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Averaged Convection In A Flat Layer At Modulated Rotation Around A Horizontal Axis

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Abstract - The averaged convection in a vertical flat layer rotating around a horizontal axis oriented perpendicular to the layer plane has been experimentally studied. The layer boundaries are isothermal and are maintained at given, different temperatures. The layer is subjected to a periodic change in the speed of rotation - librations. The frequency and amplitude of rotation speed modulation varied. It is shown that the uneven rotation of the cavity leads to the excitation of the averaged convection of the liquid from the state of quasi-equilibrium. The intensity of the average flows is determined by the pulsating Reynolds number. The flow developed in the librating layer has the form of an azimuthal vortex, the size of which increases radially with an increase in the cavity rotation rate.

***Keywords*:** heat transfer, average convection, vibrational convection, rotation, librations, oscillations.

1. Introduction

The convective movement of a fluid, caused by a temperature gradient and subject to rotation, is a common occurrence in natural processes and in industrial applications. Various ways of controlling convective flows for a horizontal layer of fluid uniformly rotating around a vertical axis have been studied in sufficient detail [1, 2]. A large number of works over the past century have been devoted to the study of classical Rayleigh-Benard convection subject to various external physical influences [3, 4], such as modulation: temperature, rotation, gravity, magnetic field. In one of the first papers [5], on the study of the stability of a horizontal liquid layer heated from below and subjected to time-periodic temperature modulation at the boundary, a linear stability analysis is presented. The obtained results showed that by choosing a certain modulation frequency, it is possible to effectively control the heat transfer. Later, the Rayleigh–Benard problem under temperature modulation was studied in a rotating layer [6, 7] using linear stability analysis. It was concluded that by certain combinations of parameters: the modulation frequency, the Taylor number and the Prandtl number, it is possible to stabilize or destabilize the stability threshold. At high frequencies, the effect of rotation and temperature modulation becomes weaker.

In rotating hydrodynamic systems, in addition to the classical (Rayleigh) mechanism of convection excitation, there is one more mechanism - vibrational [8]. In density-inhomogeneous media, as a result of the action of an oscillating external force, averaged vibrational flows are generated. The theoretical work [9] describes the averaged (vibrational) motion of a nonisothermal fluid in a uniformly rotating cavity around a horizontal axis. Rotation stabilizes vibrational convection like gravitational convection [9, 10]. Excited as a result of fluid oscillations, averaged convection has the form of a system of cells immobile relative to the cavity, arranged in a hexagonal order [11, 12]. It was shown in [12] that below the threshold of excitation of vibrational convection in a rotating layer, there are averaged flows in the form of low-intensity toroidal rolls. An analysis of these structures showed that toroidal vortices are generated by inertial waves propagating in a rotating fluid [13]. In a number of experimental studies, it was found that under a harmonic action with a constant amplitude on a rotating fluid, the development of waves [14, 15], often called inertial ones, is possible. Such waves develop in a rotating layer of liquid due to the action of the Coriolis force. The study of the region of existence of inertial waves, in the case of non-uniform rotation of the cavity, can lead to the development of new methods for controlling the behavior of fluids in conditions of reduced gravity

In the current work, for the first time, an experimental study of the effect of rotation velocity modulation (librations) on heat transfer in a flat layer rotating around a horizontal axis is presented. The study of a rotating system subjected to a periodically changing speed of rotation has a great application in engineering, where the control of instability and heat transfer in fluid systems is very important.

2. Experimental Setup and Methodology

The experimental setup is shown in fig.1. The flat layer is a cylindrical cavity formed by an annular gasket made of non-heat-conducting material *1*, located between heat exchangers *2* (made of transparent organic glass) and *3* (made of aluminum). The cylindrical cavity is filled with a working fluid – distilled water. The gasket is a ring $h = 1.0$ cm thick, with a lateral cylindrical boundary $d = 1.0$ cm wide, and an inner radius of the working cavity $R = 7.0$ cm. The ends of the heat exchangers serve as flat boundaries of a cylindrical cavity, the temperatures of which are maintained at fixed values T_1 and T_2, by pumping liquid through the heat exchangers.

