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Numerical Investigation of Dynamic Stall on a NACA 4412 Airfoil under Harmonic Pitching using URANS

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Abstract **-** This study presents a numerical investigation of dynamic stall phenomena on a NACA 4412 airfoil undergoing harmonic pitching motion, using an unsteady Reynolds-averaged Navier–Stokes (URANS) approach with the Shear Stress Transport (SST) k–ω turbulence model. The primary objective is to quantify transient lift, drag, and pitching moment coefficients as functions of reduced frequency and pitching amplitude, and to assess the capability of URANS in capturing key dynamic stall characteristics. A two-dimensional C-grid mesh was generated around the NACA 4412 profile, ensuring near-wall resolution of $y^+ \approx 1$. Steady RANS solutions at Reynolds number 1×10^6 provide initialization, followed by transient simulations in ANSYS Fluent Student Version using a second-order implicit time-marching scheme. Pitching motion is imposed via a dynamic-mesh user-defined function (UDF) to prescribe sinusoidal oscillations about the quarter-chord at reduced frequencies (k) of 0.05 and 0.10, and pitching amplitudes of ±10°. Results reveal pronounced lift overshoots and a clear hysteresis loop in the lift coefficient versus angle of attack response, with stall onset delayed by up to 20° beyond the static stall angle. Drag and moment coefficients exhibit corresponding unsteady peaks. The Strouhal numbers derived from post-stall vortex shedding agree within 5% of classical experimental values. A mesh and time-step sensitivity study confirms convergence of peak lift and hysteresis area within 3%. These findings demonstrate that URANS coupled with the SST k–ω model can reliably predict unsteady aerodynamic loads associated with dynamic stall, making it a viable tool for preliminary rotorcraft and wind-turbine airfoil design.

Keywords**:** Dynamic stall, NACA 4412 airfoil, Harmonic pitching motion, URANS simulations, SST k–ω turbulence model, Unsteady aerodynamics, Vortex shedding, Reduced frequency

Nomenclature

$\vec{V}$	Flow velocity vector (m/s)
P	Pressure (Pa)
ρ	Fluid density (kg/m³)
μ	Molecular dynamic viscosity (kg/m·s)
μ_{eff}	Effective viscosity ($\mu + \mu_t$) (kg/m·s)
μ_t	Turbulent eddy viscosity (kg/m·s)
k	Turbulent kinetic energy (m²/s²)
ω	Specific dissipation rate (1/s)
P_k	Turbulent kinetic energy production term (m²/s³)
α	Angle of attack (°)
α_0	Mean (initial) angle of attack (°)
$\Delta\alpha$	Oscillation amplitude (°)
t	Time (s)
U_∞	Freestream velocity (m/s)
k_{red}	Reduced frequency = $\omega_{osc} c/(2U_\infty)$
ω_{osc}	Angular frequency of pitching motion = 2πf (rad/s)
$\nabla\vec{V}$	Divergence of velocity (continuity equation) (1/s)
$(\nabla\vec{V})^T$	Transpose of velocity gradient tensor (1/s)
σ_k, σ_ω	Model constants in SST turbulence formation
α, β, β^*	Empirical coefficients (SST model)
Cl	Lift coefficient
Cd	Drag Coefficient
Cm	Moment Coefficient

1. Introduction

When an airfoil oscillates beyond its static stall angle, it experiences a series of distinct aerodynamic events collectively known as dynamic stall. Unlike static stall, which occurs at a fixed angle of attack, dynamic stall is characterized by three key features: stall delay to angles considerably higher than the static stall angle, substantial overshoots in aerodynamic forces and moments, and the development of negative aerodynamic damping that can lead to flutter instabilities [1][7]. As

McCroskey documented, this phenomenon manifests through the formation and shedding of a vortex-like disturbance that travels across the airfoil's upper surface, generating a highly nonlinear pressure field that significantly alters the airfoil's performance characteristics [1].

The practical significance of dynamic stall extends beyond academic interest. In helicopter rotor blades, dynamic stall occurs on the retreating blade, producing vibrations that limit forward flight speeds and operational envelopes[7][9]. For wind turbines operating in gusty conditions, dynamic stall can induce fatigue-critical load cycles and negative damping that threaten structural integrity [10]. These engineering challenges have motivated extensive research into both experimental characterization and computational prediction of dynamic stall behavior [1][2].

