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Effects of Energy Deposition Characteristics in Localised Forced Ignition of Turbulent Homogeneous Mixtures: A Direct Numerical Simulation (DNS) Analysis

Dipal Patel, Nilanjan Chakraborty
School of Mechanical and System Engineering, Newcastle University
Claremont Road, Newcastle Upon Tyne, NE1 7RU, UK
d.patel2@ncl.ac.uk; nilanjan.chakraborty@ncl.ac.uk

Abstract -The effects of the characteristic width of the energy deposition profile and the duration of energy deposition by the ignitor on localised forced ignition of stoichiometric and fuel-lean homogeneous mixtures have been analysed using simplified chemistry three-dimensional compressible Direct Numerical Simulation (DNS) for different values of root-mean-square velocity fluctuation u'. The localised forced ignition is modelled using a source term in the energy transport equation, which deposits energy in a Gaussian manner from the centre of the ignitor over a stipulated period of time. It has been shown that the width of ignition energy deposition and the duration over which ignition energy is deposited have significant influences on the success of ignition and subsequent flame propagation. An increase in the width of ignition energy deposition (duration of energy deposition) for a given amount of ignition energy has been found to have a detrimental effect on the ignition event, which may ultimately lead to misfire. Moreover, an increase in u' augments heat transfer rate from the hot gas kernel, which in turn leads to a reduction in the extent of overall burning and flame propagation rate for both stoichiometric and fuel-lean homogeneous mixtures but the detrimental effects of u' on localised ignition are particularly prevalent for fuel-lean mixture.

Keywords**:** Localised ignition, Homogeneous mixture, Width of energy deposition, Duration of energy deposition, Direct Numerical Simulation.

1. Introduction

Localised forced ignition of homogeneous ignitable mixtures in the form of spark or laser plays an important role in the design of energy-efficient and reliable Spark Ignition and Direct Injection engines. The localised ignition and subsequent flame propagation in homogeneous mixtures have been studied from analytical (He, 2000; Espi *et al*., 2001, 2002; Champion *et al*., 2002;), experimental (Bradley *et al*., 1987; Lefebvre, 1998; Dreizler *et al*., 2000; Beduneau *et al*., 2004; Huang *et al*., 2007) and numerical (Baum *et al*., 1995; Catlin *et al*., 1995; Poinsot *et al*., 1995; Kaminski et al., 2000; Klein et al., 2008) viewpoints by several authors. Interested readers are referred to Mastorakos (2008) and references therein for an extensive review on localised forced ignition of homogeneous mixtures. In recent times, Direct Numerical Simulations (DNS) has become an important tool for fundamental understanding and subsequent modelling of complex combustion phenomena including localised forced ignition (Kaminski *et al*., 2000; Chakraborty *et al*., 2007; Klein *et al*., 2008; Chakraborty and Mastorakos, 2008). In several previous analyses (He, 2000; Espi *et al*., 2001,2002; Chakraborty *et al*., 2007; Chakraborty and Mastorakos, 2008) the thermal effect of the ignitor is modelled using a source term in the energy transport equation, which deposits energy in space a Gaussian manner from the centre of the ignitor over a stipulated duration of time. However, the effects of the characteristic width of energy deposition and the duration of energy deposition on localised forced ignition and early stages of flame propagation following successful ignition are yet to be numerically analysed in detail. In the present study, three-dimensional compressible DNS have been carried out in order to investigate the effects of the characteristic width of energy deposition and the duration of energy deposition on the localised forced ignition of homogeneous stoichiometric and fuel-lean mixtures for different values of root-mean-square (rms) turbulent velocity

fluctuation. The reaction kinetics is simplified here by a single-step irreversible Arrhenius type chemical reaction mechanism. The main objectives of the present study are:

- To demonstrate the effects of the characteristic width and the duration of energy deposition on ignition performance and early stages of combustion for both stoichiometric and fuel-lean homogeneous mixtures for different values of root-mean-square turbulent velocity fluctuation u' based on three-dimensional DNS simulations.
- To provide physical explanations for the aforementioned effects.