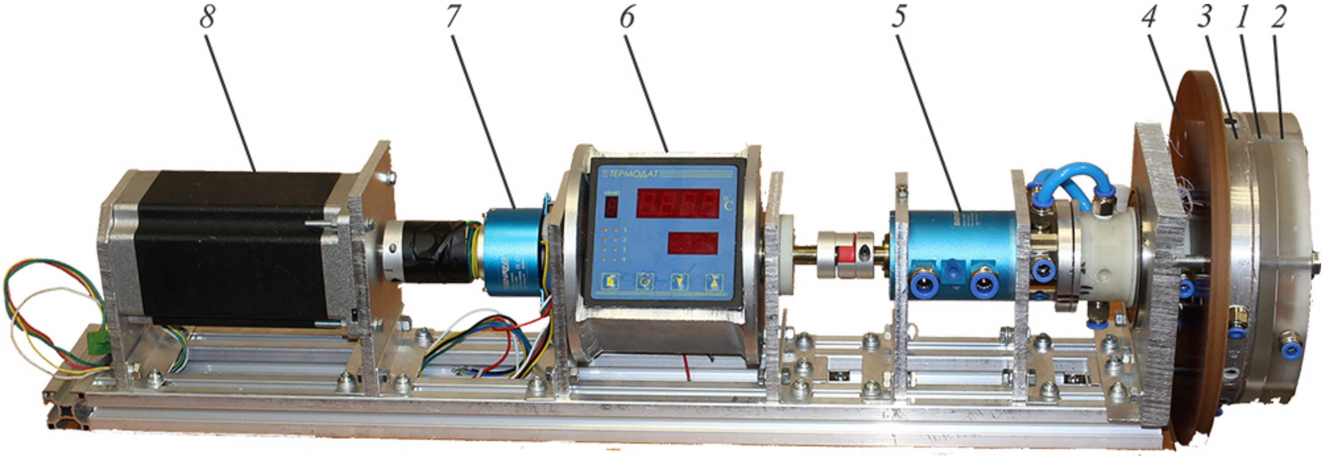


Fig. 1: Photo of the experimental setup

The cuvette is located on a table *4* rotating around a horizontal axis. Simultaneously with the rotation of the table, it is necessary to ensure that a cuvette of liquid from thermostats is supplied to the heat exchangers. For this, a hydraulic distributor *5* is used, the task of which is to circulate the thermostated liquid from the fixed thermostats to the heat exchangers *2* and *3*, mounted on a rotating table. Hydraulic manifold *5* is equipped with two pairs of holes used to supply thermostatic fluid to the heat exchangers and drain it. The high liquid pumping speed (5 liters per minute) ensures a uniform temperature over the entire surface of the heat exchanger. The temperature of the thermostated liquid circulating in the circuit from the thermostat to the heat exchanger can be set in the range from 18°C to 70°C. The temperature control accuracy of the thermostat is 0.1°C.

The measurement of the temperatures of the layer boundaries is carried out in real time. Signals from temperature sensors placed in the cuvette are transmitted via electric wires to the measuring module Termodat-13IK *6*. The module is located on the central axis and is connected on one side to the hydraulic manifold, and on the other – to the stepper motor. Thermodata-13IK is powered by an electric collector *7*. This collector uses sliding current-collecting contacts – a pair of which feeds Thermodat-13IK from a 220V network, and the other pair transmits temperature signals to a computer via an RJ-45 port.

The rotation of the table sets the stepper motor FL86STH156 *8*, controlled from the computer via the SMD-78 driver. All elements of the experimental setup are connected to each other by means of couplings, which ensure a uniform transmission of rotation from the engine to the table. The rotation speed varies in the range of 0.01-4.00 rps. The instability of the rotation speed does not exceed 0.01 rps.

