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Modulation of Heat Flux by Inertial Particles Thermal Feedback in a Turbulent Shearless Anisothermal Flow

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Abstract **-** We analyze the modulation of heat flux by particle thermal feedback in a turbulent shearless flow by employing a recently introduced decomposition of the velocity-temperature correlation in terms of particle velocity and temperature time derivative correlations. The results of a set of Eulerian-Lagrangian point-particle direct numerical simulations (DNSs) at a Taylor microscale Reynolds number equal to 56 and with the same volume fraction is used to reveal the action of thermal feedback in a wide range of thermal Stokes number and Stokes numbers. The results show that particle heat flux is influenced by thermal feedback more than fluid convective heat flux and they act in the opposite way in two-way coupling regime. We also discuss why global particle contribution to the heat flux ratio behave in a certain way and what statistics can attenuate or enhance this ratio under different particle inertia and thermal inertia.

Keywords**:** two-phase flow, turbulent mixing, heat transfer, fluid-particle thermal interaction, direct numerical simulations

1. Introduction

Suspension of solid particles, bubbles, or liquid droplets within a turbulent fluid flow can be of importance in many natural and industrial phenomena, such as water droplets in clouds [1,2] and solid fuel combustion processes [3]. There have been numerous studies to elucidate the physics behind particle suspension and its interplay with the carrier flow in different turbulent flows. However, in the most complex cases, where the dynamical and thermal behaviour of the discrete phase are simultaneously under investigation, there have been few numerical, analytical, and experimental works to date. This physical problem becomes even more complex when both phases are two-way coupled, and the carrier flow is thermally and dynamically modulated by the particle backreactions. Meanwhile, due to the difficulty in accurately measuring inertial particle velocity and temperature, even using available advanced experimental tools, Direct Numerical Simulation (DNS) is alternatively used to further the knowledge of such flow regimes. Thanks to the growing advances in computing tools in recent decades, DNS has become the main tool to investigate particle-laden turbulent flows across various disciplines, yet it has been limited to low and moderate Reynolds numbers. Heat transfer in the two-way coupling regime has mostly been investigated in unbounded statistically homogeneous turbulent flows to reveal the effect of particle feedback on fluid temperature statistics and inter-scale heat transfer [4-6]. Yet, in the literature, there have been few works on the same problem when the temperature field is inhomogeneous and statistically unsteady.

However, we recently studied an anisothermal turbulent flow laden with particles, such that a thermal mixing layer develops in a quasi-self-similar way between two homogeneous and isotropic homothermal zones. We analyzed the dynamical and thermal effects of inertial particles and the flow Taylor microscale Reynolds number in both one and two-way coupling, considering collisionless [7] and collisional regimes [8]. Furthermore, a new decomposition has been introduced in [9] to analyze the effect of particles on heat flux statistics in terms of particle and fluid velocity temperature correlations, and particle time derivatives. However, in that study, this decomposition only considered the particle effect in one-way coupling. As an extension, we aim to investigate the effect of particle thermal back-reaction by using the same decomposition and making comparisons between different two-way coupled statistical quantities with those of the one-way coupling regime. Accordingly, this study is more focused on the effect of particle back-reaction on the heat flux statistics. Note that this problem is intrinsically non-trivial even for the one-way coupling regime, but this decomposition helps us to formulate the ratio between particle to fluid heat fluxes and provides a way to gain new knowledge of particle effects in the two-way coupling regime that can be useful for further studies in two-phase flows, especially when heat transfer is the core of the investigation.