2. Mathematical Background

In combustion DNS both the three-dimensionality of turbulence and the detailed chemical structure of flame should be simultaneously taken into account. However, such simulations are still too expensive to carry out for a detailed parametric analysis so three-dimensional DNS with single step chemistry approach has been considered for the present study. The single step chemical mechanism considered here can be summarised as:

$$\text{Fuel} + s \cdot \text{Oxidiser} \rightarrow (1+s) \cdot \text{Products} \tag{1}$$

where s stands for the mass of oxidizer consumed per unit mass of fuel consumption. The fuel reaction rate is given by an Arrhenius-type expression. In the present study, specific heats for all species are taken to be equal and independent of temperature for the purpose of simplification. The Fick's law of diffusion is used to account for the diffusion velocities of different species. The transport quantities such as, viscosity μ, thermal conductivity λ and density weighted mass diffusivity ρD for all the species are considered to be the same and independent of temperature. The Lewis numbers of all species are assumed to be unity (i.e. $Le_F = Le_O = 1.0$). Combustion is assumed to take place in the gaseous phase, where all species are calorically perfect gases, which allows for following state relations:

$$E = \int_{T_{\text{ref}}}^{T^*} C_V \, dT + (u_k u_k/2); \;\; P = \rho \bar{R} T^* \tag{2}$$

where ρ is the gas density, T^* is the instantaneous dimensional temperature, T_{ref} is a reference temperature, E is the total internal energy, P is the pressure, and R is the gas constant. Standard values are chosen for the ratio of specific heats ($\gamma = C_P/C_V = 1.40$), where C_P (C_V) is the specific heat at constant pressure (volume) and Prandtl number ($Pr = \mu C_P/\lambda = 0.7$). Under the above assumptions, the energy conservation equation takes the following form:

$$\frac{\partial}{\partial t}\rho E + \frac{\partial}{\partial x_k}\rho u_k E = -\frac{\partial}{\partial x_k}u_k P + \frac{\partial}{\partial x_k}\tau_{ki}u_i + \frac{\partial}{\partial x_k}\left(\lambda \frac{\partial \hat{T}}{\partial x_k}\right) + \dot{w}_T + q''' \tag{3}$$

where τ_{ki} is the viscous shear stress, and $\dot{w}_T$ is the source term originating from heat release due to combustion, which are given by:

$$\tau_{ij} = \mu\left[\left(\partial u_i/\partial x_j\right) + \left(\partial u_j/\partial x_i\right)\right] - \delta_{ij}(2/3)[(\partial u_k/\partial x_k)]; \;\; \dot{w}_T = H|\dot{w}_F| \tag{4}$$

where $\dot{w}_F$ is the fuel reaction rate. The source term q''' in Eq. (3) accounts for heat addition by the ignitor over the energy deposition period. The source term q''' is assumed to follow a Gaussian distribution in the radial direction from the center of the ignitor (He, 2000; Espi *et al*., 2001, 2002; Chakraborty *et al*., 2007, Chakraborty and Mastorakos, 2008):

$$q'''(r) = A_q \exp(-r^2/2R^2) \tag{5}$$

In Eq. (4) $|\dot{w}_F|$ is the fuel reaction rate magnitude and H is the heat release per unit mass of fuel consumption. The source term q''' in Eq. (3) accounts for heat addition by the ignitor over the energy deposition period. The source term q''' is assumed to follow a Gaussian distribution in the radial direction from the centre of the ignitor (He, 2000; Espi *et al*., 2001, 2002; Chakraborty *et al*., 2007, Chakraborty and Mastorakos, 2008):

$$\dot{Q} = \int_V q''' \, dV \tag{6}$$

where $\dot{Q}$ is the ignition power. The ignition power is given by (Chakraborty *et al*., 2007, Chakraborty and Mastorakos, 2008):

$$\dot{Q} = a_{sp}\rho_0 C_P \tau T_0 (4/3)\pi {l_f}^3 [\{H(t-t_1) - H(t-t_2)\}/(t_2 - t_1)] \tag{7}$$

