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Dynamics of a Rotating Core-Annular Flow at Inertial Fluid Oscillations

Olga Vlasova, Ivan Karpunin, Nikolai Kozlov[1]
Perm State Humanitarian Pedagogical University
24, Sibirskaya av., Perm, Russia
[1]kozlovn@pspu.ru

Abstract - Dynamics of a centrifuged system of two immiscible liquids in a rotating cylinder is studied experimentally. In experiments, the liquids fill a horizontal cylindrical container with transparent walls that rotates about its axis. The study is carried out in the case of fast rotation when under the action of the centrifugal force the light liquid forms an axi-symmetric column on the container axis (core), while the heavy liquid is distributed along the cylindrical wall (annulus). The gravity makes the core shift radially by a small distance, which is practically invariant along the axis. This effect excites the tangential oscillations of the interface leading to the generation of azimuthal steady flows in the rotating frame of reference and to the differential rotation of the interface. The profile of azimuthal velocity has a "discontiunity", which appears on the limits of a viscous boundary layer formed at the interface. The maximum velocity is observed in the outer liquid near the interface. The analysis of the velocity profiles reveals that the liquid-liquid interface is the essential generator of the azimuthal flow in the annulus, while the Ekman pumping appears to affect the flow velocity inside the core. The results of the study may be helpful for the determination of the distribution of inclusions or species on the rotating interface.

***Keywords*:** rotation, interface, steady streaming, core-annular flow, rotating drum flow

1. Introduction

Rotation brings to hydrodynamic systems numerous non-trivial phenomena [1]. A review on the flows in horizontal rotating drums [2] demonstrates many examples of pattern formation in multiphase systems. In this regard, one of the outstanding problems is the dynamics of rotating two-fluid systems that are relevant to technological processes, as well as to geophysical and astrophysical phenomena. The role of core–annulus interface, the former being formed by the less dense phase and the latter by the denser one, is crucial in centrifuged systems as it becomes the driver for many types of flows. For example, the action of gravity on a centrifuged liquid layer with the free surface leads to two independent inertial effects: the steady displacement of the air core and the wavy disturbance of its surface [3]. Both effects result in surface oscillations relative to the container, and this leads to the generation of steady streaming in the centrifuged layer [4–6]. In the case when the two phases are immiscible liquids of different densities, these two effects persist but the dynamics becomes more complex because new parameters come into play, such as ratios of densities and of viscosities [7, 8]. The centrifuged liquid–liquid interface possesses a rich spectrum of oscillations [9]. When the cylindrical container makes translational (longitudinal or transversal) vibrations as a whole, the centrifugal waves or quasi-steady relief are excited on the interface [7, 10]. The external action perpendicular to the rotation axis generates the differential rotation of the interface [7, 8]. In the general case, in a non-rotating system, theory predicts that the steady streaming near liquid–liquid interface is characterized by a discontinuity of the tangential velocity and shear stress [11, 12]. The aim of the present study is to investigate experimentally the structure of the azimuthal steady flows that are generated due to the interplay of the rotation and oscillations of the liquid–liquid interface.

2. Problem Formulation

Two immiscible liquids fill a horizontal cylindrical container of radius R_2 that is rotated with the angular frequency Ω_r sufficiently fast, so that the interface between liquids is centrifuged and takes the form of a column with circular cross-section of radius R_1 (Fig. 1). The gravity directed perpendicularly to the rotation axis disturbs the interface, whose dynamics is determined by the ratio between the gravity and the centrifugal force $\Gamma = g / \left(\Omega_{\mathrm{r}}^2 R_1\right)$. Above some threshold value of Γ ($\Gamma > \Gamma^*$, equivalent $\Omega_{\mathrm{r}} < \Omega_{\mathrm{r}}^*$) a centrifugal wave is excited on the interface [8]. In the present study the focus is made on the subcritical and nearly critical values of Γ, when the interface maintains circular shape.