2. Physical model

The objective of this study is to investigate the heat transfer between two homogeneous zones with different temperatures, T_1 and $T_2 < T_1$, of a particle-laden flow with homogeneous and isotropic velocity fluctuations. We use point-particle Eulerian-Lagrangian direct numerical simulations. In Eulerian frame, the Navier-Stokes equations are solved for the carrier flow with a divergence-free velocity field **u**(t,**x**), pressure field p(t,**x**) and a passively advected temperature field T(t,**x**), while individual particles are tracked along their Lagrangian path. Under these assumptions, the dynamics of fluid phase is governed by incompressible Navier-Stokes equations which are given by

$$\partial_j u_j \qquad 0 \tag{1}$$

$$\partial_t u_i + u_j \partial_j u_i \quad -(1/\rho_0)\partial_i p + \nu \partial_j \partial_j u_i + f_{u,i} \tag{2}$$

$$\partial_t T + u_j \partial_j T \qquad \kappa \partial_j \partial_j T + \left(1/\rho_0\, c_{p0}\right) C_T \tag{3}$$

where ρ_0 denotes fluid density, and c_{p0} and ν are the isobaric specific heat capacity and fluid kinematic viscosity respectively. $f_{u,i}$ is an external body force introduced to maintain turbulent fluctuations in a statistically steady state, and C_T is the heat exchanged per unit time and unit mass with particles, i.e. particle thermal feedback on the carrier flow. Similar to previous works (e.g. [7-9]), we do not consider the force exerted by particles on the fluid: only fluid temperature field is two-way coupled with particles, and momentum exchange occurs only under one-way coupling regime. This assumption yields in a dilute regime and in our problem because it has been found that momentum feedback has a minor thermal effect on fluid temperature statistics [5]. Discrete phase is modeled as a monodisperse solid sphere of radius R smaller than carrier flow Kolmogorov lenghtscale η. This small material point particle has the density ρ_p much higher than the fluid density, and isobaric specific heat capacity c_{pp}. The dynamics of the particulate phase obeys the equation of motion proposed by Gatignol 1983, Maxey and Riley 1983, [10]. It is also assumed that the Stokes drag force is the dominant term in the Maxey–Riley equation for the motion of small particles in a fluid [10]. Analogous to the equation of motion of a rigid sphere in fluid, an equation for the particle temperature is derived under the same hypothesis, so that the dynamics of each individual particle is governed by the following equations in the Lagrangian reference frame

$$\frac{d}{dt}\begin{Bmatrix} X_p(t) \\ V_p(t) \\ \Theta_p(t) \end{Bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -1/\tau_v & 0 \\ 0 & 0 & -1/\tau_\vartheta \end{bmatrix} \begin{Bmatrix} X_p(t) \\ V_p(t) \\ \Theta_p(t) \end{Bmatrix} + \begin{bmatrix} 0 \\ 1/\tau_v\, u(t, X_p) \\ 1/\tau_\vartheta\, T(t, X_p) \end{bmatrix} \tag{4}$$

where X_p, V_p, and Θ_p are position, velocity and temperature of the p-th particle, respectively, and define the state of the particle. Here τ_v and τ_θ are the momentum and thermal relaxation times, given by

$$\tau_v = \frac{2}{9}\frac{\rho_p}{\rho_0}\frac{R^2}{\nu}, \tau_\vartheta = \frac{1}{3}\frac{\rho_p c_{pp}}{\rho_0 c_{p0}}\frac{R^2}{\kappa} \tag{5}$$

Note that any direct particle-particle interaction is excluded and there is no external field like gravitational field, acts on particles. The particle thermal feedback per unit time and unit volume is given by

$$C_T(t,x) = \frac{4}{3}\pi R^3 \rho_p c_{pp} \sum_{p=1}^{N_p} \frac{d\Theta_p(t)}{dt}\,\delta\left[x - X_p(t)\right] \tag{6}$$

where N_p is the total number of spherical inertial particles and $\delta(\cdot)$ is the Dirac delta function.