where a_{sp} is a parameter determining the total energy input, $l_f = D_0/S_{b(\phi=1.0)}$ is the Zel'dovich flame thickness with D_0 and $S_{b(\phi=1.0)}$ being the mass diffusivity of unburned gases, and the laminar burning velocity of the stoichiometric mixture respectively. In Eq.(7) $H(t-t_1)$ and $H(t-t_2)$ are two Heaviside functions, which ensure that the ignitor remains operational only during $t_1 < t < t_2$, where t_1 is taken to be 0 for the current analysis. The time instants t_1 and t_2 determine the ignition duration t_{sp} in following manner: $t_{sp} = t_2 - t_1 = b_{sp} t_f$ where b_{sp} is a parameter associated with the energy deposition by the ignitor and $t_f = l_f/S_{b(\phi=1.0)}$ is a characteristic chemical time scale. Experimental findings by Ballal and Lefebvre (1977) indicated that b_{sp} varies between $0.2 < b_{sp} < 0.4$ for optimum spark duration. The thermal aspect of ignition is modelled with the help of Gaussian source term in the energy transport equation in previous studies (He, 2000; Espi *et al*., 2001,2002; Chakraborty *et al*., 2007; Lou *et al*., 2007; Chakraborty and Mastorakos, 2008, Böker *et al*., 2011; Dharamshi *et al*., 2013). For the present analysis both stoichiometric (i.e. ϕ=1.0) and fuel-lean (e.g. ϕ=0.8) mixtures have been considered where the equivalence ratio ϕ is defined as:

$$\phi = (\text{AFR})/(\text{AFR})_{st} \tag{8}$$

In Eq. (8) AFR and $(\text{AFR})_{st}$ are the actual air-fuel ratio and stoichiometric air-fuel ratio respectively. Here $(\text{AFR})_{st}$ is taken to be 17.16, which yields $Y_{st} = 0.055$ and $s = 4$ (where oxygen is considered as oxidizer in Eq. 1). These values represent methane-air binary mixture. For the present analysis $\tau = (T_{ad(\phi=1)} - T_0)/T_0 = 3$ has been considered, where T_0 is the unburned reactant temperature and $T_{ad(\phi=1)}$ is the adiabatic flame temperature for the stoichiometric mixture. The simulations in this analysis have been carried out using a three-dimensional compressible DNS code SENGA (Chakraborty *et al*., 2007; Chakraborty and Mastorakos, 2008) where the conservation equation of mass, momentum, energy and mass fractions of fuel and oxidiser are solved in dimensionless form. High order finite difference and Runge-Kutta schemes are used for spatial discretisation and time advancement respectively. The boundaries in the $x_1 -$ direction are taken to be partially non-reflecting whereas the transverse directions are considered to be periodic. The simulation domain is taken to be a cube of the size $33l_f \times 33l_f \times 33l_f$ and the simulation domain is discretised by a Cartesian grid of size $200 \times 200 \times 200$ with uniform grid spacing Δx. The ignitor is placed at the geometrical centre of the domain for all cases considered here. For all the simulation, the grid spacing Δx is smaller than Kolmogorov length scale η (i.e. η=1.2Δx). The grid resolution is determined by the resolution of flame structure. In all the cases about 10 grid points are placed within the thermal thickness of stoichiometric unstrained laminar flame δ_{th} which is defined as.

$$\delta_{th} = (T_{ad(\phi=1)} - T_0)/[\max(|\nabla T^*|_L)] \tag{9}$$

The desired simulation time for decaying turbulence can be summarised as: $t_{\text{sim}} = \max(t_e, t_f)$ where $t_e = L_{11}/\sqrt{k}$ is the initial eddy turn over time, where L_{11} is the longitudinal integral length scale of turbulence, and k is the initial turbulent kinetic energy. For all turbulent cases the initial normalised value of integral length scale was taken to be $L_{11}/l_f = 3.36$ following previous analyses (Chakraborty *et al*., 2007, Chakraborty and Mastorakos, 2008). For the purpose of comparison, statistics for all cases are presented for a time $t = 1.68t_f$, which corresponds to about $2t_e$ and $3t_e$ for initial values of $u'/S_{b(\phi=1.0)} = 4.0$ and $u'/S_{b(\phi=1.0)} = 6.0$ respectively. It will be shown in the next section of this paper that one can infer whether a particular case will give rise to self-sustained combustion following successful ignition by the time $t = 1.68t_f$.

