

## Electroosmotic Flow through a Porous Media within a Microchannel

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### Extended Abstract

In this work a non-Newtonian electroosmotic fluid flow through an embedded porous media within a microchannel is analysed. The fluid is assumed to follow the well-known simplified Phan-Thien-Tanner model (sPTT) under steady state and isothermal conditions. In this investigation, the embedded porous media acquires an electrical potential  $\zeta_p$  due to its interaction with the zeta potential at the microchannel's walls  $\zeta_w$ . Depending on that interaction, we define a zeta potential ratio  $\zeta_r = \zeta_p/\zeta_w$  which allow us to modulate the influence of an electrically charged porous media on the hydrodynamics and viscoelasticity of the fluid. Unlike similar investigations, in this work the boundary condition for the electric potential at the microchannel walls changes with the electrokinetic parameter  $\kappa = (2z^2 e^2 n / \epsilon K_B T)^{1/2}$  [1]. Here,  $z$  represents the number of valences,  $e$  represents the charge of an electron,  $n$  describes the ionic distribution which follows the Boltzmann distribution [2, 3] and  $K_B$  is the Boltzmann constant. Consequently, the boundary condition for the EDL potential  $\psi$  at the microchannel walls ( $y = 1$ ) can be written as a function of the zeta potentials  $\zeta_w$  and  $\zeta_p$  [4, 5]:

$$\psi(y = 1) = \zeta_p \left( 1 - \frac{2I_1(\kappa R)}{\kappa R I_0(\kappa R)} \right) + \zeta_w - \zeta_p,$$

where  $I_N$  is the Bessel function of first kind of order  $N$  and  $R$  is the porous radius. The porous media is described by the Darcy-Forchheimer model [6], and the results are evaluated for different values of the Darcy number. On the other hand, the problem is described by the modified Cauchy equations and the Poisson equation as following:

$$\nabla \cdot V = 0,$$

$$\frac{\rho}{\varphi} \left( V \cdot \frac{\nabla V}{\varphi} \right) = -\nabla p + \frac{\nabla \cdot \tau}{\varphi} - \varphi(\rho_e + \rho_{eff})\mathbf{E} - \frac{\mu}{K}V + \rho \frac{c_F}{\sqrt{K}}|V|V,$$

$$\epsilon_0 \nabla^2 \Phi = -\varphi(\rho_e + \rho_{eff}).$$

In the above equations,  $\rho$  is the fluid's density,  $\varphi$  is the porosity of the medium,  $V$  is the velocity vector,  $p$  describes the pressure field,  $\tau$  is the shear stress tensor,  $\rho_e$  and  $\rho_{eff}$  are the charge density and effective charge density to consider the electric potential acquired by the porous media,  $\mu$  is the fluid's viscosity,  $K$  describes the permeability of the porous media,  $c_F$  is the Forchheimer coefficient and  $\mathbf{E}$  is the electric field vector. On the other hand,  $\Phi$  describes the total electric potential in the Poisson equation [7].

The resulting set equation describes a highly non-linear behaviour which is solved numerically by means of the Semi-Implicit Method for Pressure Linked Equations (SIMPLE) and Alternating Direction Implicit Method (ADI). The results illustrate the velocity fields, the EDL potential distribution and the shear stress distribution, considering different values of  $\zeta_r$ ,  $\kappa$ , the Darcy and Deborah numbers.

The results indicate that the electroosmotic velocities can be increased proportionally to  $\zeta_r$ ,  $\kappa$  and the Deborah number, however, for a very low permeability, the influence of  $\kappa$  and Deborah number is attenuated. Also, for small Darcy numbers, the EDL describes an overlapping along the axial microchannels axis, particularly for small values of  $\kappa$  and high ratios of  $\zeta_r$ . These observations allow us to deduce that a highly charged porous media can enhance the electroosmotic forces in the fluid.

## References

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