2.1. Experimental Setup

In experiments, the container is transparent, made of acrylic glass, its inner dimensions are radius $R_2 = 3.0$ cm (Fig. 1) and length $L = 7.4$ cm. R_1 is determined by the ratio of liquids' volumes $V = R_1^2 / R_2^2$. The working liquids are industrial oil ($\rho_1 = 0.834$ g/cm^3, $\nu_1 = 11.3$ mm^2/s) and aqueous solutions of glycerol (# 1: $\rho_2 = 1.24$ g/cm^3, $\nu_2 = 15.2$ mm^2/s; and # 2: $\rho_2 = 1.19$ g/cm^3, $\nu_2 = 14.8$ mm^2/s). Rotation within the range $3 < \Omega_{\mathrm{r}} < 63$ (s^{-1}) is provided by a stepper motor FL86STH118-6004A. To study the dynamics of the fluids a high-speed camera of model Optronis CamRecord CL600x2, a laser and a stroboscopic illumination are used. In order to observe flows the liquids are seeded with neutral-buoyancy fine tracer particles. The images are processed using PTV technique. Besides, a method of direct velocity measurement by synchronization with the stroboscopic light is used. For this, larger particles of the size about (0.1–0.5) mm are placed on the interface in small quantity, and the stroboscope flicker frequency is then adjusted so as to immobilize the image of particles at the interface.

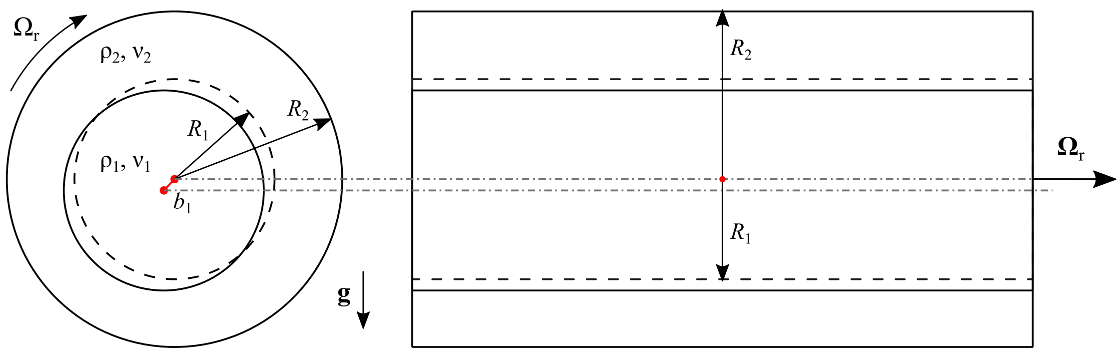


Fig. 1. Schematic of the core-annular layer at rotation, with the steady radial displacement of the light liquid column (b_1 is exaggerated): plane view along the axis of rotation (left) and from the side (right). The dashed contours depict the axisymmetric position of the core in the absence of gravity, while the solid ones show the core shifted steadily in the laboratory frame due to the action of gravity.

3. Results

Under the action of gravity the light liquid column undergoes a radial displacement, steady in the laboratory frame, of amplitude b_1 (Fig. 1). The values of b_1 found in experiments fit well to the following theoretical dependence, as it was demonstrated in [8, 13]:

$$b_1 = 0.5 \Gamma R_1 \left(1 - \frac{\rho_1}{\rho_2}\right)\left(1 - V\right). \tag{1}$$

In the reference frame that rotates with the container, this situation is equivalent to the interface oscillations with the frequency $-\Omega_{\mathrm{r}}$. Here, the minus sign indicates the rotation direction of the driving force – gravity. In the viscous boundary layer on the interface, an averaged mass force is generated that brings the fluids in the lagging differential rotation relative to the container with the angular speed $\Delta\Omega = \left(\Omega_{\mathrm{fluid}} - \Omega_{\mathrm{r}}\right) < 0$ [8]. Along the cylinder axis, the interface rotates practically uniformly, however near the flanges of the cylindrical container the difference in rotation speed between the liquid and the cylinder is slightly smaller. In Fig. 2, the interface differential rotation rates in the middle and at the edge are presented in comparison (diamond and circle symbols, respectively). Their difference is plotted with square symbols. These values are measured by the synchronization of the rotation with the stroboscopic illumination. With the decrease in Ω_r the differential rotation intensifies. In the vicinity of the wave excitation threshold Ω_r^* the flow regime changes and the experimental points fluctuate (Fig. 2). This is more pronounced near the flange and may be attributed to the wall effect.