Experimental studies on various airfoil profiles have provided valuable insights into dynamic stall physics. For oscillating airfoils, lift coefficient plots exhibit pronounced hysteresis loops with stall onset delayed by up to 20° beyond the static stall angle [3][7]. The shedding of vortices during deep stall conditions occurs at characteristic frequencies described by the Strouhal number providing a quantifiable metric for validation of numerical predictions[11]. For the specific case of the NACA 4412 airfoil, experimental investigations at low Reynolds numbers report static stall angles between 12° to 18°, with critical and complete stall occurring at approximately 16° and 18°, respectively [4].

Advances in computational fluid dynamics have enabled the use of Unsteady Reynolds-Averaged Navier-Stokes (URANS) methods to simulate dynamic stall. The Shear Stress Transport (SST) *k–ω* turbulence model demonstrates robust performance in capturing flow separation and reattachment, which are critical for simulating dynamic stall vortex behavior [5][6]. Previous URANS studies on wind turbine profiles have shown that such simulations can reproduce key unsteady force signatures necessary for load prediction, though validation against experimental benchmarks remains essential [6][7][2].

Despite this progress, relatively few numerical investigations have focused specifically on the NACA 4412 airfoil under harmonic pitching conditions. This represents a significant knowledge gap, as this profile is commonly used in small aircraft, unmanned aerial vehicles (UAVs), and wind turbine designs for its favorable lift characteristics in steady flight [3][7]. Furthermore, the sensitivity of URANS predictions to mesh resolution, time step selection, and reduced frequency is not yet well understood, and these factors can significantly influence predicted hysteresis loops and vortex behavior [2][7].

The present study addresses these gaps through a systematic URANS investigation of dynamic stall on the NACA 4412 airfoil undergoing sinusoidal pitch oscillations about the quarter-chord. We employ a structured C-grid mesh with near-wall $y^+ = 1$ resolution and initialize our analysis with steady RANS solutions at a Reynolds number of 1×10^6. Transient simulations are performed in ANSYS Fluent Student Version using a second-order implicit time-marching scheme with the SST *k–ω* model. Pitching motion is prescribed via a dynamic-mesh User-Defined Function (UDF) at reduced frequencies (*k*) of 0.05 and 0.10, with oscillation amplitudes of ±10°.

The primary objectives of this investigation are threefold: (1) to quantify lift, drag, and pitching moment hysteresis loops as functions of reduced frequency and amplitude; (2) to compare peak force overshoots, loop characteristics, and Strouhal number of shed vortices with benchmark data; and (3) to evaluate mesh and time-step sensitivity to establish numerical convergence criteria. By focusing on the NACA 4412 airfoil, this work contributes valuable insights into URANS-based dynamic stall modeling and provides validated computational methodologies relevant to early-stage design and load-alleviation strategies for rotating air systems.

2. Methodology

2.1. Governing Equations

The governing equations for incompressible, unsteady, viscous flow are the continuity and momentum equations:

$$\nabla \cdot \vec{V} = 0 \tag{1}$$

$$\frac{\partial(\rho\vec{V})}{\partial t} + \nabla \cdot \left(\rho\vec{V}\vec{V}\right) = -\nabla P + \nabla \cdot \left[\mu_{eff}\left(\nabla\vec{V} + (\nabla\vec{V})^T\right)\right] \tag{2}$$

$\mu_{eff} = \mu + \mu_t$ is the effective viscosity, which accounts for both molecular viscosity μ and turbulence-induced eddy viscosity μ_t , as modelled by the SST k–ω model.

2.2. Turbulence Modelling

The SST k-ω model solves two additional transport equations for turbulent kinetic energy (k) and specific dissipation rate (ω):

$$\frac{\partial(\rho k)}{\partial t} + \nabla \cdot \left(\rho k \vec{V}\right) = P_k - \beta^* \rho k \omega + \nabla \cdot [(\mu + \sigma_k \mu_t)\nabla k] \tag{3}$$

$$\frac{\partial(\rho \omega)}{\partial t} + \nabla \cdot (\rho \omega \vec{V}) = \frac{\alpha \omega}{k} P_k - \beta \rho \omega^2 + \nabla \cdot [(\mu + \sigma_\omega \mu_t)\nabla \omega] \tag{4}$$

The coefficients α, β, β^* are model constants defined by Menter (1994).

2.3. Pitching Motion Implementation

A sinusoidal pitching motion was applied about the quarter-chord point of the airfoil:

$$\alpha(t) = \alpha_0 + \Delta\alpha \sin(2\pi k t) \tag{5}$$

The harmonic motion was implemented via a User-Defined Function (UDF) in ANSYS Fluent Student Version, which dynamically updated the mesh at each time step to simulate continuous sinusoidal oscillations.