4. Results and Discussions

The effects of the characteristic width of energy deposition, duration of energy deposition and ignition energy has been analysed by the parametric variations of R, b_{sp} and a_{sp} respectively. The parametric variations of R, b_{sp} and a_{sp} for both stoichiometric and fuel-lean mixtures are summarised in Table 1. The temporal evolution of the maximum values of non-dimensional temperature (i.e. $T_{\max}$) and the normalized fuel reaction rate magnitude (i.e. $|\dot{w}_F|_{\max} \times l_f/\rho_0 S_{b(\phi=1.0)}$) for the cases in Table 1 are shown in Fig. 1 where non-dimensional temperature is defined as $T = (T^* - T_0)/(T_{ad(\phi=1.0)} - T_0)$ ($T = (T^* - T_0)/(T_{ad(\phi=0.8)} - T_0)$) for ϕ=1.0 (ϕ=0.8) mixture. Figure 1 demonstrates that $T_{\max}$ rises with time due to energy deposition during $t < t_{sp}$ and thermal runaway takes place when $T_{\max}$ attains a value close to $T_c \approx 1 - (1/\beta)$ (where $\beta = T_{ac}(T_{ad} - T_0)/T_{ad}^2$ is the Zel'dovich number and T_{ac} is the activation temperature) and at this point both $T_{\max}$ and $|\dot{w}_F|_{\max} \times l_f/\rho_0 S_{b(\phi=1.0)}$ increase rapidly with time. Once the energy deposition is switched off, the high thermal gradient between the hot gas kernel and the surrounding unburned gas gives rise to high rate of heat transfer from the hot gas kernel. This, in turn, leads to decrease in $T_{\max}$ with time once the ignitor is switched off but $|\dot{w}_F|_{\max} \times l_f/\rho_0 S_{b(\phi=1.0)}$ starts to decrease before the energy deposition is stopped (see Fig. 1). The maximum value of $|\dot{w}_F|_{\max} \times l_f/\rho_0 S_{b(\phi=1.0)}$ rises with time during the energy deposition period until fuel is available at the maximum temperature location to support chemical reaction but once the fuel is consumed at this location $|\dot{w}_F|_{\max} \times l_f/\rho_0 S_{b(\phi=1.0)}$ decreases with time but $T_{\max}$ continues to rise until $t = t_{sp}$ due to external energy addition. The value of $T_{\max}$ approaches the non-dimensional adiabatic flame temperature (i.e. $T \approx 1.0$) and $|\dot{w}_F|_{\max} \times l_f/\rho_0 S_{b(\phi=1.0)}$ settles to a non-negligible value, which no longer changes appreciably with time for $t \gg t_{sp}$ in the cases, where self-sustained combustion is obtained. Figure 1 shows that successful ignition has been obtained for all cases except in the ST0A, ST4A, ST4G, LT0A, LT4A, LT4B, LT4C, LT4D, LT4E, LT4F, LT0G, LT4G, LT4H and LT4I cases.

The conditions, which led to self-sustained combustion following successful ignition, depend on selection of the characteristic width of energy deposition R, duration of external energy deposition (or b_{sp}) and ignition energy (or a_{sp}). Self-sustained flame propagation has been obtained for all the quiescent cases (i.e. $u'/S_{b(\phi=1.0)} = 0.0$) except in the ST0A, LT0A, LT0G cases (see Fig. 1). For the cases LT4B, LT4C, LT4D, LT4E, LT4F, ST4G, LT0G, LT4G, LT4H and LT4I $T_{\max}$ decreases continuously for $t > t_{sp}$ and $|\dot{w}_F|_{\max} \times l_f/\rho_0 S_{b(\phi=1.0)}$ drops sharply once $T_{\max} < [1 - (1/\beta)]$. It is evident from Fig. 1 that the cases with $a_{sp} = 2.7$ are more susceptible to flame extinction than the cases with $a_{sp} = 3.5$ and 4.3 , which is consistent with the expectation that the probability of obtaining successful ignition and self-sustained combustion increases with increasing ignition energy for a given set of values of R and b_{sp}. It has been found that the ϕ=0.8 cases are more susceptible to flame extinction without the help of external energy addition than the ϕ=1.0 cases (compare cases ST4B, ST4C, ST4D, ST4E, ST4F, ST0G, ST4H and ST4I with cases LT4B, LT4C, LT4D, LT4E, LT4F, LT0G, LT4H and LT4I respectively) as the rate of heat release due to chemical reaction is weaker in the ϕ=0.8 cases than in the ϕ=1.0 cases.