To conduct numerical experiments on this problem, the governing equations are solved in a parallelepiped computational domain with dimensions $L_1 = L_2$ and $L_3 = 2L_1$ along the x_1, x_2, and x_3 directions. The temperature distribution is initialized by setting the temperature equal to T_1 in the half-domain where $x_3 < L_3/2$ and temperature equal to T_2 in the half-domain where $x_3 > L_3/2$. Periodic boundary conditions are imposed to the velocity field on all faces of the computational domain, while temperature field, which is intrinsically non periodic, is decomposed into a mean linear steady part and a fluctuating part, to which periodic boundary conditions can be applied, as described in detail in [7]. Additionally, for the sake of consistency with physics of the two-phase flow, any particles that may exit the computational domain must be reintroduced on the opposite side with the same velocity and fluctuating part of the temperature. The governing equations are all non-dimensionalized by using $L_1 = L_1/2\pi$ as reference length, a reference velocity which is deduced from the imposed kinetic energy dissipation rate ε, and the temperature difference $T_1 - T_2$ as reference temperature [7]. The flow is forced on a single length-scale, and the linear deterministic forcing, which allows to control the forced length scale and the energy injection rate, is used, as in [5, 7, 9]. To make the results more physically significant, the Taylor-microscale is used as the reference length in the definition of the Reynolds number instead of the domain size $L_1 = L_1/2\pi$. In the dimensionless form, the flow is governed by the Reynolds number $Re_\lambda = u'\lambda/\nu$, the Prandtl number $Pr = \nu/\kappa$, and the particle-to-fluid heat capacity ratio $\varphi_\vartheta = \varphi\left(\rho_p c_{pp}\right)/\left(\rho_0 c_{p0}\right)$, where φ is the particle volume fraction. In dimensionless form, particle dynamics is determined by the ratio between their relaxation times and the flow timescales. To characterize the particle dynamics in terms of local fluctuations of fluid state, the Kolmogorov timescale $\tau_\eta = (\nu/\varepsilon)^{1/2}$is used instead of the large-scale time used in the adimensionalization. Thus, the Stokes number $St = \tau_v/\tau_\eta$and the thermal Stokes number $St_\vartheta = \tau_\vartheta/\tau_\eta$ are used to describe the particle dynamical and thermal behavior. A fully dealiased pseudospectral method, using the 3/2-rule, was employed to discretize the spatial domain of the fluid phase equations (2-3). Interpolation of fluid velocity and temperature at particle positions and computation of the particle thermal feedback (6) were carried out using a recent numerical method [11, 12] based on inverse and forward non-uniform fast Fourier transforms with a fourth-order B-spline basis. Integration in time was performed for both the carrier flow equations (2-3) and the particle equations (4) using a second order exponential integrator. More details about the numerical method and flow setup can be found in [7] and [11].

All the results we present come from simulations carried out using $256^2 \times 512$ Fourier modes (after dealising, $384^2 \times 768$ grid points in physical space) with a volume fraction $\varphi = 4 \times 10^{-4}$ and a particle to fluid density ratio $\rho_p/\rho_0 = 1000$ at $Re_\lambda = 56$. Particle size and number are determined by the Stokes number. We consider the Stokes and thermal Stokes number as independent parameters, so that the particle specific heat is adjusted accordingly. The thermal Stokes number ranges from 0.2 to 10 and the Stokes number from 0.2 to 6. In dimensionless variables, the domain is $2\pi \times 2\pi \times 4\pi$, the root mean square of velocity fluctuations is $u' = 0.59$, the integral scale and Taylor microscale are $l = 0.4$ and $\lambda = 0.226$, respectively, while the Kolmogorov timescale is $\tau_\eta = 0.098$.

3. Correlation decomposition

Given the statistical inhomogeneity and unsteadiness of the temperature, in the following we consider conditional averages at a given time and position x_3 along the inhomogeneous direction, i.e., we define, for any function f of the state of the particle,

$$\langle f \rangle_p = \langle f | t, x_3 \rangle_p,$$

where $\langle \cdot \rangle_p$ is the statistical average and we define the fluctuation of f as $f' = f - \langle f \rangle_p$. The heat flux across the inhomogeneous layer, i.e. in direction x, is

$$\dot{q} = \lambda \frac{\partial \langle T \rangle}{\partial x} + \rho_0 c_{p0} \langle u'T' \rangle + \varphi \rho_p c_{pp} \left\langle V'_p \Theta'_p \right\rangle_p$$