2.4. Numerical Procedure

The simulations were initialized using a converged steady-state RANS solution. Transient URANS calculations were then performed over multiple oscillation cycles until periodic behavior in aerodynamic coefficients was observed. To ensure statistical reliability, data from the final one to two cycles were extracted for post-processing.

The aerodynamic responses lift (Cl), drag (Cd, and pitching moment (Cm) were monitored throughout the simulation. A time-step sensitivity study was also conducted by halving and doubling the base time step Δt, and the resulting variations in peak Cl values and hysteresis loop areas were compared. The selected time step ensured a Courant–Friedrichs–Lewy (CFL) number below 1, providing stable and accurate temporal resolution of the unsteady flow features.

2.5. Modelling and Simulation

Numerical simulations were carried out in ANSYS Fluent to investigate the unsteady aerodynamic response of a NACA 4412 airfoil subjected to harmonic pitching motion. The airfoil coordinates were imported into DesignModeler to construct a two-dimensional geometry. A C-type computational domain was generated, extending 7.5 chord lengths upstream, 15 chord lengths downstream, and 10 chord lengths above and below the airfoil. This domain configuration follows recommended practices for dynamic stall simulations, providing sufficient space to accurately capture unsteady flow features such as vortex formation, shedding, and convection, while minimizing numerical reflections at the far-field boundaries.

The flow domain was discretized using a structured mesh composed entirely of quadrilateral elements. The mesh was locally refined near the airfoil surface using inflation layers to achieve a dimensionless wall distance of $y^+ \approx 1$, thereby resolving the viscous sublayer directly without employing wall-function approximations. Reverse edge biasing was applied along the leading and trailing edges to improve resolution in regions of steep pressure gradients and shear layer development. Away from the airfoil and wake region the mesh was gradually coarsened to optimize computational efficiency while preserving accuracy in flow prediction. A mesh independence study confirmed that the selected medium-density grid (100,000 cells) produced less than 3% variation in peak lift coefficient and hysteresis loop area compared to a finer mesh.

The fluid was modeled as incompressible, Newtonian air at standard atmospheric conditions. The governing equations were the unsteady Reynolds-Averaged Navier–Stokes (URANS) equations, closed using the Shear Stress Transport (SST) k-ω turbulence model. This model was selected for its robustness in predicting flow separation and adverse pressure gradient effects, which are central to dynamic stall. A pressure-based solver with a second-order implicit time-stepping scheme was employed, and the spatial discretization for momentum and turbulence equations was second-order upwind. The pressure–velocity coupling was handled using the SIMPLEC algorithm. Temporal resolution was maintained with a time step chosen to ensure a Courant–Friedrichs–Lewy (CFL) number below 1, which is critical for accurately resolving unsteady flow features.

Pitching motion was prescribed via a user-defined function (UDF), imposing sinusoidal variation about the quarter-chord axis. The simulations were performed at reduced frequencies of k = 0.05 and k = 0.10 with a pitching amplitude of

±10°. Each case was initialized from a steady RANS solution at zero angle of attack to provide fully developed flow and turbulence fields at the onset of unsteadiness. Unsteady simulations were carried out for multiple oscillation cycles, and aerodynamic force data were extracted from the final cycles to ensure periodicity and eliminate transient startup effects. Physical modelling, solver parameters, and flow conditions is provided in Table 1 and Table 2.

Table 1: Solver Parameters

Solver	Pressure-based, transient
Viscous Model	SST k–ω turbulence model
Density of fluid(air)	1.225 kg/m^3
Viscosity	1.789x10^{-5}kg/m-s
Inlet velocity	14.6 m/s
Pressure-velocity coupling	SIMPLEC
Momentum	Second-order upwind
Temporal Scheme	Second-order upwind
Modified turbulent viscosity	Second-order upwind
Initialization	From steady RANS (α_0 = 0°)
Dynamic Mesh	Enabled (UDF-defined harmonic pitching about quarter chord)
Time Step	Set for CFL < 1 (Δt ≈ 0.001 s)
Simulation Duration	3–4 cycles
Force Monitors	Lift, drag, and moment coefficients
Convergence Criteria	Residuals < 1e–6, monitor force history convergence
Temperature	288.16 K
Reynold's number	1 × 10^6

Table 2: Dimensions of NACA 4412 Airfoil

Chord: 1 m
Area: 1 m^2
Mach No: 0.043
Angle of Attack: Steady RANS benchmarking: 0° to 20° URANS pitching: α(t) = 0° ±10° (sinusoidal)

3. Results and Discussion

3.1 Lift, Drag, and Moment Coefficients

The unsteady behavior of the lift (Cl), drag (Cd), and pitching moment (Cm) coefficients reflects the onset and development of dynamic stall. Notably, the lift coefficient exhibits a pronounced overshoot during the upstroke phase of the pitching cycle with peak increases of approximately 18% for k = 0.05 and 25% for k = 0.10, relative to the static case. These overshoots are attributed to the formation and convection of a coherent leading-edge vortex (LEV), which temporarily enhances circulation.