Table 1. List of parameters used to analyse the effects of R, a_{sp} and b_{sp}.

			ϕ=1.0 (S)		ϕ=0.8 (L)	
			$u'/S_{b(\phi=1)} = 0$ (T0)	$u'/S_{b(\phi=1)} = 4$ (T4)	$u'/S_{b(\phi=1)} = 0$ (L0)	$u'/S_{b(\phi=1)} = 4$ (L4)
$b_{sp} \approx 0.2t_f$ $a_{sp} \approx 3.5$	$R = 1.41l_f$	(A)	ST0A	ST4A	LT0A	LT4A
	$R = 1.10l_f$	(B)	ST0B	ST4B	LT0B	LT4B
	$R = 0.93l_f$	(C)	ST0C	ST4C	LT0C	LT4C
$R \approx 1.10l_f$ $a_{sp} \approx 3.5$	$b_{sp} = 0.1$	(D)	ST0D	ST4D	LT0D	LT4D
	$b_{sp} = 0.2$	(E)	ST0E	ST4E	LT0E	LT4E
	$b_{sp} = 0.3$	(F)	ST0F	ST4F	LT0F	LT4F
$b_{sp} \approx 0.2t_f$ $R \approx 1.10l_f$	$a_{sp} = 2.7$	(G)	ST0G	ST4G	LT0G	LT4G
	$a_{sp} = 3.5$	(H)	ST0H	ST4H	LT0H	LT4H
	$a_{sp} = 4.3$	(I)	ST0I	ST4I	LT0I	LT4I

The extent of the completion of chemical reaction can be quantified in terms of a reaction progress variable c, which can be defined as:

$$c = (Y_{Fu} - Y_F)/(Y_{Fu} - Y_{Fb}) \tag{10}$$

where Y_{Fu} and Y_{Fb} are the fuel mass fractions in the unburned gas and the fully burned gas respectively. Both Y_{Fu} and Y_{Fb} are functions of ϕ in the mixture but c rises monotonically from 0 in unburned gases to 1.0 in fully burned gases irrespective of the value of ϕ. The extent of burning can be characterized by the burned gas mass $m_b(c \geq 0.9)$. The temporal evolution of the burned gas mass normalised by the mass of unburned gas with a radius equal to l_f (i.e. $[4/3]\rho_0\pi {l_f}^3$) for different values of u' for both stoichiometric and fuel-lean mixtures are shown in Fig. 2. It can be seen from Fig. 2 that the burning rate is the highest for the quiescent cases (i.e. $u'/S_{b(\phi=1.0)} = 0.0$) for the stoichiometric mixture (i.e. ST0B, ST0C, ST0D, ST0E, ST0F, ST0G, ST0H and ST0I) for a given set of values of R, b_{sp} and a_{sp}. Moreover, the extent of burning is greater in stoichiometric (i.e. ϕ=1.0) mixture than in the fuel-lean (e.g. ϕ=0.8) mixture for a given set of values of R, b_{sp} and a_{sp} due to higher rate of chemical reaction in stoichiometric mixture.