where the last term is the contribution of particles motions, which is the focus of this work. We now express the average the heat flux in terms of the time derivatives of particle velocity (i.e., the particle acceleration) and temperature. By subtracting from (4) its conditional average, the particle temperature and velocity fluctuations can be expressed in terms of the fluctuations of the time derivatives, i.e.,

$$V'_{p,i} = u' - \tau_v \dot{V}'_{p,i} \tag{7}$$

$$\Theta'_{p,i} = T' - \tau_\vartheta \dot{\Theta}'_p \tag{8}$$

where fluid velocity and temperature are to be computed at particle position. In the following, we will skip the apex from all moments that are second order or higher to keep notations simple. Following [9]. by multiplying equations (7) and (8) and taking the conditioned statistical average, we obtains the following expression of the particle velocity-temperature correlation

$$\left\langle V_{p,i} \Theta_p \right\rangle_p = \langle u_i T \rangle_p - \tau_v \left\langle \dot{V}_{p,i} T \right\rangle_p - \tau_\vartheta \left\langle u_i \dot{\Theta}_p \right\rangle_p + \tau_v \tau_\vartheta \left\langle \dot{V}_{p,i} \dot{\Theta}_p \right\rangle_p \tag{9}$$

which expresses the particle contribution to the convective heat flux. This correlation can be conveniently divided by the fluid temperature-velocity correlation to obtain

$$\frac{\langle V_{p,i}\Theta_p\rangle_p}{\langle u_i T\rangle_p} = 1 - \tau_v \frac{\langle \dot{V}_{p,i} T\rangle_p}{\langle u_i T\rangle_p} - \tau_\vartheta \frac{\langle u_i \dot{\Theta}_p\rangle_p}{\langle u_i T\rangle_p} + \tau_v \tau_\vartheta \frac{\langle \dot{V}_{p,i} \dot{\Theta}_p\rangle_p}{\langle u_i T\rangle_p} \tag{10}$$

This ratio relates, apart for a constant coefficient (the particle to fluid heat capacity), the particle and fluid contributions to the Nusselt number (see [7]). In this way, we have decomposed the particle contribution to the heat flux in terms of the correlations between the particle derivatives and between them and the fluid velocity and temperature fluctuations. The particle derivatives account for the instantaneous heat exchanges between the two phases and strongly depend on both relaxation and thermal relaxation times, which also explicitly appear as coefficients in the decomposition.

4. Results and discussion

The decomposition (9) has been used in [9] in the two-way coupling regime for a wide range of Stokes and thermal Stokes numbers. Here, we aim to elucidate the role of particle feedback by analysing the flow at the same Reynolds number. In the flow configuration there is only one non homogeneous transport direction, the one parallel to the mean temperature gradient, so that we will skip the index. To comprehend how particle feedback modifies the fluid temperature fluctuations and, consequently, the fields experienced by the particles, we present the decomposition in terms of the ratios between the three terms of the decomposition (9) in the two-way coupling and the same terms in the one-way coupling at the same Reynolds and Stokes/thermal Stokes numbers. Since the flow is statistically almost self-similar [7], all data presented refer to the central sublayer of the interaction region, where the mean temperature and heat flux are largest.

