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Diagnosis of a Leaky Pipeline Carrying Multiphase Flow under Plug Flow Conditions

Hicham Ferroudji^{1,2*}, Wahib A. Al-Ammari³, Abinash Barooah¹, Ibrahim Hassan⁴, Rashid Hassan⁵, Ahmad K. Sleiti³, Sina Rezaei Gomari⁶, Mohammad Azizur Rahman¹

¹Department of Mechanical Engineering, College of Science and Engineering (CSE), Hamad Bin Khalifa University (HBKU), Qatar

²Laboratory of Petroleum Equipment's Reliability and Materials, Hydrocarbons and Chemistry Faculty, Boumerdes University, Algeria.

³Department Mechanical Engineering, Qatar University, Qatar. ⁴Department of Mechanical Engineering, Texas A&M University at Qatar, Qatar ⁵Department of Petroleum Engineering, Texas A&M University, College Station, United States. ⁶School of Computing, Engineering and Digital Technologies, Teesside University, United Kingdom. Corresponding author: Hicham Ferroudii *hferroudii@hbku.edu.ga*

Abstract - Multiphase flows are crucial to the oil and gas industry since most petroleum companies produce and transport both gas and oil simultaneously. Pipeline leaks are frequently caused by corrosion, aging, and metal deterioration. After an incident, the energy sector not only loses money but also raises environmental and safety concerns. Therefore, developing a successful tool for instantaneous leakage identification in pipelines becomes crucial. In the current work, a leaky pipeline carrying multiphase flow is numerically simulated using Ansys-Fluent under plug flow conditions. The obtained numerical results were validated against experimental data collected from an experimental setup. After that, Probability Density Function (PDF), Wavelet Transform (WT), and Empirical Mode Decomposition (EMD) methods were applied to the obtained time series signals. On the other hand, the analysis is complemented by the application of several machine learning models like Random Forest (RF), Support Vector Machine (SVM), and k-Nearest Neighbors (k-NN). For instance, it is observed that the Empirical Mode Decomposition exhibits better performance in leakage identification.

Keywords: Turbulent flow; Multiphase flow; Leaky pipeline; Time series; Computational fluid dynamics

1. Introduction

The simultaneous flow of gas, liquid, and occasionally solid phases in pipelines is known as multiphase flow, and it is an essential phenomenon in a number of industrial applications, such as process, chemical, and petroleum engineering. Understanding the pressure drop response in a pipeline under different conditions, such as leakage occurrence, is a crucial component of multiphase flow analysis. In addition to other elements associated with infrastructure aging, multiphase flow dynamics, and thermal stresses, pipelines may experience various phenomena, such as material defects [1], corrosion phenomena [2], and pressure surges like hydraulic hammer and pressure pulsations [3].

Various investigations were conducted about the effect of leakage on the flow behaviour in a pipeline, either for onephase or multiphase flows. In this regard, several researchers analysed the impact of leakage presence on the flow behaviour in a pipeline [4, 5]. Other studies have focused on leakage occurrence under different flow regime maps [6, 7]. In this regard, researchers performed their studies based on experimental testing, numerical modelling, and recently they started to develop machine learning models applied for leakage identification based on previously generated data. The study of multiphase flow in a leaky pipeline is crucial for guaranteeing the safe and efficient transfer of energy. Diverse methodologies, including computational fluid dynamics (CFD) simulations and experimental research, have been employed to comprehend flow properties and enhance leak detection methodologies. Experimental research has significantly advanced in characterising leakage behaviour in liquid-gas two-phase flow pipelines, particularly through flow loop experiments (Khan et al. 2024; Meng et al. 2024; Ferroudji et al. 2025) [8, 9, 10]. This research underscores the critical impact of flow regimes, leak sizes, and detection methods, including acoustic techniques (Shama et al. 2017; Ji et al. 2018; Li et al. 2022; Ferroudji et al. 2024) [11, 12, 13, 14] and time-series signal processing (Mujtaba et al. 2020; Barooah et al. 2024) [15, 16]. Over the past decade, sophisticated approaches have been integrated into experimental studies, such as signal processing methods (Mujtaba et al. 2020; Ma et al. 2023) [15, 17], which significantly enhanced leakage detection capabilities.