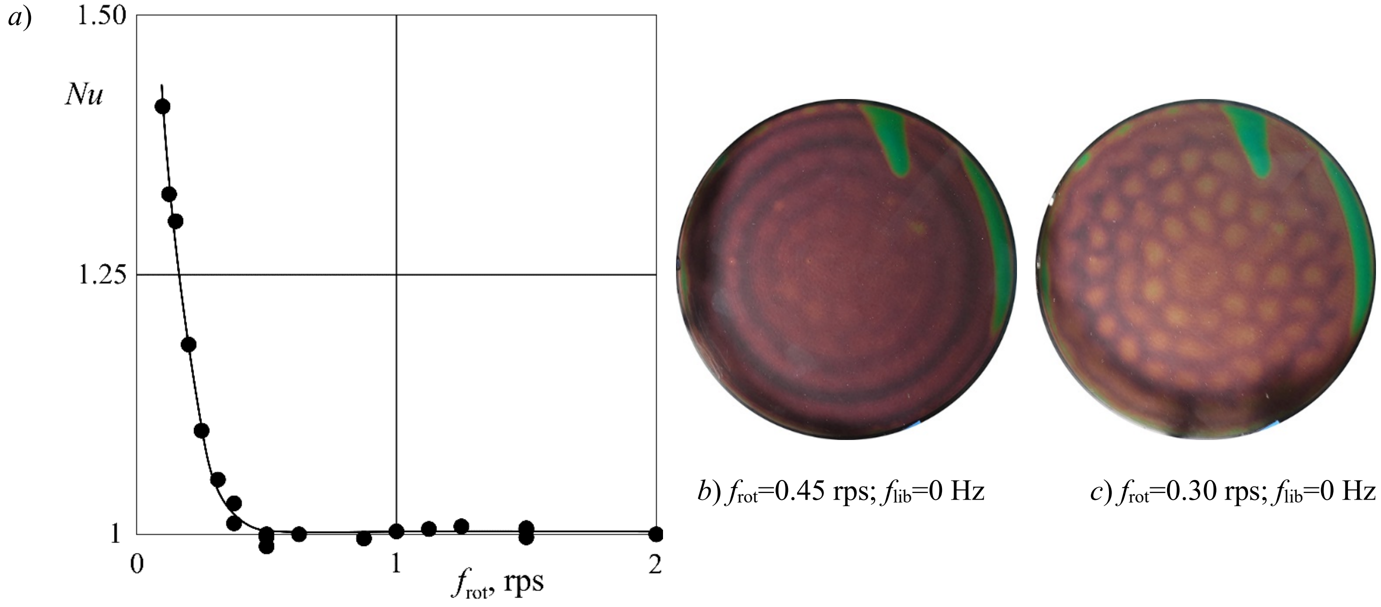
In the current work, the averaged convection in a flat liquid layer, with isothermal boundaries of different temperatures, performing a modulated rotation around a horizontal axis, is studied. The cell rotation velocity is modulated according to the law $\Omega = \Omega_{\text{rot}}\left(1 + \varepsilon \sin\left(\Omega_{\text{lib}} t\right)\right)$, where Ω_{rot} – is the average angular velocity of cavity rotation, Ω_{lib} – is the cyclic frequency of angular oscillations, and $\varepsilon = \varphi_0 \Omega_{\mathrm{lib}} / \Omega_{\mathrm{rot}}$ – is the dimensionless amplitude of rotation velocity modulation.