Figure 1(a) illustrates the particle and fluid velocity-temperature correlations. Since particles tend to accumulate in regions with high temperature gradients [5, 13], particle back-reaction tends to reduce fluid temperature gradients. Therefore, it is not surprising that the fluid velocity-correlation is reduced, and this reduction increases with particle thermal inertia, i.e., with the thermal Stokes number. This effect is more pronounced when the Stokes number is of order one because particle clustering is more intense. However, for $St_\vartheta \gg 1$ the relative reduction seems to be independent of St. Only for very high Stokes numbers is there a minor increment in fluid heat flux when $St_\vartheta \ll 1$, with a maximum around $St_\vartheta \simeq 0.5$. In such situations, we can speculate that particle motion decorrelates from fluid motions, allowing particles to transfer heat between eddies at very different temperatures easily and frequently. However, if particle thermal inertia becomes very large, the heat transfer becomes very slow, so the damping effect prevails as particles cross multiple fronts, while in the opposite limit, a vanishing particle thermal inertia produces no heat transfer, and thus the maximum effect is seen at intermediate thermal inertia. On the contrary, as far as particle heat flux is concerned, two-way coupling always increases the particle velocity-temperature correlation. However, for small Stokes numbers, the effect is negligible and almost independent of the thermal Stokes number, while a significant increase is observed when St exceeds a certain threshold. Indeed, feedback reduces the particle-to-fluid temperature difference, thus making the variation of particle temperature slower. This allows particles, which can cross eddy borders in the inhomogeneous direction, to carry their own enthalpy over longer distances. The effect of thermal feedback on the overall heat flux depends on the particle-to-fluid heat capacity ratio φ_ϑ. At the simulated volume fraction, 4×10^{-4} the heat capacity of particles is larger than that of the fluid, $\varphi_\vartheta \simeq 1.664$, and thus the overall heat flux is increased by the particle feedback.

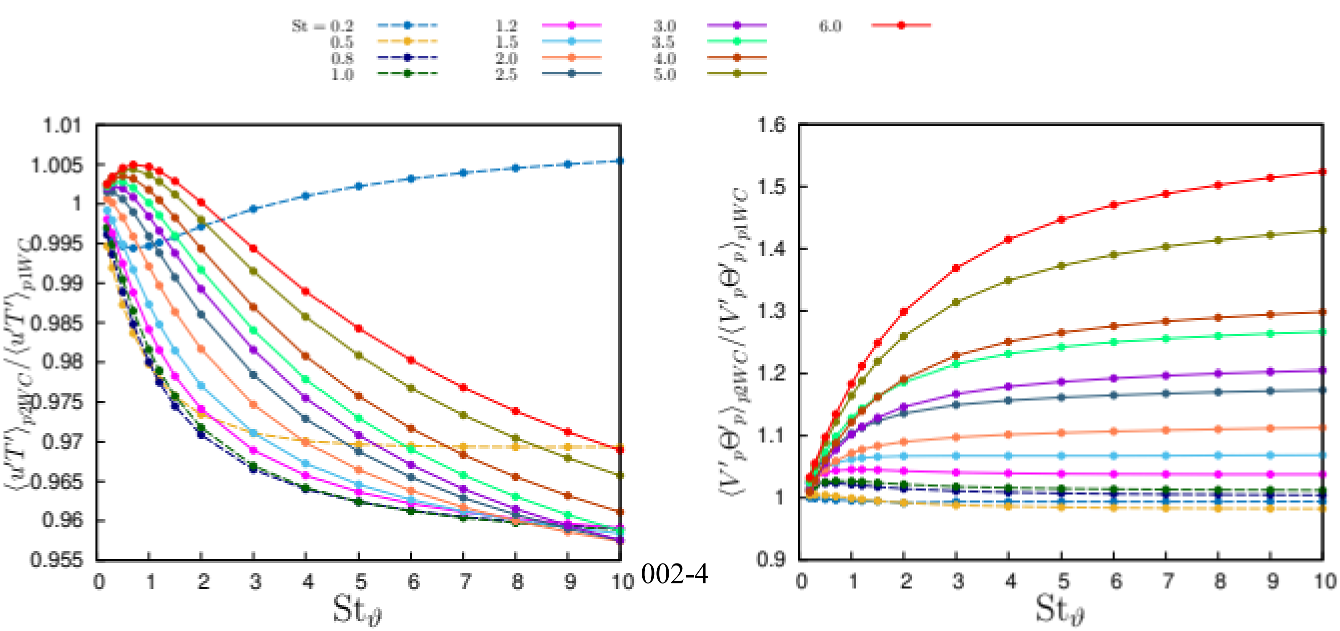

Fig. 1: Ratio between (a) fluid and (b) particle velocity-temperature correlation in one- and two-way coupling regimes.