Furthermore, acoustic and pressure-based techniques began addressing noise-sensitive issues and obstacles associated with transient two-phase flows to address real field challenges (Shama et al. 2017; Ji et al. 2018) [11, 12]. Recently, Meng et al. (2024) [9] conducted an experimental study on leakage characteristics of a horizontal pipe to evaluate leakage mass flow rate, pressure drop, and phase separation. The methodologies of Standard Deviation (SD), Probability Density Function (PDF), Cumulative Probability Density Function (CPDF), and Power Spectral Density (PSD) were utilised to analyse the experimental data. Ferroudji et al. (2025) [10] conducted an experimental study examining the effects of various leak scenarios (none, one, and three leaks) on the behaviour of a multiphase flow system (liquid-gas), focussing on dynamic pressure signals recorded by sensors located in the upstream and downstream regions. The analysis of the obtained signals utilised the pressure drop fluctuation coefficient, Probability Density Function (PDF), Cumulative Probability Density UP), The impact of leaks on the flow-regime map was assessed. This study was complemented by employing various Machine Learning (ML) models to classify leakage nature.

On the other hand, Bueno et al. (2014)) [18] performed a numerical model of stratified two-phase flow in a nearly horizontal pipeline with a leak, emphasising the effects of leak size and placement on pressure loss. Tavares (2014) utilised CFD-based simulations (Euler-Euler model) to investigate leakage in liquid-gas two-phase flow within a horizontal pipe. Their study contained and covered an analysis of pressure, volume fraction, and velocity variation. Adegboye et al. (2021)) [6] conducted a numerical modelling to investigate the dynamics of liquid-gas two-phase flow with leakage in a subsea natural gas pipeline. The simulation results have been validated against the latest experimental and numerical data from the literature.

Ferroudji et al. (2024) 14 formulated a three-dimensional numerical model utilising the Volume Of Fluid (VOF) approach with the $k-\varepsilon$ turbulence model to investigate the concurrent leakage events (first leak of 3 mm and second leak of 1.8 mm) in a pipe conveying a multiphase flow. The numerical outputs are validated by experimental data from a laboratory flow loop setup, demonstrating a significant level of agreement. Ren et al. (2024)) [7] conducted a numerical analysis of a leaky pipeline conveying multiphase flow under plug flow conditions. The authors examined the impact of superficial velocities of liquid and gas, as well as the axial position of leaks, on leakage flow and bubble morphology.

In terms of numerical modelling, most of the previous studies focused on stratified flow due to its simplicity compared to intermittent flow. For that reason, more attention needs to be paid to leakage modelling in pipelines conveying multiphase flows under intermittent flows, particularly with the enhancement of the treatment and storage capacity of super-calculators. Subsequently, the current investigation focuses on the identification of leakage occurrence in a pipeline carrying, considering various approaches such as, Probability Density Function (PDF), Wavelet Transform (WT), Empirical Mode Decomposition (EMD), and Machine Learning (ML). This study can be considered as an extension of the following works [14, 19].

2. Materials and methods

2.1. Numerical modelling

The multiphase flow with leaks is modelled using the Volume of Fluid (VOF) approach. According to previous studies (Sun and Sakai 2016; Zahedi et al. 2014) [20, 21]., this model will provide accurate results for simulating multiphase flows (water-air) in a horizontal pipeline.

This method characterises the flow domain using a single momentum equation, whereas the multiphase system is defined by correlations among the volume fractions of each phase and the parameters of the momentum equation, including density and viscosity (Ansys-Fluent 2011) [22]. The equation for mass conservation is expressed as follows:

$$\frac{\partial P}{\partial t} + \nabla . \left(\rho \vec{v}\right) = 0$$

where \vec{v} represents the velocity vector, P is the pressure, and ρ stands for the density.

