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A Numerical Study of Unsteady Natural Convection from Two-Sided Thin Horizontal Isothermal Plates

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Abstract – Unsteady natural convective heat transfer following sudden heating of two-sided, thin isothermal horizontal plates with complex shapes has been numerically studied. The plates were isothermal and exposed to air at ambient conditions. Three different plate shapes were considered. Variations of the heat transfer rates, expressed in terms of the Nusselt number with dimensionless time for various Rayleigh numbers were first obtained with the square root of the plates' surface area being used as the length scale. Then, the heat transfer rates for steady state were obtained with the same length scale being used. The transient heat transfer rate results showed that depending on the value of the Rayleigh number, the Nusselt number either gradually decreased to a steady state value or first decreased to a minimum and then increased to the steady state value. Similar Nusselt number variation with time were obtained for all plate shapes considered. Now, in studies of steady state natural convective heat transfer rates from horizontal plates with different shapes, it has been found that if 4*Area/Perimeter is used as the length scale in presenting the results, the variation of Nusselt number with Rayleigh number were essentially the same for all shapes considered. Therefore, the unsteady state results obtained in the present study were re-expressed using 4*Area/Perimeter as the length scale and it was found that the variations of Nusselt number with dimensionless time for various Rayleigh numbers for the plate shapes considered, were much closer to each other when 4*area/perimeter was used as the length scale than when the square root of the plate area was used as the length scale.

Keywords**:** natural convection; transient heat transfer; horizontal plate; two-sided; thin, complex-shaped plates

Nomenclature

x	x cartesian coordinate; m	T	Temperature; K	ν	Kinematic viscosity; m^2/s
y	y cartesian coordinate; m	t	Time; s	α	Thermal diffusivity; m^2/s
z	z cartesian coordinate; m	p	Pressure; Pa	T_H	Plate temperature; K
u	Velocity in x direction; m/s	ρ	Density; kg/m^3	T_F	Fluid temperature; K
v	Velocity in y direction; m/s	g	Gravity; m/s^2	A	Surface area, m^2
w	Velocity in z direction; m/s	β	Coeff. of thermal expansion; 1/K	P	Perimeter; m

1. Introduction

Natural convective heat transfer from plates of various configurations, shapes and dimensions have been studied by multiple researchers. Studies [1-3] have been conducted on horizontal and vertical plate configurations. Researchers [4,5] have also studied the effect of inclination on the heat transfer parameters from plates.

In case of natural convective heat transfer from horizontal plates, variations such as single sided heat transfer [6] and double-sided heat transfer [7-8] have been studied. Pretot et al. [6] studied the steady state heat transfer from a single exposed surface, to a semi-infinite medium and noted that the highest heat transfer rate was at the surface edges. Wei et al. [7] studied the heat transfer from single and dual surface(s) plate exposed to an infinite medium at steady state. They noted lower heat transfer of up to 40% from lower surface as compared to upper surface. Oosthuizen and Kalendar [8] investigated the heat transfer from upper and lower surface of isothermal circular plate with adiabatic center and found varying sensitivity of Nusselt number on plate diameter based on Rayleigh number.

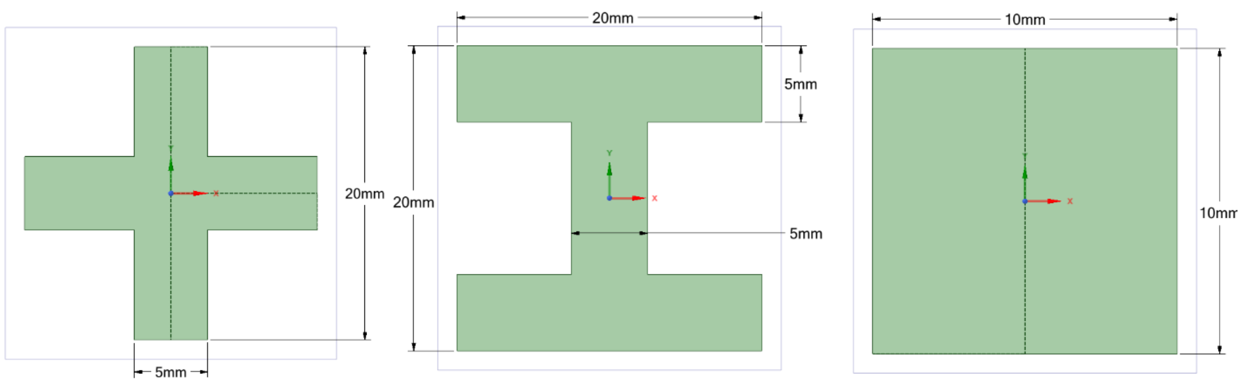
The variation in shape and dimension also affects the convective heat transfer rates and hence are important areas of study. Al-Arabi and El-Reidy [9] observed similarity in heat transfer behaviour at steady state from circular and square plates when the diameter of circular plate and edge length of square plate were same. Manna and Oosthuizen [10] compared the heat transfer between circular and square isothermal plates with varying thickness and observed high sensitivity of upper surface heat transfer on the plate thickness. Kobus and Wedekind [11] also investigated the effect of dimensions on heat transfer from a circular plate and had proposed an empirical correlation relating Nusselt number to Rayleigh number.