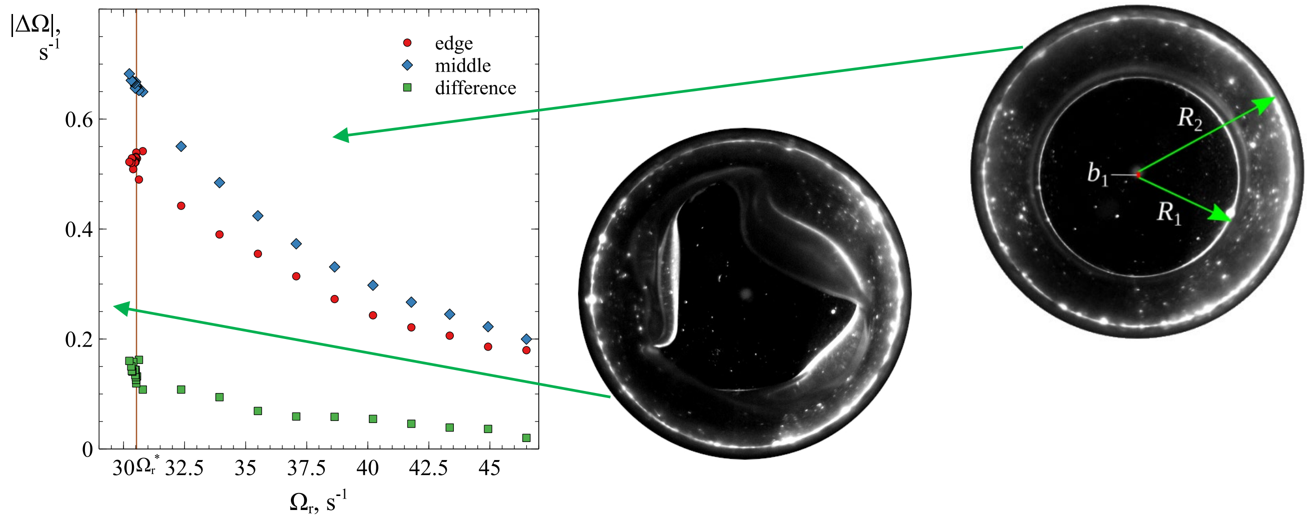


Fig. 2. In the graph: the absolute value of the average rate of the interface differential rotation as a function of the angular frequency of container rotation. The photographs on the right show the undisturbed (right-top) and the wavy interface (left-bottom).