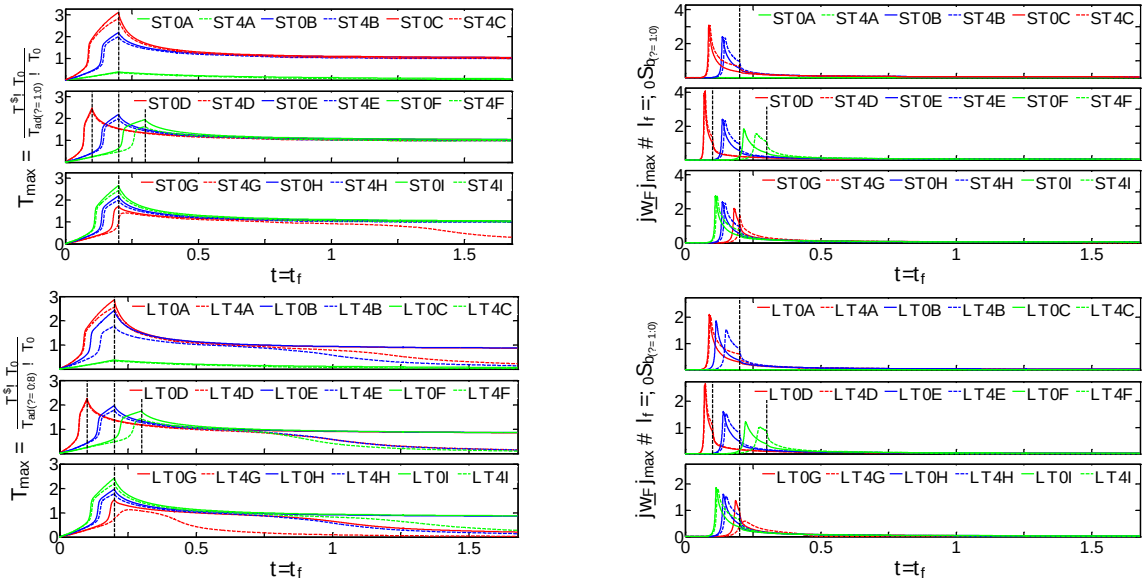

Fig. 1. Temporal evolution of non-dimensional maximum temperature (i.e. $T_{\max}$) and the normalised fuel reaction rate magnitude (i.e. $|\dot{w}_F|_{\max} \times l_f/\rho_0 S_{b(\phi=1.0)}$) for all cases listed in Table 1 (black broken like shows $t = t_{sp}$).

Figures 1 and 2 show that the cases with $R = 1.41l_f$ (i.e. ST0A, ST4A, LT0A and LT4A) fail to ignite, whereas a reduction in R gives rise to successful ignition and increases the burning rate. An increase in R for a given set values of b_{sp} and a_{sp} leads to deposition of energy over a larger volume (mass) of gas, which leads to a reduction in the probability of finding high temperature values for which chemical reaction can be sustained. This in turn reduces the overall rate of fuel consumption and heat release within the flame kernel for high values of R, and the flame eventually extinguishes when the heat transfer from hot gas kernel overcomes the chemical heat release. Figure 2 also demonstrates that the effects of b_{sp} on the extent of burning in both stoichiometric and lean mixtures. An increase in b_{sp} gives rise to a reduction in ignition power for a given set of values of ignition energy (i.e. given value of a_{sp}) and ignitor radius R. This further reduces the probability of finding high temperature to lead to high reaction rate magnitude. Thus, the probability of finding high values of $|\dot{w}_F|$ and chemical heat release decreases with increasing b_{sp}, which leads to a reduction in the extent of burning and the flames are more prone to extinction for high values of b_{sp} especially for fuel-lean cases. It can be seen from Fig. 2 that an increase in a_{sp}, for a given set of values of R, b_{sp} and u', increases the probability of finding high temperature and $|\dot{w}_F|$ values (see $T_{\max}$ and $|\dot{w}_F|_{\max}$ variations in Fig. 1), which in turn leads to higher extent of burning with increasing a_{sp}. This is especially critical for fuel-lean cases because of the smaller rate of heat release than the stoichiometric cases. It can be concluded from the results shown in Fig. 2 that higher (smaller) values of a_{sp} (b_{sp} and R) provide favourable conditions in order to achieve self-sustained combustion following successful ignition.