The experimental technique was the following. Before the start of the experiment, the cavity was filled with a working working fluid. Special attention was paid to the absence of air inclusions in the cavity. The flat layer was brought into rapid rapid rotation. After the system reaches the stationary mode (at least 10 minutes). rotation speed modulation is set. In all all experiments, the frequency of cavity librations is maintained constant $f_{\mathrm{lib}} = 1.0$ Hz. The amplitude ε changes step by by step, at each step the time to reach the stationary mode (10 minutes) is waited. Temperature measurement is carried out continuously throughout the experiment using thermal resistances made of copper wire 0.02 mm thick. The wire is fastened with a self-adhesive film to the outer boundary of the heat exchanger, which serves as the boundary of the cavity. In this way, the temperatures of the layer boundaries T_1 and T_2 are measured. To determine the heat flux passing through the working layer, the heat exchanger *2* (fig.1) is equipped with a heat flux sensor, which is a thin gasket made of a low-conductive material. In experiments, the temperature difference of the layer boundaries $\Theta = T_2 - T_1$, and the heat flux through the layer $\Delta T = T_3 - T_2$ (where T_3 is the temperature of heat exchanger *2*) are measured. As a characteristic of heat transfer, the Nusselt number $Nu = \Delta T / \Delta T_0$ is introduced, which is defined as the ratio of the heat flux through the layer ΔT to the heat flux ΔT_0 . Heat flux ΔT_0, is defined as the average value of the heat flux at high speeds of rotation of the layer (heat transfer at $f_{\mathrm{rot}} > 1$ rps is close to molecular).

To observe the structures, the heat exchanger *1* (fig.1) was made of transparent organic glass. The structures were visualized using a thermochromic film applied to the boundary of heat exchanger *2*. Hot liquid is pumped through transparent heat exchanger *1*, and cold liquid is pumped through heat exchanger *2*, on which the film is applied. Thus, when a flow occurs from the hot boundary of the transparent heat exchanger *1* to the less heated boundary of the heat exchanger *2*, it is possible to observe a change in the color of the thermochromic film. Thermochromic film operates over a narrow temperature range, with hotter areas displayed in brighter colors. The temperature range of the film was $T = (35 \pm 3)$ °C. This film, unlike finely dispersed visualizers, does not affect the heat transfer in the layer.

3. Experimental Results

Let us consider the case of uniform rotation of a flat liquid layer, with isothermal boundaries of different temperatures, around a horizontal axis. In a rotating flat layer, excitation of vibrational thermal convection is possible [9–12]. Convection and heat transfer in this case are determined by the action of the thermal vibration mechanism [8]. The experiments were carried out in a relatively thick layer, thick $h = 1.0$ cm. Figure 2*a* shows the change in the Nusselt number depending on the speed of rotation of the cavity. The temperature at the layer boundaries is set and maintained constant, $\Theta = 10.5$ °C. With a step-by-step decrease in the rotation speed from a sufficiently high ($f_{\mathrm{rot}} = 2.0$ rps), Θ and ΔT are measured. The horizontal area corresponds to the area of quasi-equilibrium, $Nu = 1$. A smooth increase in heat transfer, in the range of rotation speeds from $f_{\mathrm{rot}} = 0.5$ rps to $f_{\mathrm{rot}} = 0.3$ rps, is associated with the development of toroidal structures in the layer (fig.2*b*). The generation of toroidal structures occurs due to the development of inertial waves [12] in a thin boundary layer. A further decrease in the rotation speed leads to the development of vibrational thermal convection in the layer [8, 9]. The cavity contains a developed system of hexagonal cells of regular order (fig.3*c*), which intensifies heat transfer. It is worth noting the presence of an artifact in the photographs (a bright spot of green color) caused by a loose fit of the film to the border of the working cavity. This artifact, as can be seen from fig.2*c*, does not affect the order and symmetry of the developed convective structures in the layer.