The mass conservation equation in its general form can be used to describe an incompressible fluid flow. Moreover, the momentum conservation can be written as:

$$\frac{\partial P}{\partial t}(\rho \vec{v}) + \nabla (\rho \vec{v} \vec{v}) = -\nabla p + \nabla (\bar{\tau}) + \rho \vec{g} + \vec{F}$$

where \vec{g} and \vec{F} are the gravitational body force and external body force, respectively. The stress tensor $(\bar{\tau})$ is defined as follows:

$$\bar{\tau} = \mu \left[(\nabla \vec{v} + \nabla \vec{v}^T) - \frac{2}{3} \nabla \cdot \vec{v} I \right]$$

where I represents the unit tensor, μ is the molecular viscosity, and the impact of volume dilation is taken into consideration by the second component on the right-hand side.

Alternatively, the turbulence in the multiphase flow is modelled using the " $k - \varepsilon$ " theory, where the following equations represent the kinetic energy "k" and its rate of dissipation " ε ":

$$\frac{\partial}{\partial t}(\rho k) + \frac{\partial}{\partial x_i}(\rho k u_i) = \frac{\partial}{\partial x_j} \left[\left(\mu + \frac{\mu_t}{\sigma_k} \right) \frac{\partial k}{\partial x_j} \right] + G_k - \rho \varepsilon + S_k \tag{4}$$

and

$$\frac{\partial}{\partial t}(\rho\varepsilon) + \frac{\partial}{\partial x_i}(\rho\varepsilon u_i) = \frac{\partial}{\partial x_j} \left[\left(\mu + \frac{\mu_t}{\sigma_\varepsilon} \right) \frac{\partial\varepsilon}{\partial x_j} \right] + G_{1\varepsilon}G_k \frac{\varepsilon}{k} - G_{2\varepsilon}\rho \frac{\varepsilon^2}{k} + S_{\varepsilon}$$
(5)

The turbulent viscosity, μ_t is estimated by considering the following relationship:

$$\mu_t = \rho C_\mu \frac{k^2}{\varepsilon} \tag{6}$$

 G_k is a term that accounts for the amount of kinetic energy that turbulence produces as a result of mean velocity gradients, as follows:

$$G_k = -\rho \overline{u'_i u'_j} \frac{\partial u_j}{\partial x_i} \tag{7}$$

 $G_{1\varepsilon}$, $G_{2\varepsilon}$ and C_{μ} are constant. S_k and S_{ε} are user-define source terms. σ_k and σ_{ε} are the turbulent Prandtl numbers for k and ε , respectively. All of these constants have the following default values in the standard $k - \varepsilon$ model (Launder and Spalding, 1972):

$$G_{1\varepsilon} = 1.44, G_{2\varepsilon} = 1.92, C_{\mu} = 0.09, \sigma_k = 1, \sigma_{\varepsilon} = 1$$

2.2. Mesh generation and numerical details

This study focuses on Air-water two-phase flow through a horizontal pipe of 0.0508 m of diameter and 3 m of length with leakage in the middle region, where the flow phenomena of the inside pipe and surrounding area are considered (Fig. 1). In this context, the generated mesh is designed to match the experimental set up in the Lab for comparison purposes. On the other hand, outputs are recorded at four different points situated in the upstream and downstream regions of the pipe and separated by a distance of 0.4 m.



Fig. 1. Flow domain of the pipe.

To generate accurate results, mesh sensitivity is performed for various meshes considering extreme operation conditions of gas and liquid superficial velocities ($V_{SG} = 0.4 \text{ m/s}$ and $V_{SG} = 3 \text{ m/s}$). Fig. 2 shows that the optimum number of elements to ensure the independency of generated results is 4×10^5 elements. However, a mesh of 5×10^5 elements is adopted to capture strong gradients in the flow domain as well as to meet y+ requirements.