Limited studies have also been undertaken on unsteady natural convective heat transfer by multiple researchers, e.g., see [1,6,12,13].

Critical assessment of literature reveals that available studies have been primarily conducted on the effect of variation of spatial configurations, shapes and dimensions on steady state heat transfer. Limited work has been reported on the effect of length scale variation on transient natural convection for complex shapes. The objective of the current investigation is to study the effect of using two length scales on unsteady convective heat transfer and on other heat transfer related parameters for multiple complex shapes.

2. Plate Shapes Considered

Three plate geometries were considered. These have been shown in fig. 1, and are, a plus shape, an I shape and a rectangular shape. In the case of the rectangular shaped plate, results were obtained for Aspect Ratios (AR's) of 1, 2, 3, 4 and 6. The rectangular shape shown in fig. 1 (c) has an aspect ratio of 1, i.e., it is square.



(a) (b) (c)

Fig. 1: Configurations for (a) plus shape, (b) I shape and (c) rectangular shape with AR-1

The rectangular shapes were constructed by increasing the aspect ratio whilst keeping the surface area same.

3. Solution Procedure

The Commercial CFD software, ANSYS FLUENT, was used in obtaining the results presented in this study.

3.1 Governing equations, Dimensionless numbers and Boundary conditions

In obtaining the solution, the Boussinesq approximation was adopted, i.e., it was assumed that all the fluid properties except for density were constant, and that the density varied linearly with temperature. The governing equations used were as shown in Eqs. (1)-(5).

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right) \tag{2}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) \quad (3)$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) + g\beta(T_H - T_F) \quad (4)$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) \quad (5)$$

The Nusselt number (Nu) is a function of Rayleigh number (Ra), Prandtl number (Pr) and Fourier number (Fo) as shown in Eq. (6).

$$Nu = f(Ra, Pr, Fo) \quad (6)$$

The numerical representation for these numbers has been shown in Eqs. (7)-(10).

$$\text{Nusselt No. (Nu)} = \frac{\left[{}^{q}/_{(T_H - T_F)}\right].l}{k} \quad (7)$$

$$\text{Rayleigh No. (Ra)} = \frac{g\beta(T_H - T_F)l^3}{\nu\alpha} \quad (8)$$

$$\text{Prandtl No. (Pr)} = \frac{\nu}{\alpha} \quad (9)$$

$$\text{Fourier no. (Fo)/dimensionless time } (\tau) = \frac{t\alpha}{l^2} \quad (10)$$

In this work, the heat transfer has been assumed to be to air and Pr has been assumed to be constant and equal to 0.71.

In obtaining the results presented here, the thin horizontal plate was maintained at a constant temperature of 400K and the ambient temperature was taken to be 300K. The no slip condition was assumed on the plate-fluid interface. The pressure and temperature gradient across all outer edges of the computational domain was maintained at 0, thus facilitating free flow of fluid at ambient temperature across the domain boundary.

3.2 Grid Independence test

Extensive grid independence testing was undertaken and indicated that the grids used in this study gave results that were grid independent to better than 5%.