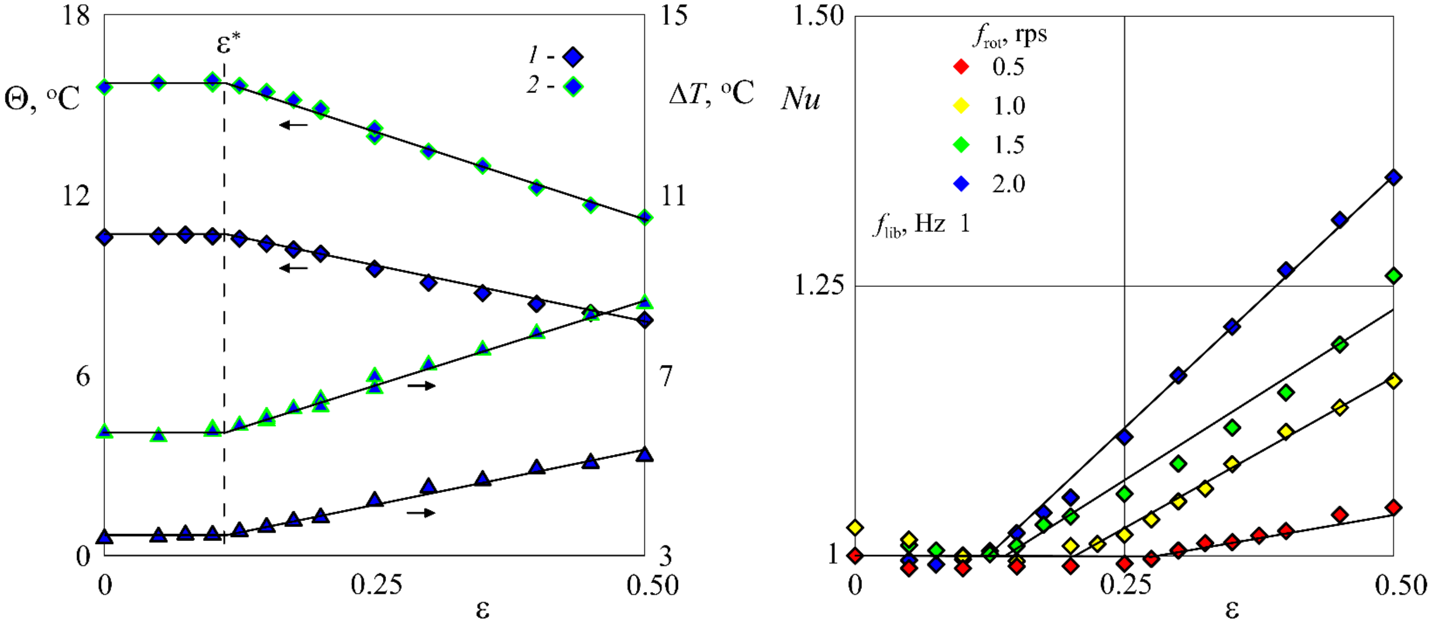
b) f_{rot}=0.45 rps; f_{lib}=0 Hz

c) f_{rot}=0.30 rps; f_{lib}=0 Hz

Fig. 2: Heat transfer in the case of uniform rotation of a flat layer around a horizontal axis, depending on the cavity rotation speed (*a*) and photographs of convective structures, the development of which is responsible for inertial waves (*b*) and the thermal vibration mechanism (*c*)

Consider the case of non-uniform rotation of the cavity around the horizontal axis. Non-uniform rotation is carried out according to the law $\Omega = \Omega_{\mathrm{rot}}\left(1+\varepsilon \sin\left(\Omega_{\mathrm{lib}} t\right)\right)$. The experiments were carried out at high rotation speeds, when the Nusselt number took on a value $Nu = 1$ (horizontal area in fig.2*a*). On the plane $\varepsilon, \Delta T$ and ε, Θ are the results of measuring the heat flux and temperature difference depending on the amplitude of the librations (fig.3*a*). At a fixed temperature difference of the layer boundaries and given $f_{\mathrm{rot}} = 2.0$ rps and $f_{\mathrm{lib}} = 1.0$ Hz, the amplitude ε was changed step by step. The temperature at the layer boundaries Θ and the heat flux ΔT were measured. Symbols *1* correspond to the case of a fixed temperature difference of the layer boundaries $\Theta = 10.6$ °C, and symbols *2* correspond to $\Theta = 15.8$ °C. Figure 3*a* shows that small-amplitude librations do not affect the heat transfer in the layer. Upon reaching a certain critical amplitude ε^*, the heat transfer in the layer increases. With a further increase in the amplitude of librations, the law of increase in heat transfer does not change significantly. It should be noted that the law of increase in heat transfer, with an increase in the temperature difference of the boundaries of the layer, remains the same. An increase in the temperature difference of the layer boundaries ($\Theta = 15.8$ °C, symbols *2*) does not lead to a shift in the threshold value ε^*. The excitation thresholds for the averaged convection at $\Theta = 15.8$ °C and at $\Theta = 10.6$ °C are within the confidence interval of the value ε^*. In the case of the forward stroke (increasing ε) and the reverse (decreasing ε), no hysteresis was observed in the layer.