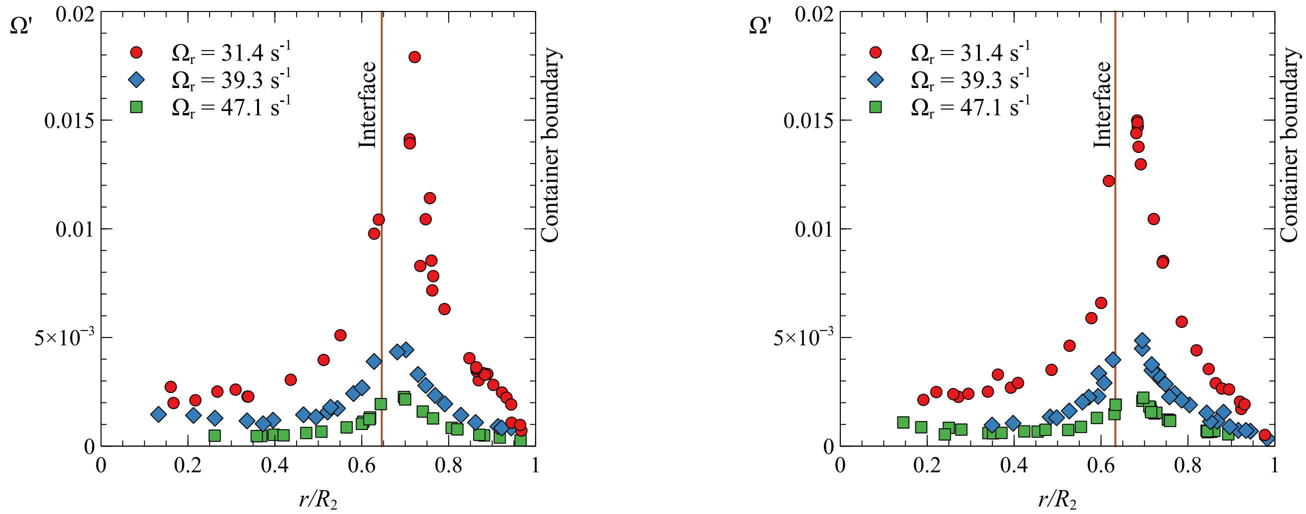
The observation shows that, in the container frame, the tracers move along circular trajectories at constant angular velocity. The radial profiles of the dimensionless steady angular velocity averaged along the azimuth are given in Fig. 3. Here, Ω' is defined so as to take into account the variations of properties of liquids: density and viscosity. It is known from experiments [8] that, below the wave excitation threshold, the differential rotation rate of the interface $\Delta\Omega_1$ is determined by the following scaling law:

$$\left|\Delta\Omega_1\right| \sim \Omega_r \Gamma^2 \left(1-\frac{\rho_1}{\rho_2}\right)^2 \frac{R_1}{\delta}\left(1-V\right). \tag{2}$$

Here, $\delta=\sqrt{2\nu_2/\Omega_r}$ is the thickness of the viscous boundary layer on the interface, on the side of the outer liquid. By dividing equation (2) by the expression $\Omega_r\left(1-\rho_1/\rho_2\right)^2 R_1/\delta$ we render it dimensionless and account for the terms with the properties of liquids. We obtain thus the following definition:

$$\Omega' = \frac{\left|\Delta\Omega\right|\left(2\nu_2\right)^{1/2}}{\Omega_r^{3/2} R_1 \left(1-\dfrac{\rho_1}{\rho_2}\right)^2}. \tag{3}$$

The measurements of the azimuthal flow shown in Fig. 3 are obtained by PTV. They reveal that the maximum velocity of the inner liquid, $|\Delta\Omega_1|$, is at the interface, while for the outer liquid, $|\Delta\Omega_2|$, it is at some distance from the interface. The flow is characterized by a "discontinuity" of the tangential (azimuthal) steady velocity $[\Delta\Omega] = |\Delta\Omega_2 - \Delta\Omega_1|$ ($[\Delta\Omega] \sim \Omega'_2 - \Omega'_1$). The radial distance between the two peaks of $|\Delta\Omega|$ is of the order of the viscous boundary layer thickness, $\delta = \sqrt{2\nu_2 / \Omega_r}$. The flow velocity within the "discontinuity" could not be measured because the tracer particles left this region. With an increase in Γ (decrease in Ω_r), the velocity of steady streaming increases, and at the same time the speed drop $[\Delta\Omega]$ becomes larger. At some critical value of Γ the centrifugal waves are excited on the interface. The velocity profiles in Fig. 3 are obtained below the wave excitation threshold.



a b

Fig. 3. Radial profiles of the normalized rate of differential rotation in cross-sections near the container flange (a) and in the middle (b).

