

# Bayesian Structural Characterization and Classification for Direct Numerically Simulated Turbulence Features

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## Extended Abstract

Oscillatory sediment-laden water flow is a turbulent process important to the fluid dynamical comprehension of tidally driven estuaries, riverine-oceanic deltas, and surface wave forced coastal waters. It is also crucial to the understanding of pollution dispersion dynamics, and heat and gas exchange processes across fluid interfaces. Significant understanding of the complex nonlinear dynamics found in this flow has been accomplished through machine learning data analysis of direct numerical simulations of sinusoidally driven particle-laden flows. Previous direct numerical simulation modeling along with machine learning-based feature extraction and Bayesian belief network modeling revealed an asymmetry in the relationship of the sub-surface stress to the surface layer pressure, velocity, and sediment concentration spatial scales [1,2]. Revisitation of the analysis of the nonlinear dynamics is performed via the application of three different Bayesian statistical models to the same direct numerically simulated data. The objective is to corroborate previous findings and evince new ones.

Generative topographic mapping [3], a nonlinear latent variable model, reveals a weak but clear demarcation between data clusters emanating from an eighty-three-point, four-dimensional data time series consisting of stress, and vertical velocity, pressure, and sediment concentration spatial scales. The hidden Markov model (HMM) [4] parameter estimation-based state transition matrix shows that high positive ( $>+1.0$  Pa) and low negative stress states ( $-4.0 - -0.5$  Pa) tend to transition to low absolute value stress states ( $-0.5 - +0.5$  Pa) which occurs near the zero phase of forcing.

The HMM emission matrix for stress and pressure spatial scales shows that low negative ( $-4.0 - -0.5$  Pa) and very high positive stress states ( $>+0.5$  Pa) tend to produce small pressure spatial scales only. Higher negative stress states ( $-0.5 - 0.0$  Pa) tend to produce small and large pressure spatial scales with small pressure spatial scales slightly dominating. Lower positive stress states ( $0.0 - +0.5$  Pa) tend to produce an array of pressure spatial scales dominated by large pressure spatial scales ( $+1.5 - +2.0$  Pa). The HMM emission matrix for stress and concentration spatial scales shows that the aforementioned low negative stress and very high positive stress states show a significant amount of statistical probability at both low and very high concentration spatial scales. Higher negative and lower positive stress states previously mentioned tend to both produce a broad range of small to large concentration spatial scales. HMM emission table results corroborate the asymmetry found in previous work where naïve Bayesian belief network modeling only was employed. This asymmetry is displayed in the response of the oscillatory turbulent system with evidence of maximum positive stress states being more clearly visible via surface sediment concentration spatial scale structure than extreme negative stress states.

Naïve and traditional covariance-based Bayesian classification was performed in which stress is modeled as the causal node and the remaining three variables as effect nodes. Comparison of naïve Bayesian classification to covariance-based Bayesian classification [5] results reveals a lower error for the former. This suggests that the prediction of stress state from surface scale dynamics is best performed through treating pressure, velocity, and particle concentration spatial scales as independent effect variables.

## References

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