The experiments were carried out at different, fixed speeds of rotation of the cavity (fig.3*b*). The plane ε, Nu (fig.3*b*) shows the heat transfer curves for the following fixed rotation speeds: $f_{\mathrm{rot}} = 2.0,\ 1.0,\ 0.5$ rps at $\Theta = 10.6$ °C. A decrease in the rotation speed leads to a shift of the threshold value ε^* to the region of large cavity libration amplitudes. Decreasing the rotation speed of the cavity reduces the intensity of heat transfer, the slope of the heat transfer curves changes. With the same value of $\varepsilon = 0.5$, the value of the number Nu for the $f_{\mathrm{rot}} = 0.5$ rps case is 1.35 times smaller compared to the $f_{\mathrm{rot}} = 2.0$ rps case.



a) *b*)

Fig. 3: Heat transfer in the case of modulated cavity rotation: *a*) at a fixed rotation speed and libration frequency, but at different given temperature differences of the layer boundaries; *b*) at a fixed libration frequency and temperature difference of the layer boundaries, but at different rotation speeds

The use of a thermochromic film makes it possible to detect the flow and visualize the temperature change at the boundary of the working layer. Figure 4 shows photographs of convective structures in the case of a fixed frequency and amplitude of the cavity librations at different rotation speeds. The photograph shown in fig.4*a* corresponds to the case of uniform rotation of the cavity, when there is no heat transfer in the layer. The film in the central part has a black color, which indicates the absence of flow from the hot boundary of the cavity to a less heated one. An increase in the rotation speed leads to a change in the color of the thermochromic film from black to light orange, which indicates the appearance of a bulk flow near the lateral cylindrical boundary and the excitation of averaged convection. The temperature distribution of this flow at the less heated boundary of the working layer is a toroidal structure, the center of which is at a distance of the order of the thickness of the working layer from the side boundary of the cavity (fig.4*b*). A further increase in the rotation speed (fig.4*c*) leads to a narrowing of the central cold region (black), and an increase in the width of the hotter (orange) region at the layer boundary. The width of the toroidal shaft has grown. From the change in the color of the thermochromic film (color transition from light orange to orange) in the case of $f_{\text{rot}} = 1.5$ rps (fig.4*c*), it can be said that the temperature manifested in the cold increased compared to the temperature of the observed structures at $f_{\text{rot}} = 1.0$ rps (fig.5*b*).

In the case of a higher rotation speed (fig.5*d*), the area of the thermochromic film near the side boundary acquires a yellow tint. The change in color (from orange to yellow) indicates an increase in the intensity of the flow from the hot layer boundary to a less heated one. The heat flux in the central region also increased, the film acquired a reddish tint instead of black.

For photographs taken in the presence of librations, a green area is visible near the lateral cylindrical boundary. This color on the cold wall characterizes the hottest region of the flow that occurs in the layer. Near the lateral oscillating boundary of the cavity, the liquid is transferred more intensively than in the central part of the cavity.