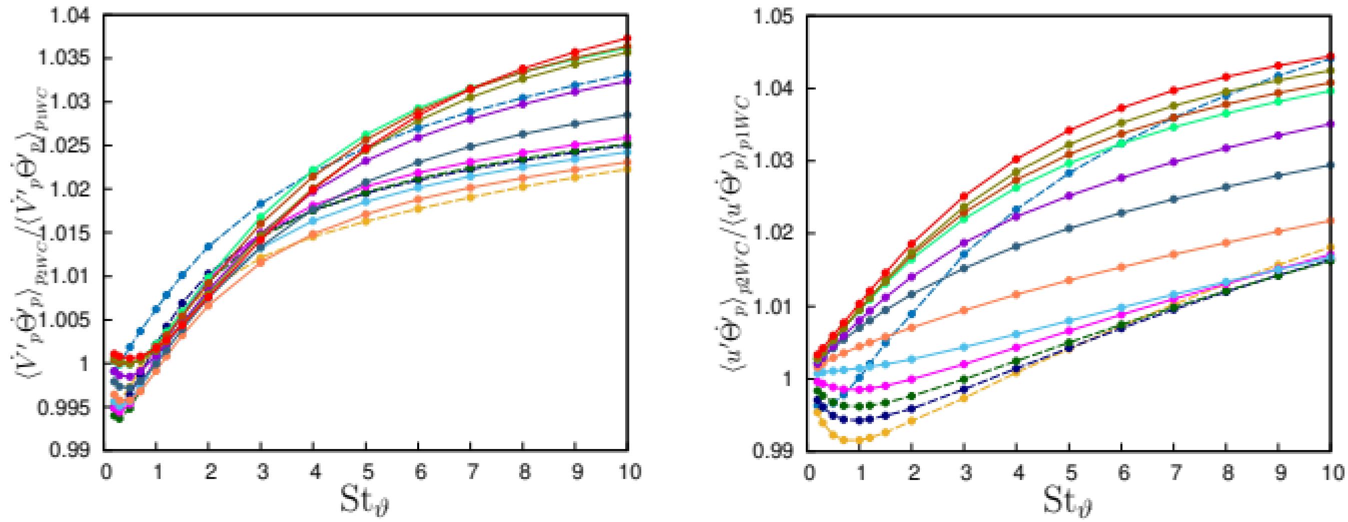

Fig.
2: (a) Ratio between normalized particle velocity and temperature derivative correlation and (b) normalized fluid velocity-particle temperature derivative correlation in one- and two-way coupling regimes. Legend as in Figure 1.