Fig. 2. Mesh sensitivity for P1 and P2.

For the boundary conditions, a pressure outlet is adopted for the outlet, and mass flow rate at the inlet is applied for both water and air inlets (gas and liquid superficial velocities vary from 0.1 to 0.4 m/s and from 1 to 3 m/s, respectively, to produce plug flow regime in the main pipe). Additionally, the water tank is assumed to be full of water to mimic underwater conditions. Additionally, input parameters of the numerical approach are exhibited in Table 1.

| I able 1. Parameters of numerical simulations | | | |
|---|--|--|--|
| Parameter | Selected option | | |
| Solver | Pressure-based - Transient time scheme | | |
| Time step | 0.0001 s | | |
| Multiphase Model | Volume Of Fluid (VOF) - Sharp Interface Modeling | | |
| Phase-Interaction | Surface Tension Force Modelling (CSF) | | |
| | Constant Surface Tension Coefficient = 0.073 N/m | | |
| Turbulence Boundary | standard $k - \varepsilon$ model | | |
| Conditions | Inlet: imposed mass flow rate – Outlet: Outflow | | |
| | Walls: Non-slip conditions | | |
| Material Properties | Water: $\rho_w = 998.2 \ kg/m^3$, $\mu_w = 0.001 \ kg/(m \cdot s)$ | | |
| | Air: $\rho_A = 1.2 \ kg/m^3$, $\mu_A = 1.7894e - 05 \ kg/(m \cdot s)$ | | |
| Solution methodology | Coupling of pressure-velocity: PISO | | |
| | Pressure Spatial Discretization: PRESTO! | | |
| | Volume Fraction Spatial Discretization: Geo-reconstruct | | |
| | Gradients: Least-squares cell-based | | |
| | Other Variables: first and second-order upwind | | |

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2.3. Validation against experimental data

The pressure drop gradient in the upstream section is taken into account to compare numerical outputs with experimental data, as can be observed in Fig. 4. With a mean error of roughly 10%, the calculated numerical findings and the experimental data exhibit an acceptable fit, where, the disparity can be ascribed to the numerical model's simplifications, including those in the valves that control leaks. The capacity of the numerical model to produce accurate outputs and forecast the gas-liquid flow in a leaky pipeline is then validated.



Figure 3. Schematic representation of the experimental setup.



Fig.4. Validation of the numerical results against experimental data (pressure drop gradient).

3. Results and discussion

Pressure variation during a flow time of 15 seconds is recorded in positions P_1 , P_2 , P_3 , and P_4 , as can be seen in Figure 5. As can be observed, the multiphase flow takes around 2 seconds to be fully developed in the main pipe from its entrance. On the other side, the last 8 seconds of the flow time are considered for the investigation to ensure that the flow is under fully developed conditions.



Figure 5. Pressure variation during flow time in positions P1, P2, P3, and P4.

To investigate more about the effect of a leaky pipeline carrying multiphase flow, the gas phase void fraction is determined in both upstream and downstream regions of the main pipeline. To consider the transient nature of the plug flow, the gas void fraction is estimated to be over 8 seconds of the time flow (from 7s to 15s) using the flowing formula:

$$\alpha = \frac{1}{V} \int_{7}^{15} V_i \, dt$$

where α stands for the void fraction of the gas phase, V is the pipeline total volume, V_i represents the equivalent gas phase volume at the instant (t).

Moreover, the leakage effect is evaluated in terms of the percentage of gas void fraction decrease from the upstream to downstream region (β), as follows:

$$\beta = \frac{\alpha_{Up} - \alpha_{Down}}{\alpha_{Up}} \times 100$$

where α_{Up} and α_{Down} represent the gas volume fraction in the upstream to downstream regions, respectively.