a) f_{rot}=1 rps; f_{lib}=0 Hz *b)* f_{rot}=1 rps; f_{lib}=1 Hz

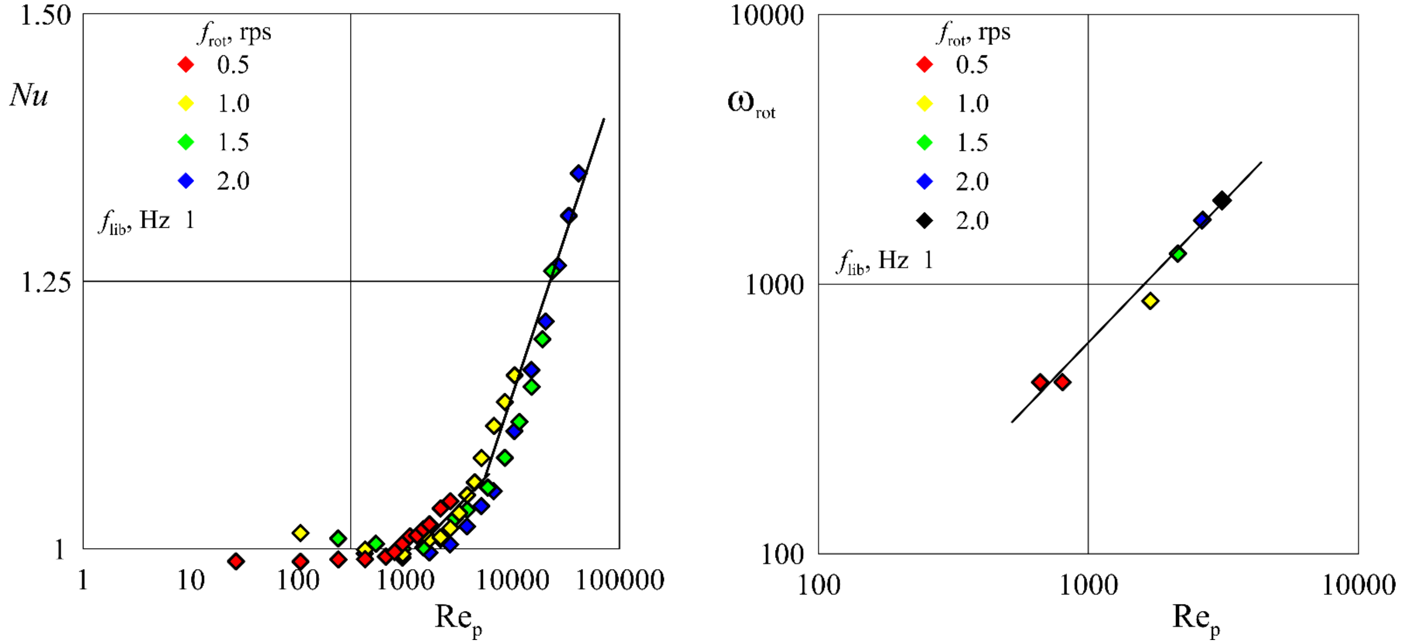
c) f_{rot}=1.5 rps; f_{lib}=1 Hz *d)* f_{rot}=2.0 rps; f_{lib}=1 Hz

Fig. 4: Photographs of convective structures with uniform (*a*) and modulated (*b*, *c*, *d*) rotation of the cavity around the horizontal axis

4. Discussion of the Results

Liquid oscillations caused by librations of the lateral cylindrical boundary lead to the appearance of an average flow in thin boundary layers [16]. With an increase in the intensity of librations, more and more liquid is involved in the oscillatory process. Fluid mixing propagates from the side boundary along the radius to the center of the layer. To describe the system under study, the following dimensionless control parameters (introduced in [17]) are used: the pulsating Reynolds number $\mathrm{Re}_{\mathrm{p}} = \varphi^2 R^2 \Omega_{\mathrm{lib}} / \nu$ determines the intensity of the average flows; dimensionless rotation speed $\omega_{\mathrm{rot}} = \Omega_{\mathrm{rot}} h^2 / \nu$ - the ratio of the Coriolis force to the viscous forces; dimensionless cavity vibration frequency $\omega_{\mathrm{lib}} = \Omega_{\mathrm{lib}} h^2 / \nu$ - characterizes the ratio of the cavity size to the thickness of the Stokes layer. In these parameters, the following values are used: φ - amplitude of the angular oscillations of the cavity; ν - is the kinematic viscosity of the liquid; geometric dimensions of the cavity, where R is the radius, h is the layer thickness; as well as rotational speed $\Omega_{\mathrm{rot}} = 2\pi f_{\mathrm{rot}}$ and vibration frequency $\Omega_{\mathrm{lib}} = 2\pi f_{\mathrm{lib}}$.