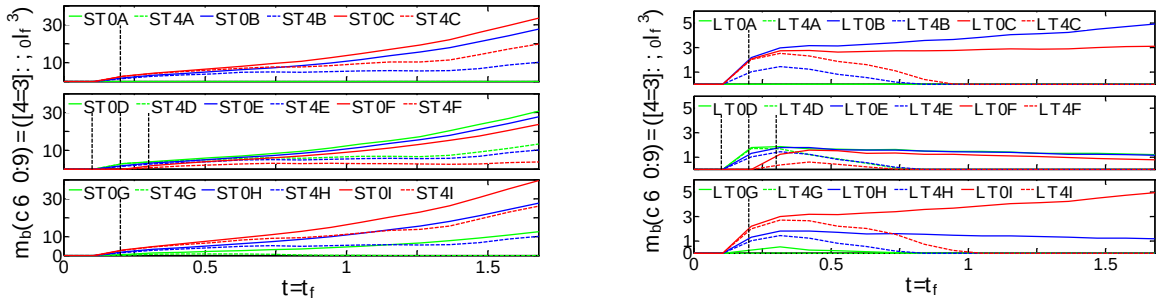

Fig. 2. Temporal evolution of burned gas mass (i.e. $m_b(c \geq 0.9)/[4/3]\rho_0\pi{l_f}^3$) for all cases listed in Table 1 (black broken like shows $t = t_{sp}$).

The findings from Fig. 2 further indicate that m_b decreases with increasing u' for both stoichiometric and fuel-lean mixtures. Furthermore, Fig. 2 suggests that it is possible to obtain misfire or flame extinction for increasing values of u'. The heat release due to chemical reaction needs to overcome heat transfer from the hot gas kernel to surrounding cold gas in order to the have the growth of the flame kernel. By contrast, the hot kernel shrinks once the heat transfer overcomes the chemical heat release and eventually flame extinguishes. The eddy thermal diffusivity scales as $D_t \sim u' L_{11}$ for the turbulent flows, which indicates that an increase in u' for a given value of L_{11} augments the rate of heat transfer from the hot gas kernel. Thus, the mass of the hot gas kernel decreases with increasing u' and large values of u' give rise to either misfire or an eventual extinction of flame. As the rate of heat release in fuel-lean cases is smaller than in the stoichiometric cases, the rate of augmented heat transfer for turbulent cases are more likely to overcome chemical heat release in the fuel-lean cases than the stoichiometric cases. Thus, the fuel-lean cases are more susceptible to flame extinction than the stoichiometric cases and this behaviour is particularly prominent especially for turbulent flows. The adverse effects of u' on ignition and early stages of flame propagation are consistent with previous experimental (Lefebvre 1998; Huang *et al*., 2007) and computational (Catlin *et al*., 1995; Chakraborty *et al*., 2007; Chakraborty and Mastorakos, 2008; Klein *et al*., 2008) findings.

Table 2. List of parameters which leads to self-sustained combustion with $b_{sp} = 0.2t_f$

		ϕ=1.0 (S)			ϕ=0.8 (L)		
		$\frac{u'}{S_{b(\phi=1)}} = 0$ (T0)	$\frac{u'}{S_{b(\phi=1)}} = 4$ (T4)	$\frac{u'}{S_{b(\phi=1)}} = 6$ (T6)	$\frac{u'}{S_{b(\phi=1)}} = 0$ (L0)	$\frac{u'}{S_{b(\phi=1)}} = 4$ (L4)	$\frac{u'}{S_{b(\phi=1)}} = 6$ (L6)
$R = 1.10l_f;\ a_{sp} = 5.5$	(Q)	ST0Q	ST4Q	ST6Q	LT0Q	LT4Q	LT6Q
$R = 1.55l_f;\ a_{sp} = 13.0$	(R)	ST0R	ST4R	ST6R	LT0R	LT4R	LT6R