For instance, $\beta = 100\%$ means that the totality of the gas phase is released through the leakage. Figure 6 outlines the percentage of gas void fraction decrease (β) for various gas and liquid superficial velocities considering different leakage scenarios, including single leakage of 6 mm, single leakage of 15 mm, double leakage of 6 mm, and double leakage of 15 mm. As can be seen, the lowest value of the V_{SG} shows the highest percentage of gas void fraction decrease for all the leakages scenarios, indicating that all the gas amount in the main pipeline is released through the leakage, particularly for the cases of single leakage of 15 mm, double leakage of 6 mm, and double leakage of 15 mm. However, with the increase of the V_{Sg} , the percentage of gas void fraction decrease (β) diminishes, particularly for high values of the V_{SL} where this effect is more prominent in the case of a single leakage of 6 mm. This behaviour can be explained by the fact that once the flow regime map is close to the bubbly flow (high values of liquid and gas superficial velocities), most of the gas amount is carried within the main pipeline (Ferroudji et al. 2025). This indicates that in the case of a leaky pipeline carrying multiphase flow, higher flowrates gas and liquid phases are required to reduce gas release through leakages. On the other hand, it is noticed that the case of a double leakage of 6mm shows a higher gas release compared to the case of a single leakage of 15mm.



Figure 6. Behaviour of the percentage of gas void fraction decrease as a function of the liquid superficial velocity for various leakage scenarios.

Moreover, standard deviation, Wavelet Transform (WT), and Empirical Mode Decomposition (EMD) methods were applied to the obtained time series signals. As can be seen, the calculated root mean square for the Empirical Mode (Decomposition

Intrinsic Mode Functions (IMF1)) shows better performance to identify leakage occurrence in a horizontal pipe conveying multiphase flow.

| Method | Mean deviation percentage from the reference case (no leakage) |
|-----------------------|---|
| Standard deviation | 77.78 % |
| CWT coefficients mean | 78.15 % |
| DWT RMS approximate | 64.75 % |
| DWT RMS details | 89.34% |
| EMD RMS IMF1 | 73.56 % |
| EMD RMS IMF2 | 69.19 % |
| EMD RMS IMF1 | 92.15 % |
| EMD RMS IMF2 | 82.56 % |

Table 2. Comparison between various methods.

The influence of input features on model performance is depicted in **Table 3**. As the complexity of input data increases (e.g., inclusion of P3, P4, DP, and NF1), test accuracy significantly improves leak detection. Group #6, which incorporates all available features, achieves the highest accuracy of 0.9870 for leak detection. The addition of derived features such as NF1 provides valuable insights into flow dynamics, enhancing classification accuracy for leakage identification.

Table 3. Test accuracy for leak detection across varying input configurations and groupings.

| Input(s) | Group # | Test accuracy (Leak detection) |
|---|---------|-----------------------------------|
| V _{SL} , V _{SG} | 1 | 0.6067 |
| V_{SL} , V_{SG} , P_3 | 2 | 0.5353 |
| V_{SL} , V_{SG} , P_3 , P_4 | 3 | 0.7265 |
| V_{SL} , V_{SG} , P_3 , P_4 , DP | 4 | 0.5952 |
| V _{SL} , V _{SG} , P ₄ , DP, NF1 | 5 | 0.9600 |
| V _{SL} , V _{SG} , P ₃ , P ₄ , NF1 | 6 | 0.9870 |

4. Conclusion

- The following points outline the main concluding remarks:
- The multiphase flow (plug flow) in a horizontal pipe with leakage is successfully modelled.
- The Empirical Mode Decomposition Method (EMD) shows better performance in identifying leakage occurrence in a horizontal pipe conveying multiphase flow, considering the root mean square of the Intrinsic Mode Function (IMF1).
- The addition of derived features such as NF1 (new features) provides valuable insights into flow dynamics, enhancing classification accuracy for leakage identification of the machine learning models.

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