Heat transfer as a function of the ripple Reynolds number is shown in fig.5*a*. With an increase in the amplitude of cavity librations (see fig.3*b*), the Reynolds number increases, heat transfer through the layer increases, and an averaged flow appears in the form of a toroidal vortex located azimuthally (fig.4.*a*). The heat transfer curves on the plane Re, Nu for different rotation speeds are consistent - they have a similar slope angle. Heat transfer is only a passive indicator of the beginning of the volumetric movement of the liquid. The Nusselt number is determined by the average fluid motion, which is formed under the action of the libration mechanism.



a) *b*)

Fig. 5: Heat transfer depending on the pulsating Reynolds number - *a*). Dependence of the dimensionless pulsation parameter on the dimensionless rotation speed – *b*)

On the parameter plane $\mathrm{Re}_{\mathrm{p}}, \omega_{\mathrm{rot}}$, symbols show the critical values of the number Re_{p} corresponding to an increase in heat transfer in the layer. As the cavity rotation speed increases, the intensity of the mean flow increases monotonically (fig.5*b*). The dark symbol in fig.5*b* corresponds to the value of the temperature difference $\Theta = 15.8$ °C (see symbols *2* in fig.3*a*), for the other symbols the temperature difference takes the value $\Theta = 10.6$ °C. As was shown above (see fig.3*a*), a change in the temperature difference of the layer boundaries by a factor of 1.49 does not affect the threshold for the increase in heat transfer. With the development of an average flow in the layer, the temperature difference of the layer boundaries Θ decreases, the dimensionless parameter $Ra_{\mathrm{c}} = \Omega_{\mathrm{rot}}^2 R\beta\Theta h^3 / \nu\chi$ decreases, and the contribution of the centrifugal mechanism weakens. Centrifugal convection does not affect the development of the average flow in the layer. The intensity of the average motion is determined by the frequency and amplitude of librations, as well as by the geometry of the cavity.

5. Conclusion

For the first time, the excitation thresholds of averaged convection in a flat layer rotating around a horizontal axis during cavity librations have been experimentally studied. It is found that cavity librations generate an averaged flow in the form of a toroidal vortex in the layer near the lateral boundary. It is shown that with an increase in the rotation speed, the effect of librations on heat transfer increases - the threshold of excitation of averaged convection shifts to the region of lower cavity libration amplitudes. With an increase in the rotation speed, the intensity of the average motion increases, which leads to an increase in the radial size of the flow, while the average temperature of the flow increases. It is shown on the plane of control parameters that the intensity of the average fluid motion is determined by the dimensionless rotation speed and the pulsating Reynolds number. In this case, the contribution of centrifugal convection is insignificant.

The nature of this problem reflects the competition and non-linear interactions between the centrifugal and vibratory convective mechanisms, as well as the instability caused by large-scale fluid circulation due to the modulated cavity The complexity of studying the flow caused by a periodically changing rotation speed lies in the fact that the mechanism sets the fluid in motion manifests itself most intensively at high rotation speeds, thereby causing oscillatory instability in a thin boundary layer. This makes it difficult to determine the contribution to the development of the average flow of each mechanism separately. Further experimental study of the effect of rotation velocity modulation on the stability of a flat liquid layer will expand the data on the nature of this mechanism and predict the response of the liquid to the action of librations. Further study of the averaged convection threshold is planned: at other (different) cavity libration frequencies, in a cavity with a different aspect ratio, as well as on other working fluids. The study of the instability caused by cavity librations in a flat layer rotating around a horizontal axis, for various values of the quantities listed above, will make it possible to supplement and noticeably expand the boundary of the existence of new regimes on the plane $\mathrm{Re}_\mathrm{p}, \omega_\mathrm{rot}$ of control parameters. An interesting case is the search for resonant regions, with the presence of inertial waves in a simultaneously rotating and librating flat layer.

Acknowledgements

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