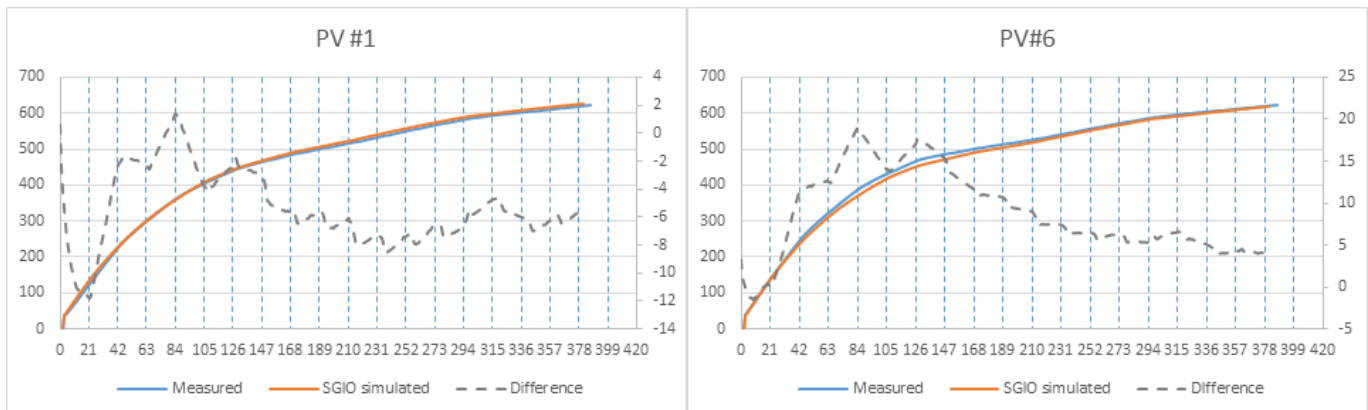

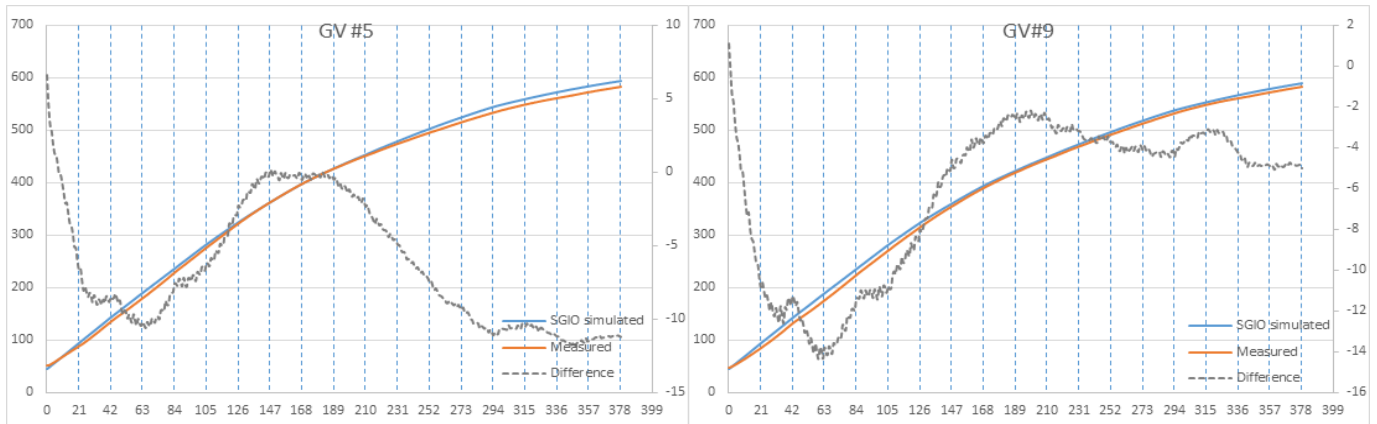Fig. 6: Results of simulation and measurements for inner ply (PV).

Fig. 7: Results of simulation and measurements for outer ply (GV).

4. Conclusion

The application of iterative sub-structuring methods on the CIN thermal network method gave way to the multi-CIN model for large thermal problems. The multi-CIN model of establishing the thermal network of finite elements and volumes proved to be efficient. The model reproduced well the thermal behaviour of the windshield passing through 18 thermoforming cells, and gave an acceptable precision of +/- 9 degrees at the end of the cycle. Each sub-structure was solved in parallel. Parallelization of the direct solver, the communication between solvers and interface resolution was held by a new code, SGIO, that acts as an extension to the already existing solver. Predicting thermoforming for a given heating recipe is promising, and gives way to other studies that serve the purpose of energy optimization.

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