3.3 Length Scales

Initially, the length scale used was the square root of the surface area of the plate. However, it is thought that by using a length scale of four times total surface area divided by total perimeter length (4A/P), heat transfer results for the different plate shapes will be relatively close together. The criteria for choosing 4A/P as alternate length scale would be discussed in the subsequent section. Therefore, after first getting the results using square root area as length scale, the results would be obtained using length scale 4A/P.

3.4 Model Testing

Prior to undertaking the numerical study, the soundness of the current method was validated by obtaining results for situations that had been in the experimental study by Kobus and Wedekind [14]. They had experimentally measured heat transfer from circular plates of various diameters and thicknesses at multiple Ra. A comparison of the Nu obtained numerically, with those obtained experimentally are presented in fig. 2.

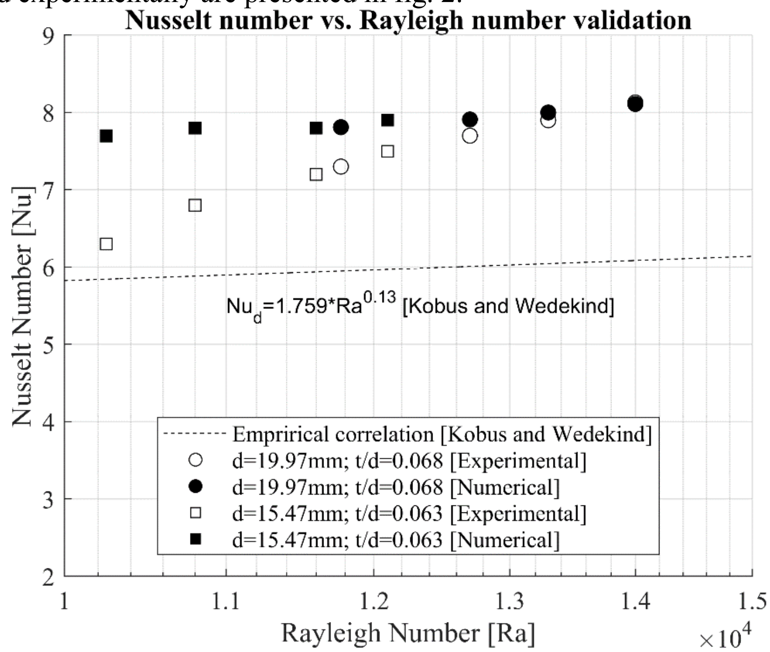


Fig. 2: Comparison between experimental and computational data obtained for circular plates of multiple diameters 'D'

It will be seen from Fig. 3 that except at low Rayleigh numbers, a good agreement between the numerical and experimental results is obtained. The average deviation of numerical results from experimental calculations was 2.6% for 19.97mm plates and 12.6% for 15.47mm plates respectively. The numerical results also compared well to the experimental uncertainty of 10%.

4. Results and Discussion

The heat transfer results obtained in the current numerical study for square root area as length scale have been presented first, followed by those achieved for 4A/P.

4.1 Unsteady state (transient) heat transfer

The results obtained for unsteady natural convective heat transfer for the multiple shapes and two length scales have been presented in this section.

4.1.1 Length Scale- Square root area

The transient heat transfer from the plate in form of Nu variation with dimensionless time, while using square root area as length scale has been shown in fig. 3.