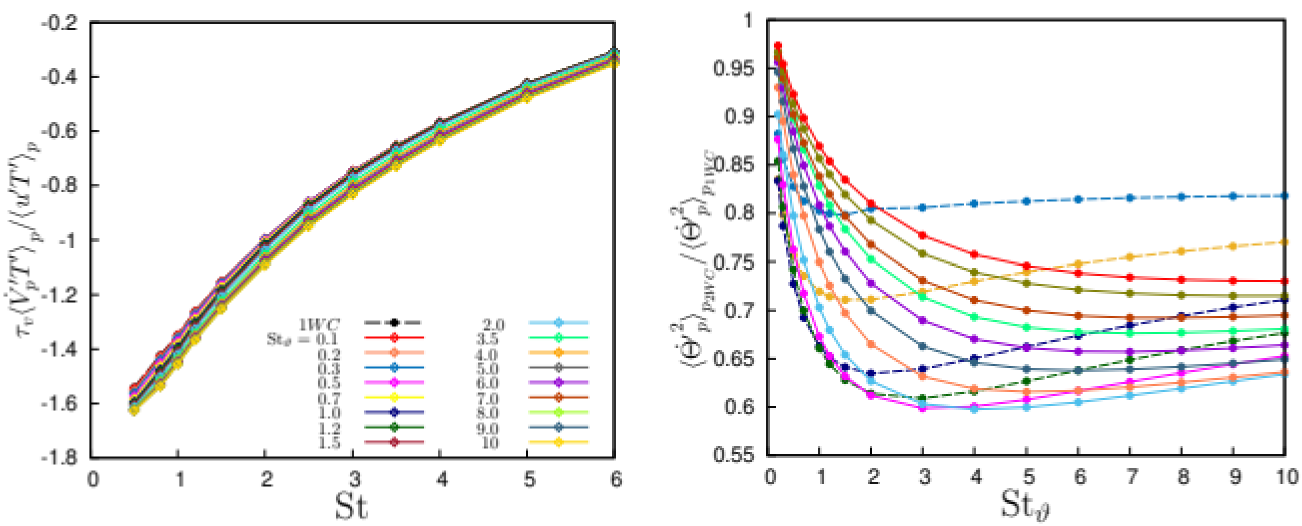

Fig. 3: (a) Particle acceleration-temperature correlation normalized with fluid velocity-temperature correlation, and (b) Ratio between particle temperature derivative variance in one- and two-way coupling regimes. Legend as in Figure 1.

We can now analyse the three terms on the right-hand side of equation (9) Note that all these terms are negative, so that the first two actually build the correlation while the third one tends to damp it [7]. Indeed, a positive velocity fluctuation produces a positive temperature fluctuation, because, as intuitive, the particle is moving in a region colder in the average, but consequently a negative temperature derivative. A similar simple argument cannot be used for correlation involving particle acceleration, even if higher accelerations are expected for particles which are in the higher strain zones [14], because the velocity field is isotropic and no preferential direction is present. Figure 2(a) shows the acceleration-temperature derivative correlation, last term of equation (9), which increases with St_ϑ, with a minor dependence on St. This contributes toward a reduction of the particle heat flux, which, however, can be mainly appreciable for large Stokes and thermal Stokes numbers, because the term is multiplied by the product of the relaxation times, $\tau_v\tau_\vartheta$, so that it could become relevant and maybe dominant only for very large St and St_ϑ. However, for any given Stokes number, i.e. for any fixed τ_v, it never dominates the other ones, at least up to $St = 6$. Only if the ratio $\tau_\vartheta/\tau_v = S\,t_\vartheta/S\,t$is kept fixed, it could possible to observe a reduction in the heat flux. This could explain the reduction in heat flux observed in some flow configurations [15].

Figure 3(a) illustrates the first term in the decomposition, and shows that in this correlation particle feedback has almost no influence. Indeed, since particles are momentum one-way coupled only, fluid velocity and particle accelerations are independent from any thermal effect, and the only effect of thermal feedback is on T. The main effect depends only on particle inertia, which makes particle motion gradually to become independent from fluid, and thus reduces the correlation $\langle \dot{V}_p T\rangle_p$. This implies that the effect of the two-way coupling is to increase the sum of the last two terms in (9), as to overcome the observed reduction in $\langle uT\rangle_p$. Since their sum is equal to $-\tau_\vartheta\langle V_p\dot{\Theta}_p\rangle_p$, we can infer that thermal feedback tends to increase the modulus of the correlation between particle velocity and temperature derivative. At the same time, we can observe that the variance of particle temperature derivative is always reduced by thermal feedback. This, which can be attributed to the reduction in the difference between the temperature of the particle and of the surrounding fluid operated by the thermal feedback on the carrier fluid phase, is more marked at higher thermal Stokes number and when the Stokes number is of order one. In this situation the smoothing of the fluid temperature gradient where particle cluster can be the explanation.

In conclusion, we have observed that thermal feedback produces an overall increase of the heat flux, which is due to the enhancement of particle velocity-temperature correlation, which overcomes the reduction of the fluid one due to the smoothing of fluid temperature gradients and variance. This effect is mainly generated by the increased correlation between the particle velocity and its time derivative. This effect, which highlights the importance of the temperature path history, is further enhanced by the presence of $\tau_\vartheta$as coefficient. Further insight could be achieved by looking at all terms in the enthalpy and velocity-temperature balance equations.

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