3.1. Discussion

It was demonstrated experimentally in [8] that in the domain subcritical as related to the centrifugal waves the rotation rate at the interface, $|\Delta\Omega_1|/\Omega_r$, is proportional to Γ_q^2 (see equation (2)). Here,

$$\Gamma_q \equiv \Gamma\left(1 - \frac{\rho_1}{\rho_2}\right)\sqrt{\frac{R_1}{\delta}(1-V)}\,. \tag{4}$$

This proportionality holds until the threshold of centrifugal wave excitation and then changes quite rapidly with Γ_q^2 [8].

Let us consider the impact of Γ_q on the velocity drop $[\Delta\Omega]/\Omega_r$. The experimental data for both the edge and the middle cross-sections is shown in Fig. 4. The dashed line is plotted according to the scaling law

$$\frac{[\Delta\Omega]}{\Omega_r} \sim \Gamma_q^2, \tag{5}$$

while the solid vertical line is traced at the value Γ_q^*, which corresponds to the threshold of wave excitation. At lower amplitudes of forcing ($\Gamma_q < 0.45$) the experimental points follow the relation (5) quite closely, while at higher amplitudes of forcing a discrepancy between the scaling law and the experimental data establishes and gradually increases. This means that at weak forcing the ratio $[\Delta\Omega]/\left|\Delta\Omega_1\right|$ is nearly constant, however with the increase in Γ_q^* the two parameters return the response of different intensity to the external forcing. It is interesting to mention that in terms of velocity drop at the interfacial boundary layer the dynamics becomes clearly non-linear quite long before the threshold of wave excitation.

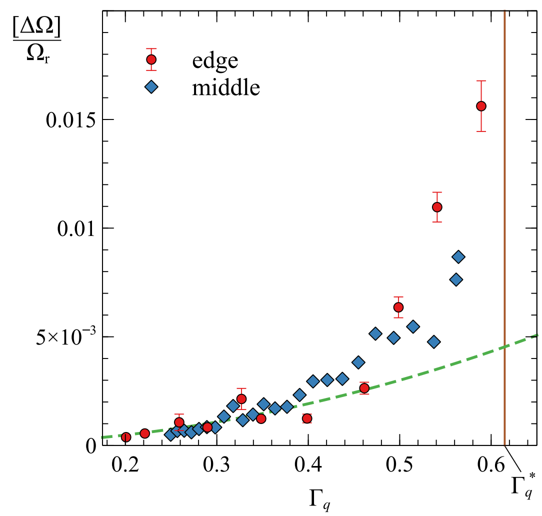


Fig. 4. The dependence of the dimensionless velocity drop at the interface on the dimensionless acceleration.

4. Conclusion

The dynamics of a rotating horizontal core-annular flow has been studied experimentally in the case when due to the action of gravity oscillating in the rotating reference frame the inertial fluid oscillations generate the steady streaming. In the experiments the rotation rate was kept above the threshold of wave excitation, hence the mechanism of forcing applied to the fluid consisted in the steady (as seen from the laboratory frame) radial displacement of the core due to the gravity. Two independent optical methods have been applied to measure the flow velocity: the stroboscopic illumination has provided the rate of the interface rotation, while the PTV has allowed obtaining the radial velocity profiles.

The results obtained reveal that the fluid flow follows the circular paths around the core. The azimuthal steady streaming, generated in the rotating cylinder by oscillations of liquid–liquid interface, has a “discontinuity” in the profile of its angular velocity that is related to the oscillating viscous boundary layer. The streaming may be characterized by two quantities: the differential rotation rate of the interface and the velocity drop (“discontinuity”) at the boundary layer. The boundary layer (and the “discontinuity”) is located in the outer liquid. The flows in the outer liquid are more intensive than in the inner one, at the same time the former is also more perceptive to the onset of centrifugal waves. This is manifested by a more rapid evolution of the scaling law, by which the velocity drop parameter depends on the forcing amplitude. The comparison with previous results demonstrates that the scaling law of the interface differential rotation is altered only at the emergence of the centrifugal waves. This is preceded with the increase in the velocity drop at smaller amplitudes.

Results of the present study may be applied for the development of methods of heat and mass transfer control by oscillations.

Acknowledgements
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