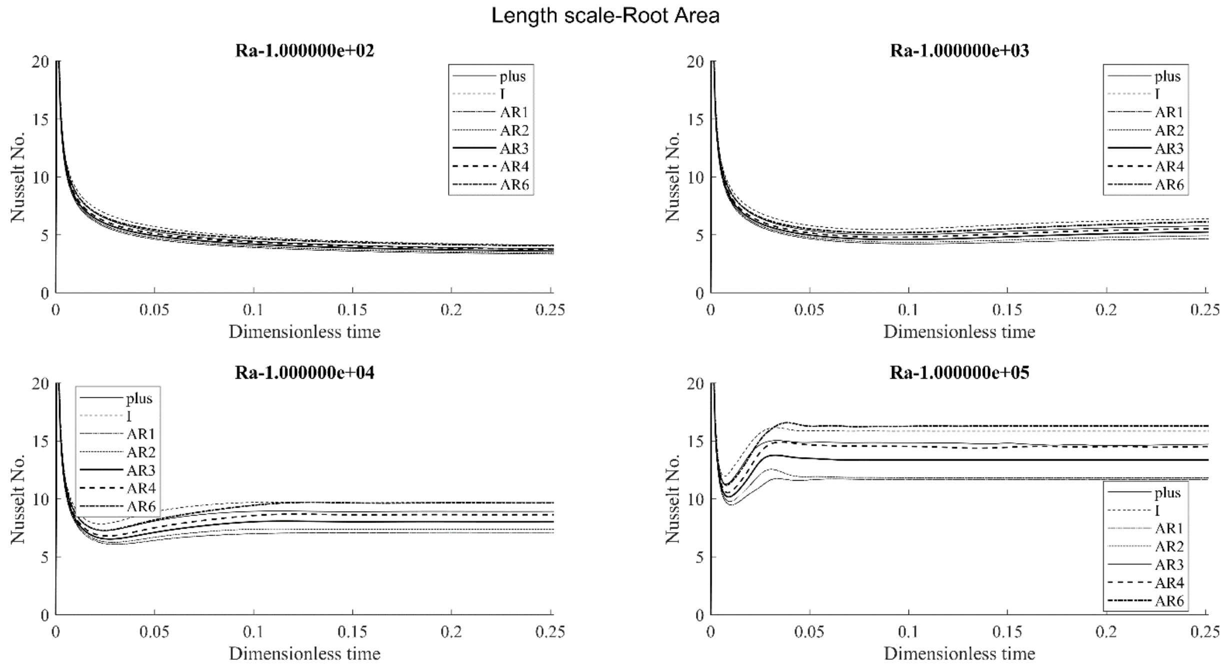


Fig. 3: Nu variation with dimensionless time for different geometries at length scale square root area

It will be seen. from fig. 3, that at the lower Ra values of 102 and 103, the heat transfer sharply decreases from the initial high value at early stage before attaining an almost steady state value. However, a somewhat different behaviour will be seen to occur at higher values of Ra, in this case Nu will be seen to first decrease, from initial high values to a minimum value before increasing to the steady state value. The minimum Nu attained during the unsteady heat transfer increases with increasing Ra. As such, the trend was found to be similar for all shapes.

Fig. 4 summarizes and compares the dimensionless time required to reach steady state Nu value as a function of Ra.

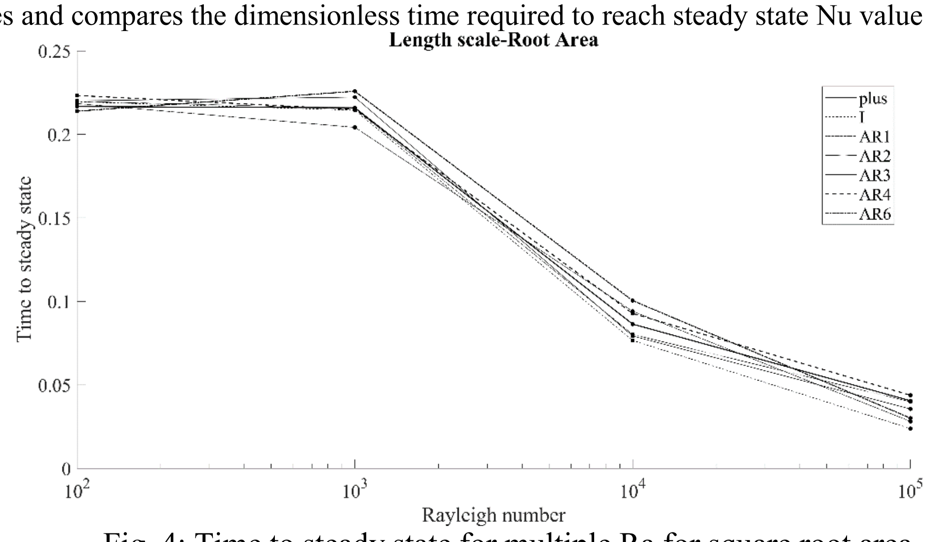


Fig. 4: Time to steady state for multiple Ra for square root area

At lower Ra (between 102 and 103), time to steady state remained almost constant, however, decreases subsequently with increasing Ra. Higher Ra is expected to increase intensity of convective flow, facilitating higher rate of convective heat transfer and thus lowering of time to reach steady state.