The turbulent fuel-lean cases shown in Fig. 1 and 2 do not exhibit self-sustained combustion but it is possible to obtain self-sustained combustion for turbulent fuel-lean cases subject to the proper choices of ignition parameters R, b_{sp} and a_{sp}. In order to demonstrate this, the cases summarised in the Table 2 (i.e. cases Q and R) have been considered. The temporal variations of $T_{\max}$ and m_b for these cases (i.e. cases Q and R) are shown in Fig. 3. It can be seen from Fig. 3 that $T_{\max}$ settles to the adiabatic flame temperature (i.e. $T = 1.0$) for $t >> t_{sp}$ and m_b continues to rise with time for all cases, except in the LT6Q case which shows failed self-sustained combustion following successful ignition for large values of u' for the fuel-lean mixture. However, successful self-sustained combustion has been obtained for same level of u' for the stoichiometric mixture. Figure 3 further shows that the choice of $R \approx 1.55l_f$ and $a_{sp} \approx 13.0$ yields successful self-sustained combustion following ignition for both stoichiometric and fuel-lean mixtures for all values of u' considered here. Thus, it is possible to obtain successful ignition and self-sustained combustion even for fuel-lean cases by judicious selections of R, b_{sp} and a_{sp} and these parameters can be altered independently of each other in case of laser ignition. The present findings are consistent with recent experimental findings (Beduneau *et al*., 2004; Lou *et al*., 2007; Böker *et al*., 2011; Dharamshi *et al*., 2013), which demonstrated that laser ignition can be effective for fuel-lean mixtures by altering the laser power (i.e. equivalent to the variation of a_{sp}), laser beam radius (i.e. equivalent to the variation of R) and pulse durations (i.e. equivalent to the variation of b_{sp}).

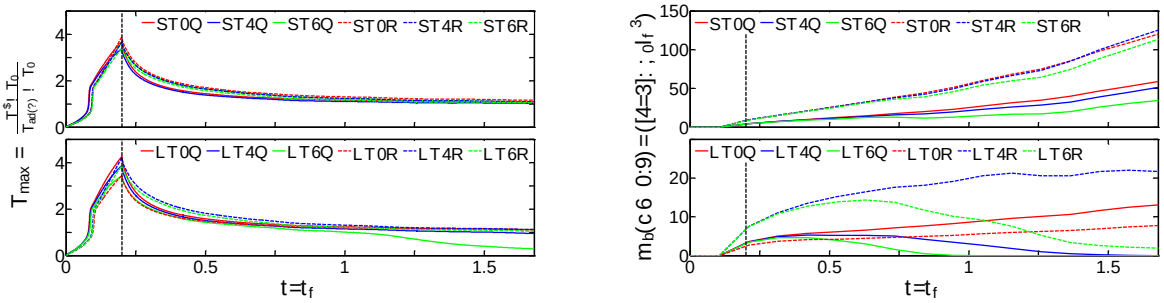

Fig. 3. Temporal evolution of non-dimensional maximum temperature (i.e. $T_{\max}$) and burned gas mass (i.e. $m_b(c \geq 0.9)/[4/3]\rho_0\pi{l_f}^3$) for all cases listed in Table 2 (black broken like shows $t = t_{sp}$).

5. Conclusion

The effects of ignition energy, the characteristic width of the energy deposition profile and the duration of energy deposition on localised forced ignition of both stoichiometric (i.e. ϕ=1.0) and fuel-lean (e.g. ϕ=0.8) homogeneous mixtures have been investigated for different values of rms turbulent velocity fluctuation based on three-dimensional DNS simulations. The ignition process is modelled by a Gaussian power source in the energy conservation equation, which deposits energy at a given location over a stipulated time interval. It has been found that higher amount of ignition energy is necessary to obtain self-sustained combustion in fuel-lean mixture than in the case of stoichiometric mixture. Moreover, an increase in rms turbulent velocity fluctuation decreases the extent of burning and may lead to misfire for large values of u'. The detrimental effects of u' are particularly critical in the fuel-lean cases than in the stoichiometric mixture because of weaker heat release effects in the fuel-lean mixture. An increase in

energy deposition width (duration of energy deposition) for a given amount of input ignition energy has been found to have detrimental effects on the extent of burning and these effects can be critical for the fuel-lean cases. The above findings clearly indicate that laser ignition can be advantageous for obtaining successful ignition and self-sustained combustion especially in the fuel-lean cases because the energy deposition width, duration of energy deposition and input ignition energy can be altered independently of each other in the case of laser ignition. The present analysis has been carried out using simple chemistry DNS for moderate values of turbulent Reynolds number. Although the qualitative nature of the present findings are unlikely to change in the presence of detailed chemistry and transport, further analysis based on three-dimensional detailed chemistry DNS for higher values of turbulent Reynolds number will be necessary to achieve more comprehensive physical insight and accurate quantitative predictions.

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