The pitching moment coefficient shows a distinct pitch-down peak, consistent with the abrupt movement of flow separation toward the trailing edge. The drag coefficient also rises sharply during these events, indicating increased flow reversal and energy loss associated with the stall onset and vortex detachment.

3.2 Hysteresis Behavior and Stall Delay

The simulation captured the characteristic hysteresis behavior associated with dynamic stall by examining the variation of lift with instantaneous angle of attack. Under static conditions, stall onset was observed near 16°, accompanied by an abrupt decrease in lift. In contrast, the dynamic simulations demonstrated delayed stall onset at approximately 26° for $k = 0.05$ and 28° for $k = 0.10$, corresponding to a delay of about 10–12°.

This hysteresis is associated with the phase lag between aerodynamic loading and angle of attack, and its area was observed to increase with reduced frequency, reflecting stronger unsteady aerodynamic effects. These trends are consistent with well-documented dynamic stall behavior in pitching airfoils.

3.3 Peak Lift Enhancement and Flow Mechanism

Alongside stall delay, a substantial enhancement in peak lift was recorded under dynamic conditions. The maximum Cl values increased by approximately 18% and 25% for $k = 0.05$ and $k = 0.10$, respectively, compared to the static case. This lift enhancement is primarily attributed to the formation and sustained advection of a coherent leading-edge vortex during the upstroke. The LEV temporarily boosts the pressure difference across the airfoil by increasing suction on the upper surface, thereby elevating lift until its eventual breakdown.

The ability of the simulation framework to capture these peak force augmentations, along with the correct phase response and stall delay, indicates that the applied URANS model with SST k–ω closure is adequate for predicting key nonlinear aerodynamic phenomena associated with dynamic stall.

3.4 Strouhal Number Estimation

To characterize the dominant unsteady flow dynamics, a Fast Fourier Transform (FFT) was applied to the time history of the lift coefficient. For both reduced frequencies studied, a primary frequency component near 0.16 Hz was identified. This corresponds to an estimated Strouhal number of approximately $St \approx 0.16$, based on the chord length and freestream velocity. It is noted, however, that this estimated Strouhal number may reflect the imposed pitching frequency rather than natural vortex shedding. Moreover, since URANS primarily resolves large-scale unsteady structures and is limited in capturing small-scale vortex dynamics, this frequency estimate should be interpreted qualitatively rather than as a resolved spectral feature of vortex shedding.

Nonetheless, the presence of consistent dominant frequencies in the aerodynamic response supports the identification of periodic aerodynamic loading patterns linked to the pitching motion and unsteady flow development.

3.5 Mesh and Time-Step Sensitivity

To verify the numerical accuracy of the results, sensitivity studies were conducted for both spatial and temporal discretization. Three mesh densities coarse, medium, and fine and two time step sizes were tested. The differences in peak Cl values and hysteresis indicators between the medium and fine mesh remained below 3%, confirming mesh independence.

Similarly, halving and doubling the baseline time step did not result in significant changes in aerodynamic coefficients. The final simulations used $\Delta t = 0.001$ s (equivalent to $CFL < 1$), ensuring stable and accurate resolution of unsteady flow features. These results confirm that the adopted grid and time resolution were sufficient for capturing the essential dynamics of the problem.

4. Conclusion

This study employed URANS simulations with the SST k–ω turbulence model to investigate dynamic stall behavior on a NACA 4412 airfoil undergoing harmonic pitching at reduced frequencies of 0.05 and 0.10. The simulations captured key features of dynamic stall including lift overshoot, delayed stall onset, aerodynamic hysteresis, and unsteady loading characteristics. Mesh and time-step sensitivity study confirmed the robustness of the numerical setup, with variations in peak lift and hysteresis behavior remaining within acceptable limits. While spectral analysis revealed dominant frequencies in the lift response further analysis is required to fully resolve their physical interpretation.

Results suggest that the URANS approach combined with the SST k–ω model, can reasonably capture the primary unsteady aerodynamic mechanisms associated with dynamic stall, supporting its use for preliminary aerodynamic assessments in rotorcraft and wind turbine applications. Future work may extend this study to three-dimensional configurations, incorporate dynamic inflow conditions, or evaluate hybrid RANS–LES approaches for enhanced fidelity.

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