4.1.2 Length scale- 4A/P

From the results presented above, it is apparent that for Ra used in this study, the unsteady heat transfer parameters for shapes differ noticeably when using square root area as length scale. Therefore, an investigation has been performed to check if an alternate length scale could be identified to produce closer Nu values.

The alternate length scale was derived from the findings of Al-Arabi and El-Reidy [9] on similarity of heat transfer behaviour between square and circular plates. Accordingly, it was noted that the edge length of a square and diameter of a circle were same when four times total surface area was divided by total perimeter (4A/P), which was used as the alternate length scale.

The transient heat transfer characteristics for all shapes using 4A/P were captured and have been presented in fig. 5.

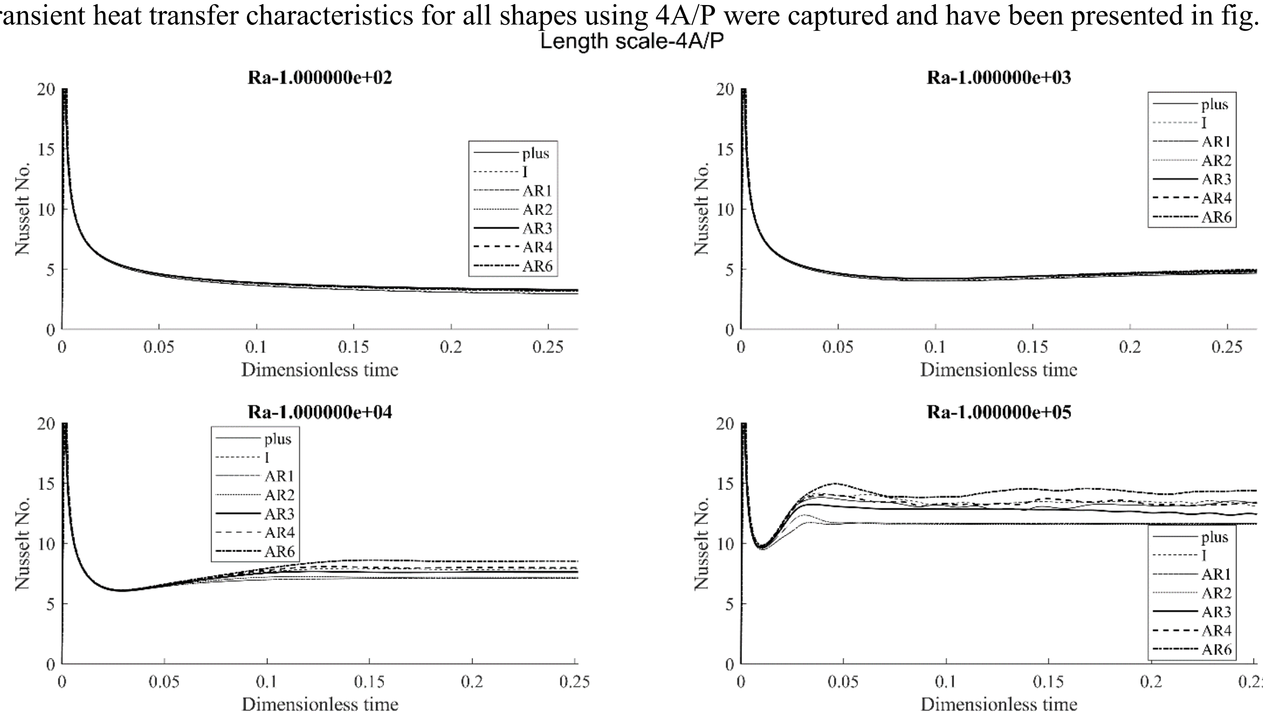


Fig. 5- Nu variation with dimensionless time for different geometries at length scale 4A/P

From fig. 5, it was noted that the general trend of Nu variation with dimensionless time appears similar to that of length scale square root area. However, Nu variation with dimensionless time for all the shapes were much closer to each other at all Ra and almost merged into a single line at lower Ra of 102 and 103. At higher Ra, however, the difference increased until the flow became turbulent for some shapes at Ra of 105. Even then, Nu for shapes were relatively much closer together compared to root of surface area.

The results obtained from study of time to steady state for multiple Ra has been shown in fig. 6.

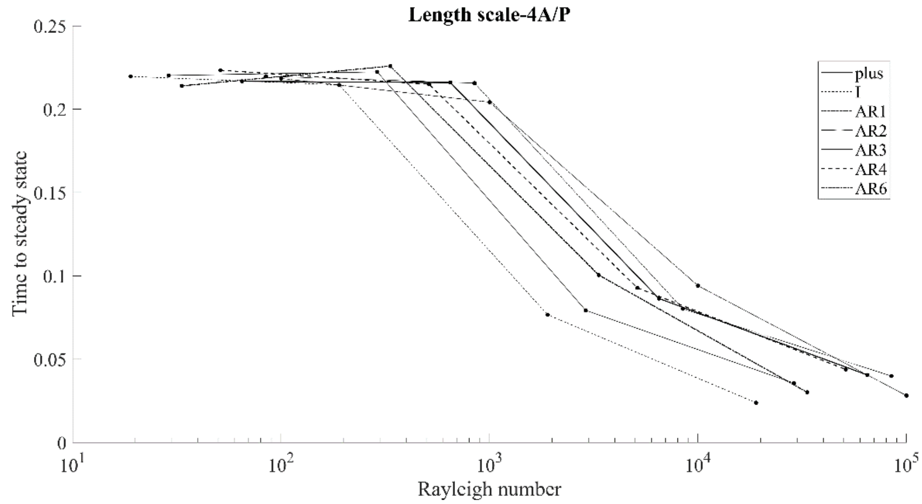


Fig. 6: Time to steady state for multiple Ra for 4A/P

Fig. 6 shows that the time to steady state decreased with increasing Ra, similar to the previous length scale. However, the time to steady state was more spread out with 4A/P compared with square root area. This implied that use of square root area as length scale is a better option while calculating time to steady state of heat transfer, but 4A/P produced much better similarity between various plate shapes for unsteady heat transfer estimation.

3.2 Steady State

Nu values at steady state for multiple Ra has been shown in fig. 7.

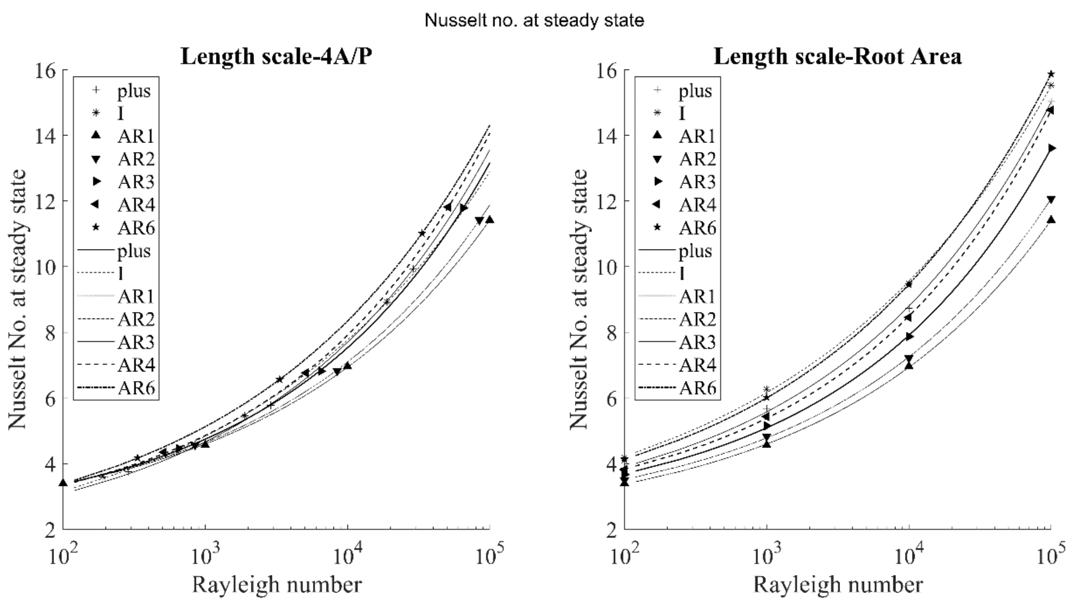


Fig. 7: Figure showing the steady state Nusselt No. distribution for complex shapes at multiple Rayleigh nos. and both length scales

As seen from the of findings of Al-Arabi and El-Reidy [9], using 4A/P as length scale brought the Nu values at steady state closer to each other for simple shapes. In the current study, similar procedures were conducted for complex shapes. It may be noted from fig. 7, that the steady state Nu values were closer for all shapes considered when 4A/P was used as length scale compared to square root area.

Conclusion

Upon comparison of unsteady & steady heat transfer results at both length scales for multiple complex shapes, the following observations have been noted-

- Nu value progression with dimensionless time was similar, but distinctly separate for all shapes at low Ra for square root area length scale. At higher Ra however, the difference in Nu between the shapes was high.
- Unsteady heat transfer representation by using Nu with 4A/P as length scale showed higher similarity between shapes. At low Ra, the Nu progression with dimensionless time for all shapes merged together to almost form a single line. Though the difference in Nu increased at higher Ra, the difference was much lower compared to square root area.
- Time to steady state comparison between the two length scales revealed that square root area produced higher similarity than 4A/P.
- Steady state Nu values confirmed that 4A/P produced higher similarity than square root area for complex shapes as well.

References

[1]- A.M. Clausing, J.J. Berton, "An experimental investigation of natural convection from an isothermal horizontal plate", J. Heat Transfer, 111(4): 904-908, 1989

[2]- B. Chambers, T.Y.T. Lee "A numerical study of local and average natural convection Nusselt numbers for simultaneous convection above and below a uniformly heated horizontal thin plate", J. Heat Transfer, 119(1): 102-108, 1997

[3]- S.W. Churchill, H.H. Chu, "Correlating equations for laminar and turbulent free convection from a vertical plate" Int. J. Heat Mass Transfer, 18(11): 1323-1329, 1975

[4]- J.J. Wei, B. Yu, H.S. Wang, W.Q. Tao, "Numerical study of simultaneous natural convection heat transfer from both surfaces of a uniformly heated thin plate with arbitrary inclination", J. Heat and Mass Transfer, 38(4-5): 309-317, 2002.

[5]- R. Manna & P. Oosthuizen, "A Numerical Study of the Simultaneous Natural Convective Heat Transfer from the Upper and Lower Surfaces of a Thin Isothermal Diagonally Inclined Square Plate" 2018

[6]- S Pretot, B Zeghmati, G Le Palec, "Theoretical and experimental study of natural convection on a horizontal plate", Applied Thermal Engineering, 20 (10): 873-891, 2000.

[7]- Wei, J. J., Yu, B., & Kawaguchi, Y, "Simultaneous natural-convection heat transfer above and below an isothermal horizontal thin plate" Numerical Heat Transfer; Part A: Applications, 2003.

[8]- P. Oosthuizen, and A. Kalendar, "A numerical study of the simultaneous natural convective heat transfer from the upper and lower surfaces of a thin isothermal horizontal circular plate," Proc. 2016 ASME International Mechanical Engineering Congress and Exposition (IMECE2016), vol. 8: Heat Transfer and Thermal Engineering, Phoenix, Arizona, November 2016.

[9]- M. Al-Arabi, & M. El-Riedy, "Natural convection heat transfer from isothermal horizontal plates of different shapes", International Journal of Heat and Mass Transfer, 19(12), 1399–1404, 1976

[10]- R. Manna & P. Oosthuizen, "A Numerical Study of Natural Convective Heat Transfer from Two-Sided Circular and Square Heated Horizontal Isothermal Plates having a Finite Thickness", 2018

[11]- C J. Kobus, & G.L. Wedekind, "An experimental investigation into natural convection heat transfer from horizontal isothermal circular disks", International Journal of Heat and Mass Transfer, Volume 44, Issue 17, Pages 3381-3384, 2001

[12]- J. Atkinson, & P. Davidson, "The evolution of laminar thermals" Journal of Fluid Mechanics, 878, 907-931, 2019

[13]- T. Hattori, N. Bartos, S.E. Norris, M.P. Kirkpatrick, S.W. Armfield, "Experimental and numerical investigation of unsteady behaviour in the near-field of pure thermal planar plumes", Experimental Thermal and Fluid Science, 139-150, 2013.